Key

Unit 5 CH 1 SEQUENCES & SERIES

1.1 Arithmetic Sequences

I. Sequences.

– set of numbers in some order.

– each number in a sequence is called a **term**

II. Notation

$t_1 = a = 1^{st}$ term

$t_2 = 2^{nd}$ term.

$t_n = n^{th}$ term or general term (any term).

The sequence 8 , 13, 18, . . . is an infinite sequence as there is no end to the number of terms.

The sequence 8, 13, 18, . . . , 63 is a finite sequence, since the last term is specified.

III. Arithmetic Sequence.

+ , −

– sequence in which the difference (common difference) between consecutive terms is a constant.

Examples:

1. Which of these are arithmetic sequences? What is d? (Adding/subtracting next term)

   a. 2, 6, 12, 20, . . . No

   b. 9, 13, 17, 21, . . . yes $d = 4$

   c. 3, 6, 12, 24, . . . No

   d. –3.8, –1.7, 0.4, 2.5 . . . Yes $d = 2.1$

   e. 11, 5, –1, –7, . . . yes $d = -6$

IV. Developing a formula

7,	10,	13,	16,	19, . . .
	(7 + 3)	(7 + 3 + 3)	(7 + 3 + 3 + 3)	(7 + 3 + 3 + 3 + 3)
t_1	$(t_1 + d)$	$(t_1 + 2d)$	$(t_1 + 3d)$	$(t_1 + 4d)$

$t_{13} = 7 + (13 - 1)(3)$

$$t_n = t_1 + (n - 1)d$$

common difference

term

1st term

number of term

Examples:

1. Use a formula **to find specific term.**

a. 11, 18, 25, . . .

Find t_{22}

$t_n = t_1 + (n - 1)d$

$n = 22$

$d = 7$

$a = t_1 = 11$

$t_{22} = 11 + (22-1)(7)$

$\boxed{t_{22} = 158}$

more room?

b. $\frac{13}{12}, \frac{5}{6}, \frac{7}{12}, \ldots$

Find t_{18}

$t_n = t_1 + (n - 1)d$

$\frac{10}{12} - \frac{13}{12} = \frac{-3}{12} = \frac{-1}{4}$

$n = 18$

$d = -\frac{1}{4}$

$t_{18} = \frac{13}{12} + (17) - \frac{1}{4} \to \frac{13}{12} \quad \frac{-17}{4}$

$\frac{13}{12} - \frac{51}{12} = \frac{-38}{12} = \frac{-19}{6}$

$\boxed{t_{18} = \frac{-19}{6}}$

c. −5, −1, 3, . . .

Find t_{10}

$t_n = t_1 + (n - 1)d$

$a = -5$

$n = 10$

$d = 4$

$t_{10} = -5 + (9)4$

$\boxed{t_{10} = 31}$

d. 2x, 5x, 8x, . . .

Find t_{62}

$t_n = t_1 + (n - 1)d$

$a = 2x$

$n = 62$

$d = 3x$

$t_{62} = 2x + (63)3x$

$\boxed{t_{62} = 185x}$

$t_1 + (n-1)d$

2. Find a **simplified form of t_n** (general term) for the following.

a. 6, 13, 20, . . .

t_1 $a = 6$

$n = n$

$d = 7$

$t_n = 6 + (n-1)7$

$= 6 + 7n - 7$

$\boxed{t_n = 7n - 1}$

b. 2, −3, −8, . . .

$n = n$

$d = -5$

$t_1 = 2$

$t_n = 2 + (n-1)-5$

$= 2 + -5n + 5$

$\boxed{t_n = -5n + 7}$

3. Find **the number of terms** in each. *Treat t_n as "last term" - Finite set.*

a. 6, 10, 14, . . . , 170

$t_n = t_1 + (n-1)d$

$n = n$
$d = 4$
$t_n = 170$
$t_1 = 6$

$170 = 6 + (n-1)4$

$170 = 6 + 4n - 4$

$170 = 4n + 2$

$168 = 4n$

$\boxed{n = 42}$

Note

★

b. 8, 1, −6, −11, . . . , −146

-13

$n = n$
$d = -7$
$t_n = -146$
$t_1 = 8$

$-146 = 8 + (n-1)(-7)$

$-146 = 8 - 7n + 7$

$-161 = -7n$

$\boxed{n = 23}$

c. $\frac{1}{3}, \frac{5}{6}, \frac{4}{3}, \ldots, \frac{16}{3}$

$\frac{5}{6} - \frac{1}{3}$

$\frac{5}{6} - \frac{2}{6}$

$\frac{3}{3} - \frac{1}{2}$

$n = n$
$d = \frac{1}{2}$
$t_n = \frac{16}{3}$
$t_1 = \frac{1}{3}$

$\frac{16}{3} = \frac{1}{3} + (n-1)\left(\frac{1}{2}\right)$

$\frac{16}{3} = \frac{1}{3} + \frac{1}{2}n - \frac{1}{2}$

$\frac{32}{6} = \frac{2}{6} + \frac{3}{6}n - \frac{3}{6}$

$\frac{33}{6} = -\frac{1}{6} + \frac{3}{6}n$

$\frac{33}{6} = \frac{1}{2}n$

$\frac{2}{1} \cdot \frac{33}{6} = n$

$\boxed{11 = n}$

4. Find the missing terms in each of the following.

a) 29 <u>22</u> <u>15</u> <u>8</u> <u>1</u> −6
(How many terms?) 6

$n = 6$
$d = ?$
$t_1 = 29$
$t_n = -6$

$-6 = 29 + (6-1)d$

$-6 = 29 + 5d$

$-35 = 5d$

$\boxed{d = -7}$

b) <u>21.6</u> <u>20.5</u> <u>19.4</u> 18.3, 17.2

$d = -1.1$

c) 23 19 <u>15</u> <u>11</u> 7 (How many terms?...a little tricky, here)

$* \ n = 4$
$d = ?$
$t_1 = 19$
$t_n = 7$

$7 = 19 + (4-1)d$

$7 = 19 + 3d$

$-12 = 3d$

$d = -4$

5. Given $t_{16} = -18$ and $d = 4$ for an arithmetic sequence, find t_1.

$n = 16$

$d = 4$

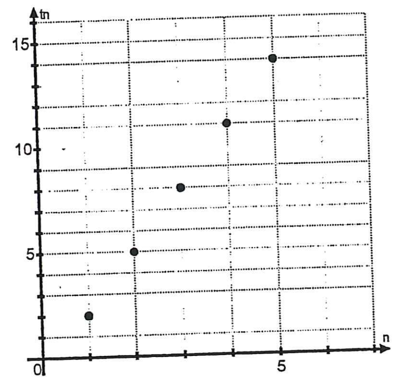
$t_{16} = -18$

$-18 = t_1 + (15)4$

$-18 = t_1 + 60$

$\boxed{t_1 = -78}$

6. a) Draw a graph for the following arithmetic sequence: 2, 5, 8, 11, 14 . . .



b) Find t_n?

