Polynomials

1. **Polynomial**

is an expression consisting of variables and real numbers . The exponents of the variables must be whole numbers.

Ex. Circle the polynomials within the expressions below:

$3x + 11$ $\qquad x + \sqrt{2}$ $\qquad \sqrt{x}$ $\qquad 7x^2y - 3$ $\qquad \frac{x}{y^2}$

$-3x^{\frac{1}{3}} + x - 7$ $\qquad 1.02x^3 + x^2 + 2.54$ $\qquad (x - 7)(x + 3)$

2. **Coefficient**

is the number immediately preceding a variable in a term.

Ex. a) $-4x^2 + 7x$, the coefficients are -4 , $+7$.

b) $x^3 - x^2 - 3$, the coefficients are 1, -1, -3.

3. **Degree of a term**

The sum of the exponents of the variables in the term.

Ex: $9x^4 + 3x^2 + 7$ $\qquad$ ex.2 $9x^4 + 3x^2y^2 + 7$

Degree = 4 2 0 $\qquad$ 4

4. **Degree of a polynomial.**

Degree of a polynomial is determined by the term with the highest degree.

Ex: $4y^{10} - 6y^7 + 2y^3 - 4$

Degree = 10 7 3 0 $\qquad$ Degree of this polynomial is 10.

5. **Constant term.**

The term which has no variable.

Ex: a. $3x^3 + 7x^2 - 6$ Constant term is -6

b. $7x^5$ Constant term is 0

Multiples

3 → 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33

11 → 11, 22, 33, 44 ..

The *least common multiple* (LCM) of two or more numbers is the smallest number that can be **divided** by each number.

Again, determine the prime factorization of each number. Determine the *prime multiplicity* for each number; three 2's, two 5's, etc. The LCM will have at least one of all the primes, with the quantity of each prime equal to the largest prime multiplicity (look at exponent) found in any of the numbers. Example:

Find the LCM of 72 and 132.

72 = 2 x 2 x 2 x 3 x 3   Primes are 2 and 3:

- three 2's ← 2^3
- two 3's ← 3^2

132 = 2 x 2 x 3 x 11   Primes are 2, 3 and 11:

- two 2's (2^2)
- one 3 (3^1)
- one 11 ← 11^1

LCM will contain at least one of 2, 3, and 11. LCM will need to have:

- three 2's
- two 3's
- one 11

LCM = $2^3 \times 3^2 \times 11 = 792$

Or – list the multiples of each number until the same multiple appears in all three lists (Method 1, page 137.)

Practice - pg. 140

6. **Classifying polynomials according to the degree of the polynomial.**

Degree	Name	Examples
0	constant	-11
1	linear	$7x + 2$
2	quadratic	$2x^2 - 7x + 3$
3	cubic	$5x^3 + 8x^2 - 9x - 6$

Questions

	Number of terms	Degree of polynomial	Name according to # of terms	Name according to Degree
$7m+2$	2	1	linear	binomial
$5x^3 + 21x^2 - x$	3	3	cubic	trinomial
454	1	0	Constant	Constant
$6x^3 - 7x^2 + 8x - 66$	4	3	Cubic	~~Quadratic~~ polynomial
$n^2 + 5n + 6$	3	2	quadratic	trinomial
$7x$	1	1	linear	monomial

ADDING & SUBTRACTING POLYNOMIALS

A. To **add polynomials**, group like terms, then combine the terms by adding their coefficients.

- To add: $(8x + 7) + (3x^2 + 2x - 3)$

$(8x + 7) + (3x^2 + 2x - 3)$	1 Remove the brackets.
$= 8x + 7 + 3x^2 + 2x - 3$	2 Collect like terms.
$= 3x^2 + 8x + 2x + 7 - 3$	3 Combine like terms.
$= 3x^2 + 10x + 4$	

B. To **subtract polynomials**, use the properties of integers.

Subtracting an integer is the same as adding the opposite integer.
So, to subtract a term, add the opposite term.

- To subtract: $(3n^2 + 7n) - (2n^2 - 4n)$

$(3n^2 + 7n) - (2n^2 - 4n)$	Subtract each term.
$= 3n^2 + 7n - (2n^2) - (-4n)$	**Multiply the -1 through the brackets!**
$= 3n^2 + 7n - 2n^2 + 4n$	Collect like terms.
$= 3n^2 - 2n^2 + 7n + 4n$	Combine like terms.
$= n^2 + 11n$	

Check Your Understanding

1. Add or subtract.

a) $(6x + 3) + (2x + 5)$ **b)** $(2x^2 + 6x - 5) + (-4x^2 - 3x + 7)$

c) $(5a - 8) - (2a + 3)$ **d)** $(3a^2 - 2a + 6) - (-2a^2 + 7a - 9)$

e) $(-7 + 3d^2 - 2d) + (8 - 4d^2 + 3d)$ **f)** $(5e - 9 + 2e^2) - (2e^2 - 9 + 5e)$

g) $(10v - 5v^2 - 2) + (3v - 7v^2 - 1)$ **h)** $(m - 3m^2 - 5) - (3m^2 + 5 - m)$

Answers

1. **a)** $8x + 8$ **b)** $-2x^2 + 3x + 2$
c) $3a - 11$ **d)** $5a^2 - 9a + 15$
e) $1 - d^2 + d$ **f)** 0
g) $13v - 12v^2 - 3$
h) $-6m^2 + 2m - 10$