$n = n$

$d = 3$

$t_1 = 2$

$t_n = 2 + (n-1)3$

$= 2 + 3n - 3$

$t_n = 3n - 1$

c) What is the slope of this graph?

$m = 3$

d) What is the difference between $t_n = 3n - 1$ and $y = 3x - 1$?
Is this a continuous or a discrete graph?

7. Determine x so that $2x + 3$, $5x - 1$ and $7x + 4$ form an arithmetic sequence.

$a = 2x + 3$

$n = 3$

$d =$

$t_n = 7x + 4$

$5x - 1$
$-(2x + 3)$
$3x - 4$

$7x + 4$
$-5x + 1$
$2x + 5$

$7x + 4 = (2x + 3) + (2)(3x - 4)$

$7x + 4 = 2x + 3 + 6x - 8$

$\boxed{9 = x}$

or

$7x + 4 - (5x - 1) = 5x - 1 - (2x + 3)$

$7x + 4 - 5x + 1 = 5x - 1 - 2x - 3$

$2x + 5 = 3x - 4$

$x = 9$

8. Given $t_3 = 27$ and $t_7 = -5$ find t_1 and d.

$a = 27$

$n = 5$

$d =$

$t_n = -5$ (t_5)

43 35 $\underset{(t_3)\ t_1}{27}$ $\underset{2}{19}$ $\underset{3}{11}$ $\underset{4}{3}$ $\underset{(t_7)\ t_5}{-5}$

$-5 = 27 + (4)d$

$-32 = 4d$

$d = -8$

$\boxed{t_1 = 43 \quad d = -8}$

Assign up to # 11

9. **Question 23 on page 19.**

The Diavik Diamond Mine is located on East Island in Lac de Gras East, Northwest Territories. The diamonds that are extracted from the mine were brought to surface when the kimberlite rock erupted 55 million years ago. In 2003, the first production year of the mine, 3.8 million carats were produced. Suppose the life expectancy of the mine is 20 years, and the number of diamond carats expected to be extracted from the mine in the 20th year is 113.2 million carets. If the extraction of diamonds produces an arithmetic sequence, determine:

2003 2023

3.8 113.2

$n = 20$

$d =$

$t_1 = 3.8$

$t_{20} = 113.2$

a) the common difference.

$113.2 = 3.8 + (19)d$

$d \doteq 5.76 \quad (5.757894731\ldots)$

b) what this value means.

The amount of carats in millions, extracted / year

10. Determine the number of multiples of 7 between:

a. 14 and 210 inclusive

$a = 14$

$n = ?$

$d = 7$

$t_n = 210$

$210 = 14 + (n-1)7$

$210 = 14 + 7n - 7$

$203 = 7n$

$\boxed{n = 29}$

b. –51 and 348

$n = ?$

$d = 7$

$t_1 = -51$

$t_n = 348$

$348 = -51 + (n-1)7$

$348 = -51 + 7n - 7$

$406 = 7n$

$\boxed{n = 58}$

$406 \div 7 = 58$

Assignment

Pages 16 – 19 1. a – c 2. b, d 3. c 4. a, c 5. a, b | 7, 9, 11, 16 , **24

Plus: $t_4 = -5$ and $t_9 = 10$. find t_1 and d. ($t_1 = -14$ and d = 3)

$\boxed{d = 3}$

1.2 Arithmetic Series

Arithmetic series is the **sum** of the terms of an arithmetic sequence.
In order to understand the formula, consider:

$3 + 10 + 17 + 24 + 31$ Find S_5

$S_5 = 3 + 10 + 17 + 24 + 31$
$S_5 = 31 + 24 + 17 + 10 + 3$

$2S_5 = 34 + 34 + 34 + 34 + 34$

$2S_5 = 5(34)$

$S_5 = \dfrac{5(3+31)}{2}$

2nd formula on sheet First formula on sheet

$$S_n = \frac{n(t_1 + t_n)}{2} \quad \text{or} \quad \frac{n[t_1 + t_1 + (n-1)d]}{2} = \frac{n[2t_1 + (n-1)d]}{2}$$

2a

n = number of terms; t_1 = first term; t_n = last term; d = common difference

**** You use the first formula when you know the last term or have to find the last term (t_n)**

Examples:

1. Find the following sums.

a) S_{34} for $14 + 20 + 26 + \ldots$

$a = 14$
$n = 34$
$d = 6$
t_n - N/A
$S_{34} =$

$S_{34} = \dfrac{34[2(14) + (34-1)6]}{2}$

$= 17(28 + 198)$

$\boxed{S_{34} = 3842}$

b) S_{24} for $\dfrac{1}{12}, \dfrac{1}{2}, \dfrac{11}{12} \ldots$ (6 written above; 2 corrected to $\frac{6}{12}$)

$a = 1/12$
$n = 24$
$d = \dfrac{5}{12}$
t_n: N/A
$S_{24} =$

$S_{24} = \dfrac{24(2(\frac{1}{12}) + 23(\frac{5}{12}))}{2}$

$= 12(\frac{2}{12} + \frac{115}{12})$

$= 12(\frac{117}{12})$

$S_{24} = 117$

$a = 13\sqrt{7}$ ** c. S_{11} *for* $13\sqrt{7}+10\sqrt{7}+7\sqrt{7}+\ldots$

$n = 11$

$d = -3\sqrt{7}$

$S_{11} =$

$$S_{11} = \frac{11\left(2(13\sqrt{7}) + (10)(-3\sqrt{7})\right)}{2}$$

$$= \frac{11\left(26\sqrt{7} - 30\sqrt{7}\right)}{2}$$

$$= \frac{11(-4\sqrt{7})}{2}$$

$$= \frac{-44\sqrt{7}}{2}$$

$$\boxed{S_{11} = -22\sqrt{7}}$$

$\boxed{a = -8}$ 2. Find the sum of $-8 - 4 + 0 + 4 + \ldots + 72$

$n = ?$

$d = 4$

$t_n = 72$

$S_n =$

$$S_n = \frac{n(-8+72)}{2}$$

→ Need to find n first, so use:

$72 = -8 + (n-1)4$

$72 = -8 + 4n - 4$

$84 = 4n$

$n = 21$

$$S_n = \frac{21(64)}{2}$$

$S_n = 672$

$S_{21} = 672$

$a = 3$ 3. Find the sum of $3 + 7 + 11 + \ldots + 143$.

$n = ?$

$d = 4$

$t_n = 143$

Find n:

$143 = 3 + (n-1)4$

$143 = 3 + 4n - 4$

$144 = 4n$

$\boxed{n = 36}$

$$S_{36} = \frac{36(3+143)}{2}$$

$= 18(146)$

$\boxed{S_{36} = 2628}$

$a = 24$ 4. Find the sum of all the multiples of 6 between 24 and 396 inclusive.

$n = ?$

$d = 6$

t_1

$t_n = 396$

S_n

Find n:

$396 = 24 + (n-1)6$

$396 = 24 + 6n - 6$

$378 = 6n$

$n = 63$

$$\frac{63(48 + 62(6))}{2}$$

$$S_{63} = \frac{63(24+396)}{2}$$

$$S_{63} = \frac{63(420)}{2}$$

$\boxed{S_{63} = 13230}$

- pg 27 1, 2, 3,

S8 - [illegible]

trouble . .

$a=17$

$n = \cancel{53}/38$

$d = ?$

$t_1 = 17$

$t_n = 128$ t_{38}

5. Given $t_1 = 17$ and $t_{38} = 128$, find S_{53}.

Find d, using $t_{38} = 128$

$128 = 17 + (37)d$

$111 = 37d$

$d = 3$

Now we can use:

$a=17$ $n = 53,\ d = 3,\ t_1 = 17,\ S_{53} =$

with 1st formula

$S_{53} = \dfrac{53\,[2(17) + (53-1)3]}{2}$

$= \dfrac{53\,(34 + 156)}{2}$

$\boxed{S_{53} = 5035}$

6. A stack of logs has 3 in the 1st row, 7 in the 2nd row, 11 in 3rd row and so on. If there are 271 logs in the last row, how many rows? logs?

$a = 3$

$n =$ rows

$d = 4$

$t_1 = 3$

$t_n = 271$

$\dfrac{3}{t_1}$ $\dfrac{7}{t_2}$ $\dfrac{11}{t_3}$, ... $\dfrac{271}{t_n}$

$271 = 3 + (n-1)4$

$271 = 3 + 4n - 4$

$4n = 272$

$n = 68$

$\boxed{68 \text{ rows}}$

Use ~~first~~ short formula:

$S_{68} = \dfrac{68\,(3 + 271)}{2}$

$= 34(274)$

$\boxed{S_{68} = 9316 \text{ logs}}$

$a = 3$

$n = 68$

$d = 4$

$t_n = 271$

$S_n = S_{68}$

7. The sum of the first 12 terms of an arithmetic sequence is 228. The common difference is 4. Find the first 3 terms.

$a = a$

$n = 12$

$d = 4$

t_1

$t_n =$

$S_{12} = 228$

What do we have? What can we use?

1st formula $228 = \dfrac{12\,(2t_1 + (11)4)}{2}$

$228 = 6\,(2a + 44)$

$228 = 12a + 264$

$-36 = 12a$

$a = -3$

First 3 terms: $\boxed{-3,\ 1,\ 5}$

8. In an arithmetic series $t_1 = 11$, $t_n = 119$ and $S_n = 1235$. Find 'n'

have

$a = 11$

$n = ?$

$d = ?$ - don't need

$t_1 = 11$

$t_n = 119$

Use 2nd formula:

$1235 = \dfrac{n\,(11 + 119)}{2}$

$2470 = 130n$

$\boxed{n = 19}$

$a = a$

$n = 30$

d: don't need

$t_{30} = 58$

$S_{30} = 1095$

9. In an arithmetic series $t_{30} = 58$ and $S_{30} = 1095$. Find t_1.

Use 2nd formula without 'd'

$$1095 = \frac{30(a+58)}{2}$$

$$1095 = 15a + 870$$

$$225 = 15a$$

$$\boxed{a = 15}$$

$\underline{23}_4$ $\underline{\ }_5$ $\underline{\ }_6$ $\underline{\ }_7$ $\underline{\ }_8$ $\underline{\ }_9$ $\underline{41}_{10}$ 7 terms

10. If in an arithmetic sequence, $t_4 = 23$ and $t_{10} = 41$, find t_1 and d.

Temporarily make: $t_1 = 23$ $t_7 = 41$ find d

$a = 23$

$n = 7$

$d = ?$

t_1

$t_n = 41$

$$41 = 23 + (6)d$$

$$18 = 6d$$

$$\boxed{d = 3}$$

$\underline{14}_{t_1}$ $\underline{17}_{t_2}$ $\underline{20}_{t_3}$ $\underline{23}_{t_4}$

$$\boxed{t_1 = 14}$$

$\underline{\ }_3$ $\underline{\ }_4$ $\underline{\ }_5$ $\underline{\ }_6$ $\underline{\ }_7$ = 5 terms

11. In an arithmetic series where $t_3 = \frac{5}{3}$ and $t_7 = 3$, what is t_1 and d? (don't worry about sum)

$t_1 = \frac{5}{3}$ $t_5 = 3$

$n = 5$

$d = ?$

$t_1 = 5/3$

$t_n = 3$

$$3\left[3 = \frac{5}{3} + (4)d\right]$$

$$9 = 5 + 12d$$

$$4 = 12d$$

$$\boxed{d = \frac{1}{3}}$$

$\underline{\frac{3}{3} = 1}_{t_1}$ $\underline{\frac{4}{3}}_{t_2}$ $\underline{\frac{5}{3}}_{t_3}$

$$\boxed{t_1 = 1}$$

* System of EQUATIONS!