Assignment:

Practice Adding & Subtracting Polynomials (MathAids)

Or Algebra 2

3.7 MULTIPLYING POLYNOMIALS

√ **A. Multiplication Monomial by Monomial**

Ex: 1. $(3x^3)(2x^2) = 6x^5$

2. $(-4a^2b^3)(2ab) =$ $-8a^3b^4$

3. $(8x^3y^2)\left(\frac{1}{2}xy\right) =$ $4x^4y^3$

4. $(-4x^3y^2)(2xy^3)(-2xy) =$ $16x^5y^6$

B. Distributive Property - Monomial multiplied by a Binomial/Trinomial.

1. $3x^2(2x - 7y + 2)$

$= 6x^3 - 21x^2y + 6x^2$

2. $3(2x^2 - 7) + x(2x + 5) - (8x^2 + 5x - 7)$

$6x^2 - 21 + 2x^2 + 5x - 8x^2 - 5x + 7$

$= -14$

3. $9 - 2(3x + 5)$

$9 - 6x - 10$

$= -6x - 1$

4. $4a(a + 3) + 2a(a - 1) - a(2a + 4)$

$4a^2 + 12a + 2a^2 - 2a - 2a^2 - 4a$

$4a^2 + 6a$

5. $3x - 2[3 + 4(2x + 6)] - [2 + 3(x + 1)]$

$3x - 2(3 + 8x + 24) - (2 + 3x + 3)$

$3x - 6 - 16x - 48 - 2 - 3x - 3$

$= -16x - 59$

Complex Question!

Distribute the 4 and the 3.
Distribute the –2 and the –.
Clean up.

B ( ) [ ]
E
D
M
A
S

C. Multiplying Binomial x Binomial

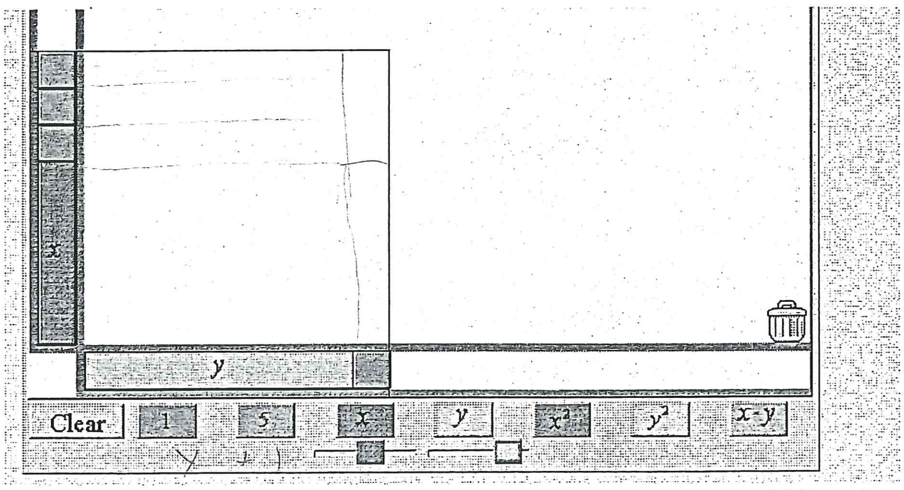
Multiply $(x+y)(a+b)$

<u>Method 1</u> – Algebra Tiles

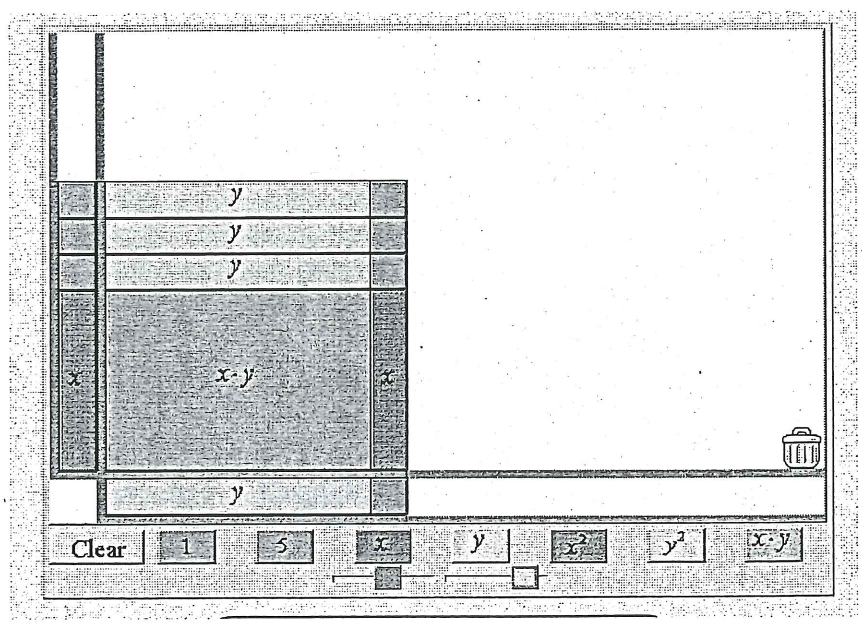
a) $(x+3)(y+1)$

Step 1 – Draw (x + 3) on the vertical axis and (y + 1) on the horizontal axis.