12. In an arithmetic series, $S_2 = -1$ and $S_6 = 81$, find t_1 and d. - long formula

$a =$
$n = 2$
$d = ?$
t_1
$t_n = ?$
$S_2 = -1$

$a =$
$n = 6$
$d =$
$t_n =$
$S_6 = 81$

Use 1st formula

$-1 = \frac{2[2a + (1)d]}{2}$

$\boxed{-1 = 2a + d}$

$81 = \frac{6(2a + 5d)}{2}$

$81 = 3(2a + 5d)$

$\boxed{81 = 6a + 15d}$ Solve system

$81 = 6a + 15d$
$3 = -6a - 3d$
$84 = 12d$
$\boxed{d = 7}$

if $S_2 = -1$
$a + a + 7 = -1$
$2a = -8$
$\boxed{a = -4}$

13. **Question 9 on page 28.**

A training program requires a pilot to fly circuits of an airfield. Each day, the pilot flies three more circuits than the previous day. On the fifth day, the pilot flew 14 circuits. How many circuits did the pilot fly:

$\underset{1}{2}, \underset{2}{5}, \underset{3}{8}, \underset{4}{11}, \underset{5}{14}$

t_1 t_{2} $a = 2$
$n = 5$
$d = 3$
$t_1 ?$
t_n 14
$S_5 = ?$

a. on the first day?

$14 = t_1 + (4)3$

$\boxed{t_1 = 2}$

b. in total by the end of the fifth day?

use 2nd

$S_5 = \frac{5(2 + 14)}{2}$

$S_5 = \frac{5(16)}{2}$

$\boxed{S_5 = 40}$

c. in total by the end of the nth day?

$a = 2$
$n = ?$
$d = 3$
$t_n = t_n$
S_n

$S_n = \frac{n[2(2) + (n-1)3]}{2}$

$= \frac{n[4 + 3n - 3]}{2}$

$S_n = \frac{n[3n + 1]}{2}$

$S_n = \frac{3n^2 + n}{2}$

ASSIGNMENT
Pages 27 - 30 1. b, c, d 2. b, d 3. c, e 4. b, c 5. a, b 6 a, c 7, 13, 15

1.3 Geometric Sequences

Geometric sequences: sequence of numbers where the **ratio** (r) (common ratio) between consecutive terms is a constant.

eg. 5, 15, 45, 135, . . . $\frac{15}{5}=3,\ \frac{45}{15}=3,\ \frac{135}{45}=3;$ r = 3

$r=\frac{t_2}{t_1}=\frac{t_3}{t_2}=\frac{t_4}{t_3}$, etc

Look at 5, 10, 20, 40, . . .

$5,\ 5\times2,\ 5\times2\times2,\ 5\times2\times2\times2,\ldots$

$5,\ 5\times2,\ 5\times2^2,\ 5\times2^3,\ldots$

change → $t_4 = 5(2)^{4-1}$ or $5(2)^3$

Therefore $\boxed{t_n = t_1 r^{n-1}}$ or $t_n = ar^{n-1}$

t_n = term; t_1 = first term; r = common ratio; n = number of the term

Examples:

$t_n = t_1 r^{n-1}$

$a = 4$
$n = ?$
$r = 2$
$t_1 = 4$
$t_n = ?$

1. Which are geometric sequences? Find t_1, r and t_n for those that are.

a. 2, 4, 6, . . . $\frac{4}{2} \neq \frac{6}{4}$ no

b. 4, 8, 16, 32, . . . $\frac{8}{4} = \frac{16}{8} = \frac{32}{16}$ yes $t_n = 4(2)^{n-1} = 2^2 2^{n-1} = \boxed{2^{n+1}}$ $2+n-1$

$r = 2 = 2 = 2$

c. 5, −15, 45, . . . $\frac{-15}{5} = \frac{45}{-15}$ yes $\boxed{t_n = 5(-3)^{n-1}}$

$r = -3 = -3$

d. $\frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}, \ldots$ $\frac{2}{3} \times \frac{3}{1} = 1 \times \frac{3}{2} = \frac{4}{3} \times 1$ No

$2 \neq \frac{3}{2} \neq \frac{4}{3}$

e. $\frac{15}{16}, \frac{3}{8}, \frac{3}{20}, \ldots$ $\frac{3}{8} \times \frac{16}{15} = \frac{3}{20} \times \frac{8}{3}$ yes $\boxed{t_n = \frac{15}{16}\left(\frac{2}{5}\right)^{n-1}}$

$r = \frac{2}{5} = \frac{2}{5}$

$t_n = ar^{n-1}$

2. Find the indicated term in each of the following geometric sequences.

$a = 4$
$n = 6$
$r = -3$
$t_n = t_6$

a) t_6 for $4, -12, 36 \ldots$

$t_6 = 4(-3)^5$
$= 4(-243)$
$\boxed{t_6 = -972}$

b) t_3 for $\frac{2}{3}, \frac{4}{5}, \ldots$

$\frac{4}{5} \times \frac{3}{2} = \frac{6}{5}$

$n = 3$
$r = \frac{6}{5}$
$t_1 = \frac{2}{3}$

$t_3 = \frac{2}{3}\left(\frac{6}{5}\right)^2$
$= \frac{2}{3}\left(\frac{36}{25}\right)$
$\boxed{t_3 = \frac{24}{25}}$

c) t_9 for $2, 2\sqrt{3}, 6, \ldots$

$\frac{2\sqrt{3}}{2} = \sqrt{3}$

$r = \sqrt{3}$

$\frac{3\sqrt{6}}{2\sqrt{3}}$ $\frac{2\sqrt{3}}{2}$ $\frac{3}{\sqrt{3}}$

$n = 9$
$r = \sqrt{3}$
$t_1 = 2$

$t_9 = 2(\sqrt{3})^{9-1}$
$= 2(\sqrt{3})^8$
$= 2(81)$
$\boxed{t_9 = 162}$

$3^{\frac{1}{2} \cdot 8} = 3^4$

3. Find the missing terms in each of these geometric sequences:

a. 7, 14, 28, 56, 112, 224.

$r = \frac{224}{112} = 2$

b. 20, 40, 80, 160, 320

$n = 5$
$r =$
t_1
$t_5 = 320$

$320 = 20(r)^4$
$16 = r^4$
$r = 2$

4. In a geometric sequence $t_2 = 48$ and $t_5 = 162$. Find a and r.

Let 48 be the first term 48 ____ ____ 162. Then 162 becomes the fourth term.

1 2 3 4

$n = 4$
$r = ?$
$t_1 = 48$
$t_4 = 162$

$162 = 48(r)^3$
$3.375 = r^3$
$\boxed{r = 1.5} = \frac{3}{2}$
$\therefore \boxed{a = 32}$

$a\left(\frac{3}{2}\right) = 48$

$48\left(\frac{2}{3}\right)$

$t_n = ar^{n-1}$

Charge.

principle ↓ interest

0.04 ∴ 1.04

t_1 t_2 after one year

5. The value of a house increases by 4% each year. The value of a house now is \$500 000.

104% after 1 year

After 1 year After 2 years

a) What is the value of the house after: i. 1 year? ii. 2 years?

500000		
t_1	t_2	t_3

$a = t_1 = 500\,000$

$* n = 2$

$r = 1.04$

$t_2 = 500\,000(1.04)^1$

$t_2 = 520000$ after 1 year

$t_1 = 500000$

$r = 1.04$

$* n = 3$

$t_3 = t_3$

$t_3 = 500000(1.04)^2$

$= 540\,800$ after 2 year.

b) Is this a geometric sequence? What is r?

yes. $\boxed{1.04}$

think of birthdays.

First term is before any changes.

c) What is the value of the house after 8 years (nearest dollar)?

t_9 ↳ after 8 "changes"

500 000 change 1

t_1 t_2 3 4 5 6 7 8 t_9

Beginning of 9th

$n = 9$

$r = 1.04$

$t_1 = 500000$

t_9

$t_9 = 500\,000(1.04)^8$

$= \$684\,284.53$

d) After how many years will the house be worth at least \$900 000?

$n = ?$

$r = 1.04$

$a\ t_1 = 500\,000$

$t_n = 900\,000$

$900\,000 = 500\,000(1.04)^{n-1}$

$1.8 = (1.04)^{n-1}$

$n = 16$ (trial + error)

Show TVM solver

or logs:

$\log 1.8 = (n-1)\log 1.04$

6. **Question 10 on page 40.**

The colour of some clothing fades over time when washed. Suppose a pair of jeans fades by 5% with each washing.

0.05 t_1 original t_2 after 1

$n=2$
$r=0.95$
$t_1 = 100\%$
t_2

a) What percent of the colour remains after one washing?

$t_2 = 100(0.95)^{2-1}$

$= 95\%$ remains

b) If t_1 = 100 (100%), what are the first four terms.

100% 95% 90.25% 85.7375% 81.4%

t_1 t_2 t_3 t_4 t_4

c) What is the value of r for your geometric sequence?

$r = 0.95$

d) What percent of the colour remains after 10 washings?

$n=11$
$r=0.95$
$t_1 = 100$
$t_{11} = ?$

$t_{11} = 100(0.95)^{11-1}$ or $t_{11} = 1(0.95)^{10}$

$= 59.87\%$ remains

e) How many washings would it take so that approximately 25% of the original colour remains in the jeans?

$25\% = 100\%(0.95)^{n-1}$ or. $0.25 = 1(0.95)^{n-1}$

$.25 = .95^{n-1}$

Guess $= .95^{27} = .25034\ldots$

$\therefore$ after 28 washings

ASSIGNMENT

Pages 39 – 43 1. a, c, d 2. a, c 3. d 4. 5. c, d 6 a, e 7. 19.

1.4 Geometric Series

Geometric Series – is the **sum** of the terms of a geometric series.

Formulas for finding the sum of a geometric series are:

$$S_n = \frac{t_1(r^n - 1)}{r - 1} \quad \text{or} \quad \frac{rt_n - t_1}{r - 1}$$

have last

S_n = sum of n terms; t_1 = first term; r = common ratio; t_n = the last term

Use the **second formula** when you know the **last term** or if you have to find the **last term** (t_n).

Examples:

change

1. Which of the following series are geometric? Determine indicated sum for those that are. (hundredth if necessary)

a) $5 - 30 + 180 - 1080 \ldots\ S_7$

Geom. $a = 5$, $n = 7$, $r = -6$, $S_n = S_7$, $\cancel{t_n}$

$$S_7 = \frac{5((-6)^7 - 1)}{-6-1} = \frac{5(-279937)}{-7}$$

$$\boxed{S_7 = 199955}$$

b) $3 + 5 + 7 + 9 + \ldots\ S_{13}$

not geometric

$$\frac{5}{3} \neq \frac{7}{5}$$

nearest hundredth

c) $4 + 6 + 9 + 13.5 + \ldots\ S_9$

$a = 4$, $n = 9$, $r = \frac{3}{2}$, $S_9 = 4$

$\frac{6}{4} = \frac{9}{6}$ yes

$$S_9 = \frac{4((\frac{3}{2})^9 - 1)}{\frac{3}{2} - 1} = \frac{149.7734\ldots}{.5}$$

$$\boxed{S_9 \doteq 299.546875 \doteq 299.55}$$

d) $27 + 18 + 12 + 8 + \ldots\ S_8$

(exact) $\frac{18}{27} = \frac{12}{18}$, $r = \frac{2}{3}$

$a = 27$, $n = 8$, $r = \frac{2}{3}$, $\cancel{t_1 = 27}$, S_8

$$S_8 = \frac{27((\frac{2}{3})^8 - 1)}{\frac{2}{3} - \frac{3}{3}} = \frac{27\left(\frac{256}{6561} - \frac{6561}{6561}\right)}{-\frac{1}{3}}$$

$$= \frac{27\left(\frac{-6305}{6561}\right)}{-\frac{1}{3}} = -81\left(\frac{-6305}{6561}\right) = \frac{510705}{6561}$$

$$\boxed{S_8 = \frac{6305}{81}}$$

$a = 5$
$n = ?$
$r = 3$
$t_n = 10935$
S_n

2. Determine sums of the following geometric series.

a) $5 + 15 + 45 + \ldots + 10\,935$

We know last term, use 2nd formula

$$S_n = \frac{3(10935) - 5}{3-1}$$

$$= \frac{32800}{2}$$

$$\boxed{S_n = 16400}$$

fix
$a = -686$
$n = ?$
$r = -\frac{1}{7}$
$t_n = \frac{-2}{343}$
S_n

nearest hundredth

b) $-686 + 98 - 14 + \ldots + \frac{-2}{343}$

$$\frac{98}{-686} = -\frac{1}{7}$$

$$S_n = \frac{-\frac{1}{7}\left(\frac{-2}{343}\right) + 686}{-\frac{1}{7} - 1}$$

$$= \frac{686.008\ldots}{-\frac{8}{7}}$$

$$S_n = -600.25$$

or $\frac{-205886}{343}$

fix
$a =$
$n = ?$
$r = 3$
t_1
$t_n = 405$
$S_n = 605$

3. In a geometric series $S_n = 605$, r = 3 and $t_n = 405$. Find t_1.
Since we know the last term (t_n), we can use the second formula.

$$605 = \frac{3(405) - t_1}{2}$$

$$1210 = 1215 - t_1$$

$$\boxed{t_1 = 5}$$

$n = ?$
$r = 5$
$t_1 = 2$
$S_n = 39062$

4. $2 + 10 + 50 + \ldots = 39\,062$. How many terms are in this series? need to find n

Use 1st form.

- guess + check
- or logs.

$$39062 = \frac{2(5^n - 1)}{4}$$

$$78124 = 5^n - 1$$

$$78125 = 5^n$$

$$n \log 5 = \log 78125$$

$$\boxed{n = 7}$$ or guess + check.

032

5. 9, 15, 3x + 7 . . . form a geometric sequence. Find the value of x.

$$\frac{15}{9} = \frac{(3x+7)}{15}$$

$\boxed{x = 6}$

or $\frac{5}{3} = \frac{3x+7}{15}$

$$9(3x+7) = 225$$
$$27x + 63 = 225$$
$$27x = 162$$

Change to

6. In a geometric series, r = –2, S_7 = ~~–254~~ 258. Find t_1.

$n = 7$

$r = -2$

$t_1 = ?$

$S_7 = \cancel{-254}$ 258

First form.

$$-258 = \frac{t_1(-2^7 - 1)}{-3}$$
$$-774 = t_1(-129)$$
$$\boxed{t_1 = 6}$$

7. **Question 13 on page 55.**

periods ∴ 11 (t_1 is 1000, no increase)

n = 100 days = 10

r = 1.4

$t_1 = 1000$

$S_n = S_{11}$

An advertising company designs a campaign to introduce a new product to a metropolitan area. The company determines that 1000 people are aware of the product at the beginning of the campaign. The number of new people aware increases by 40% every 10 days during the advertising campaign. Determine the total number of people who will be aware of the product after 100 days.

t_1 | 2 3 4 5 6 7 8 9 10 11

0 days

$$S_{11} = \frac{1000(1.4^{11} - 1)}{1.4 - 1}$$
$$= \frac{1000(39.495\ldots)}{0.4}$$
$$S_{11} \doteq 98739 \text{ people}$$

98740

a = 1000

n = 100 ÷ 10 = 10 + 1

r = 1.4 (100 + 40%)

$\cancel{t_1}$

$S_n = S_{11}$

1000 + 1400 + 1960 + 2744 + 3841.6 + 5378.64 + 7531.496 + 10544.0944 + 14761.73 +

20 666.4

+

28 933

ASSIGNMENT

→ Pages 53 – 57 1. all 2. a, b 3. a, c 4. a, d 5. a, b 6. 7, 8, 11, 15, 16

5.1 Infinite Geometric Series

2 + 6 + 18 + 54 + . . .

$S_1 \quad S_2 \quad S_3 \quad S_4$

Its sequence of partial sums is $2, \; 8, \; 26, \; 80, \dots$

It is rather obvious that the sum of this geometric series gets larger and larger as the number of terms increases. The sequence of partial sums does not approach a specific value. The geometric series is therefore **divergent.**

$2+1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\dots$ $\quad S_1 \quad S_2 \quad S_3 \quad S_4 \quad S_5 \quad S_{25} \quad S_{55}$

Its sequence of partial sums is $2, \; 3, \; 3.5, \; 3.75, \; 3.875, \; \dots, \; 3.999999881, \; \dots \; 4$

As the number of terms increases the sequence of partial sums approaches a fixed value of 4. This geometric series is **convergent** and its sum is **4.**

(collapses)

A geometric series will be convergent whenever **–1 < r < 1**

When **n** is infinite and **–1 < r < 1** then $S_\infty = \dfrac{t_1(r^n - 1)}{r-1}$

$$S_\infty = \frac{t_1(0-1)}{r-1}$$

$r^n = 0$ for very large values of n.

$$S_\infty = \frac{-t_1}{r-1} \; or \; \frac{t_1}{1-r}$$

$0.9^{200000} = 0$ on calculator

Examples:

1. State whether the following infinite geometric series are divergent or convergent.

a. $6 + 5 + \frac{25}{6} + \dots$ $r = \frac{5}{6}$ ∴ Convergent $\quad -1 < r < 1$

b. $2 + 2.4 + 2.88 + \dots$ $r = \frac{2.4}{2}$ ∴ divergent

c. $54 - 36 + 24 + \dots$ $r = \frac{-36}{54} = -\frac{2}{3}$ ∴ Convergent $\quad -1 < r < 1$

2. Determine the sum of these geometric series.

a. $t_1 = 48;\ r = \frac{-3}{5}$

convergent
use S_∞

$S = \frac{t_1}{1-r}$ $\quad S = \frac{48}{1\frac{3}{5}}$ $= \frac{48}{\frac{8}{5}}$ $\quad 48 \times \frac{5}{8}$

$\boxed{S = 30}$

b. $21 + 21\left(\frac{2}{5}\right) + 21\left(\frac{2}{5}\right)^2 + \ldots$

Convergent
use S_∞

$S = \frac{21}{1-\frac{2}{5}} = \frac{21}{\frac{3}{5}} = 21 \cdot \frac{5}{3} = \boxed{35}$

Fix

$\frac{8}{9} = .888\overline{8}$

$\frac{5}{9} = .\overline{555}$

$\frac{44}{99} = .\overline{44}$

Show that

c. ~~$0.\overline{72}$~~ $0.\overline{72} = 0.72 + 0.0072 + 0.000072 + \ldots$

$t_1 = 0.72$

$r = 0.01$

$0.\overline{72} = \frac{.72}{1-0.01}$

$= \frac{.72}{.99} = .\overline{72}$

3. $S_\infty = \frac{28}{15},\ r = \frac{1}{4};$ Find t_1.

$\frac{28}{15} = \frac{t_1}{.75}$

$21 = 15t_1$

$\boxed{t_1 = \frac{21}{15} = \frac{7}{5} \text{ or } 1.4}$

4. $S_\infty = -15,\ t_1 = -24;$ Find r.

$-15 = \frac{-24}{1-r}$

$-15 + 15r = -24$

$15r = -9$

$\boxed{r = \frac{-3}{5}}$

Fix

5 ~~4.~~ $S_\infty = 9$ and $t_1 = 15$. Find r.

$9 = \frac{15}{1-r}$

$9 - 9r = 15$

$-9r = 6$

$\boxed{r = \frac{-2}{3}}$

5. $2 + 10x + 50x^2 + \ldots$ has a sum of 8. What is the value of x? r? $r = 5x$

$n = ?$

$r = 5x$

$t_1 = 2$

$S_n = 8$

impossible to find 'n', ∴ use S_∞ formula.

$$S = \frac{a}{1-r}$$

$$8 = \frac{2}{1-5x}$$

$$8 - 40x = 2$$

$$6 = 40x$$

$$x = \frac{6}{40} = \frac{3}{20}$$

$$\boxed{x = \frac{3}{20}}$$

$$r = 5x$$

$$= \frac{5(3)}{20}$$

$$r = \frac{15}{20} = \boxed{\frac{3}{4}}$$

6. **Question 8 on page 63.**

In its first month, an oil well near Virden, Manitoba produced 24 000 barrels of crude. Every month after that, it produced 94% of the previous month's production.

$n =$

$r = 0.94$

$t_1 = 24000$

$S_n = ?$

a. If this trend continues, what would be the lifetime production of this well? S_∞

$$S_\infty = \frac{24000}{1-0.94}$$

$$= \frac{24000}{0.06}$$

$$S_\infty = 400\,000 \text{ barrels}$$

b. What assumptions are you making? Is your assumption reasonable?

That the rate would not change over its lifetime

ASSIGNMENT:

Pages 63 – 65 1. a, b, c 2. a, b, d 3. a 5. c 6. 7. 9. 16.

MATH 20-1
CHAPTER 1: SEQUENCES & SERIES REVIEW

1. Which sequence has a common difference of –5?

   a. –18, –23, –24 . . .   b. 18, 23, 28 . . .   c. –3, 7, –11 . . .   d. 17, 12, 7 . . .

2. Which of the following is a geometric sequence?

   $\frac{11}{4} \neq \frac{18}{11}$

   a. 4, 11, 18, 25, . . .

   b. 3, –6, –12, 24, . . . No

   $\frac{-6}{3} \neq \frac{-12}{-6}$

   c. $\frac{5}{18}, \frac{5}{12}, \frac{5}{8}, \frac{15}{16}, \ldots$ yes $r = \frac{3}{2}$

   $\frac{3}{2}$, $\frac{5}{12} \cdot \frac{18}{5}$, $\frac{15}{8} \cdot \frac{12}{8}$

   d. $2y, 10y^4, 50y^8, 250y^{13}, \ldots$ No

   $\frac{10y^4}{2y}$ $5y^3$ $\frac{50y^8}{10y^4}$

3. For the arithmetic sequence 7, 4, 1, . . . find:

   n = ?
   d = –3
   $t_1 = 7$

   a. a simplified form of t_n

   $t_n = 7 + (n-1)\cdot -3$

   $= 7 + -3n + 3$

   $\boxed{t_n = 10 - 3n}$

   b. t_{35} $t_{35} = 10 - 3(35)$

   $= -95$

   c. $S_{28} = \frac{28[2(7) + 27(-3)]}{2}$

   $= 14(14 - 81)$

   $\boxed{S_{28} = -938}$

4. In an arithmetic sequence $t_{14} = 96$ and d = 5. Find t_1.

   n = 14
   d = 5
   t_1 = ?
   $t_{14} = 96$

   $96 = t_1 + (13)5$

   $\boxed{t_1 = 31}$

   change

5. Find the sum of the multiples of 5 between 13 and 537? 538 +5

   n = ?
   d = 5
   $t_1 = 13$
   $t_n = 538$
   S_n = ?

   Find n first

   $538 = 13 + (n-1)5$

   $538 = 13 + 5n - 5$

   $5n = 530$ n = 106

   $S_{106} = \frac{106(13 + 538)}{2}$

   $S_{106} = 29203$

   a = 15
   $t_n = 535$
   n = 105
   $S_{105} = 28875$

6. In an arithmetic series the first term is $25x^2$ and the last term is $85x^2$. If the sum of this series is $715x^2$, find n, the number of terms.

   n = ?
   d
   $t_1 = 25x^2$
   $t_n = 85x^2$
   $S_n = 715x^2$

   $715x^2 = \frac{n(25x^2 + 85x^2)}{2}$

   $1430x^2 = n(110x^2)$

   $\boxed{n = 13}$

7. In an arithmetic series $t_1 = 17$, $t_{38} = 128$. Find S_{53}

   d = ? = 3
   n = 53
   $t_1 = 17$
   $t_{38} = 128$

   Find d first

   $t_1 = 17$, $t_{38} = 128$

   $128 = 17 + (38-1)d$

   $\boxed{d = 3}$

   a = 17
   n = 53
   d = 3
   S_{53}

   $S_{53} = \frac{53[2(17) + 52(3)]}{2}$

   $S_{53} = 5035$

8. $\frac{9}{10}+\frac{6}{5}+ \ldots +$ is a geometric series. Find:

$\frac{6}{8} \times \frac{10}{9} = \frac{4}{3}$

a. r $= \frac{4}{3}$

b. t_{23} (±0.01)

$t_{23} = \frac{9}{10}\left(\frac{4}{3}\right)^{22}$

$t_{23} = 504.54$

c. S_{17} (±0.01)

$S_{17} = \frac{\frac{9}{10}\cdot\left(\frac{4}{3}^{17}-1\right)}{\frac{4}{3}-1}$

$S_{17} = 356.49$

9. Find the missing terms in each of the following geometric sequences.

a. 4, −12, 36, −108, 324

$r = -3$

b. 62.5, 50, 40, 32, 25.6

$n = 4$
$r = ?$
$t_1 = 50$
$t_4 = 25.6$

$25.6 = 50(r)^3$

$\frac{512}{1000} = r^3$

$r = \frac{8}{10} = \frac{4}{5} = .8$

10. Find the sum of 5 + 10 + 20 + . . . + 10 240 to the nearest tenth.

$n = ?$
$r = 2$
$t_1 = 5$
$t_n = 10240.$

$10240 = 5(2)^{n-1}$

S_{12}

$2048 = 2^{n-1}$

$\boxed{n=12}$

2^{11}

$11 = n-1$

11. A photocopier was set to increase the dimensions of a drawing by 15%. If the increase was repeated, until the final dimensions were at least 3 times as large as the original, how many times was the increase carried out?

$t_1 = 100(1)$
$t_n = 300(3)$
$r = 1.15$
$n = ?$

$300 = 100(1.15)^{n-1}$

or $3 = 1(1.15)^{n-1}$

8 times

$n = 9$

$\log 3 = (n-1)\log 1.15$

12. Mrs. Sereda makes a deal with her class of 28 students. She will give each student \$100 on the first day and increase this by \$100 (for each student) each day for a 30-day period. In return the class will give Mrs. Sereda 1¢ on the first day, 2¢ on the second day, 4¢ on the third day and so on for the same 30-day period. Who has made the better deal? Explain.

$n = 30$
$t_1 = 2800$
$t_n = ?$
S_n
$d = 2800$

$n = 30$
$t_1 = 0.01$
$t_n = ?$
$S_n = ?$
$r = 2$

$S_{30} = 15\left(2(2800) + (29)2800\right]$

$15(5600 + 81200)$

$S_{30} = \$1,302,000$

gives

$S_{30} = \frac{.01(2^{30}-1)}{1}$

\$10 737 418.23

gets

$y = \frac{n}{2}(2(2800) + (x-1)2800)$

$y = \frac{n}{2}(5600 + 2800x - 2800)$

$n(2800 + 2800x)$

$S_n = \frac{a}{1-r}$

13. If $0.0\overline{54}$ is written as an infinite geometric series what is:

a. t_1 0.054

b. r = 0.01

c. S_∞ as a fraction $\frac{0.054}{1-0.01} = \frac{0.054}{.99} = \frac{54}{990} = \frac{3}{55}$

14. Find the sum of $18-12+8-\frac{16}{3}+\ldots$ (exact)

$\frac{-12}{18} = \frac{8}{-12}$ $r = -\frac{2}{3}$ convergent.

$S_\infty = \frac{18}{1+\frac{2}{3}} = \frac{18}{\frac{5}{3}} = 18 \cdot \frac{3}{5} = \frac{54}{5}$

15. If the sum of an infinite geometric series is –30 and $r = \frac{-2}{5}$, find t_1.

$-30 = \frac{a}{1+\frac{2}{5}}$ $-30 = \frac{a}{\frac{7}{5}}$ $\boxed{a = -42}$

$\frac{7}{5}(-30) = a$

16. An oil well produces 37 000 barrels of oil every week. Its production decreases by 5% each week. If this trend continues what is the expected lifetime production of this well?

.95

$S_\infty = \frac{37000}{0.05}$

740000 barrels

ANSWERS:

1. d 2. c 3. a. –3n + 10 b. –95 c. –938 4. 31 5. 28 875 6. 13 7. 5035 8. a. $\frac{4}{3}$ b. 504.54 c. 356.49 9. a. 4, –12 b. 62.5, 40, 32 10. 20 475 11. 8 12. Mrs. Sereda; Mrs. Sereda would have to give $1 302 000 and the students would have to give $10 737 418.23.

13. a. 0.054 b. 0.01 c. $\frac{3}{55}$ 14. 10.8 54/5 15. –42 16. 740 000 barrels

TEXTBOOK REVIEW
Chapter 1 Review: pp. 66 – 68
Chapter 1 Practice Test: pp. 69-70

100 105

MATH 20-1

CHAPTER 1: SEQUENCES & SERIES REVIEW (CONTINUED)

1. How many integers are divisible by 3 between 27 and 1953 inclusive? (ANS: 643)

$n =$
$d = 3$
$t_1 = 27$
$t_n = 1953$

$1953 = 27 + (n-1)3$
$= 27 + 3n - 3$
$= 24 + 3n$
$1929 = 3n$

$n = 643$

2. The sum of the first two terms of an arithmetic sequence is 9. The sum of the first six terms is 63. Determine the value of the first term t_1 and the common difference d. (ANS: $t_1 = 3$, $d = 3$)

$t_1 + t_2 = 9$
$S_2 = 9$
$S_6 = 63$

$S_2 = 9$
$9 = \frac{2}{2}(2a + (1)d)$
$(9 = 2a + d) \cdot -3$

$S_6 = 63$
$63 = 3(2a + 5d)$
$63 = 6a + 15d$
$-27 = -6a - 3d$
$36 = 12d$

$\boxed{d = 3}$
$9 = 2a + 3$
$6 = 2a$
$\boxed{a = 3}$

3. The sum of the first two terms of an arithmetic sequence is -14. The sum of the first seven terms is -154. Determine the value of the first term t_1 and the common difference d. (ANS: $t_1 = -4$, $d = -6$)

$S_2 = -14$
$-7[-14 = 1(2a + d)]$

$S_7 = -154$
$-154 = \frac{7}{2}(2a + 6d)$
$-308 = 14a + 42d$
$98 = -14a - 7d$
$-210 = 35d$

$\boxed{d = -6}$

$-14 = 2a - 6$
$-8 = 2a$
$\boxed{a = -4}$

4. A bacteria colony starts with a population of 2000, and increases at a rate of 12% per hour.
a. Determine the number of bacteria after 15 hours. (Round to the nearest whole number.) (ANS: 10 947 bacteria)

Geometric 112% / hour

$t_n = 2000(1.12)^{15}$

10 947 bacteria

b. How many hours will pass before the bacteria population first reaches 1 000 000? (ANS: 55 hours)

$1{,}000{,}000 = 2000(1.12)^{n-1}$
$500 = 1.12^{n-1}$
$\log 500 = (n-1)\log 1.12$

$\log 500 = n \log 1.12 - \log 1.12$

$\frac{\log 500 + \log 1.12}{\log 1.12} = 55.837\ldots$

-1

54.837

5. A car depreciates in value by 20% per year. If the car is now worth \$40 000, how many years will pass before the car first becomes worth less than \$10 000? (ANS: 7 years)

$10000 = 40000(0.80)^{n-1}$

$\frac{1}{4} = (.80)^{n-1}$

$\log 0.25 = (n-1)\log(.8)$
$\log 0.25 = n\log(.8) - \log(.8)$

$n = 7.212$ -1

7 years