3 + x



Step 2 – Multiply the vertical and horizontal values and fill in the rectangle.



Step 3 – Determine your final product.

There is *1 xy*, *1 x*, *3 y's* and *3 ones*. So, the answer is $xy+x+3y+3$

Method 2 – Area model

Step 1 – Create a square grid with (x + 3) on the vertical axis and (y + 1) on the horizontal axis.

	y	+1
x		
+3		

Step 2 – Fill in the grid

	y	+1
x	xy	x ~~xy~~
+3	3y	3

Step 3 – Determine your final product.

There is 1 xy, 1 x, 3 y's and a 3. So, the answer is $xy+x+3y+3$

Write in the order of *x, then xy, then y.* $(x+3)(y+1) = x + xy + 3y + 3$

Method 3 – FOIL (First, Outer, Inner, Last)

$(x+3)(y+1)$

$(x+3)(y+1)$ **First** x *times* y = xy

$(x+3)(y+1)$ **Outer** x *times* 1 = x

$(x+3)(y+1)$ **Inner** 3 *times* y = 3y

$(x+3)(y+1)$ **Last** 3 *times* 1 = 3

You now collect your terms.

There is 1 xy, 1 x, 3 y's and a 3. So, the answer is $x + xy + 3y + 3$

Method 4

$(x+3)$
$(y+1)$

$xy + x + 3y + 3$

$x + xy + 3y + 3$

Practice

$(x+3)(x-11)$

$x^2 - 8x - 33$

$(x+5)(x+5)$

$x^2 + 10x + 25$

$(x+4)(x-4)$

$(2x+3)(x-1)$

$2x^2 + x - 3$

SkBP **Multiply** $(x+y)(x-y)$ **Two binomials that "are the same" but different signs.**

a) $(2a-3b)(2a+3b)$

$(2a-3b)$
$(2a+3b)$

$4a^2+6ab-6ab-9b^2$

$4a^2-9b^2$

FOIL (First, Outer, Inner, Last)

$(2a-3b)(2a+3b)$

$(2a-3b)(2a+3b)$ **First** 2a times 2a = $4a^2$

$(2a-3b)(2a+3b)$ **Outer** 2a times 3b = 6ab

$(2a-3b)(2a+3b)$ **Inner** –3b times 2a = –6ab

$(2a-3b)(2a+3b)$ **Last** –3b times 3b $\mp 9b^2$ error

Fix

Answer: $4a^2-9b^2$

b) $(x-3)(x+3)$

When dealing with **conjugates** (that's what $(x+y)(x-y)$ are called), you can save some time by using the "shortcut"

$(x-3)(x+3)$

$x^2+3x-3x-9$

x^2-9

$(x-3)(x+3)$

$\boxed{x^2-9}$ ← Perfect square

c) $(5x^3+4y)(5x^3-4y)$

$25x^6-16y^2$

Multiply $(x+y)^2$ **Squaring a binomial.**

a) $(2m+5n)^2$

Let's realize that $(2m+5n)^2$ is no different than $(2m+5n)(2m+5n)$ and we know how to multiply two binomials

$(2m+5n)(2m+5n)$

$4m^2 + 10mn + 10mn + 25n^2$

$4m^2 + 20mn + 25n^2$

But wait! Squaring a binomial $(x+y)^2$ has a shortcut!

Let's try the previous question again.

a) $(2m+5n)^2$ When you are squaring a binomial, here is the shortcut... square the first, square the last, multiply the two terms and double the coefficient. Easy, right?

$2(10)$

$4m^2 + 20mn + 25n^2$

b) $(\cancel{7c^6-8d^3})^2$ $2(-56)$ ← $(x-3)^2$

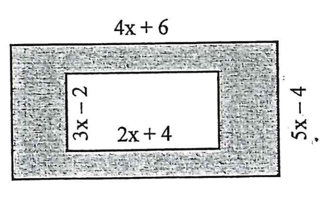
$49c^{12} - 112c^6d^3 + 64d^6$

Homework pg 186.

Youtube?

Applications

Find the area of the **shaded** region.



A = A~Large~ − A~Small~

$(4x+6)(5x-4) - [(2x+4)(3x-2)]$

$20x^2 + 14x - 24 - (6x^2 + 8x - 8)$

$20x^2 + 14x - 24 - 6x^2 - 8x + 8$

$\boxed{A = 14x^2 + 6x - 16}$

Multiplication and simplifying of:

✓ 1. Monomial x Polynomial

✓ 2. Binomial x Binomial

3. Polynomial x Polynomial

Examples

1. $(3x-2)(x^2+5x-7)$ **Distribute** the $(3x-2)$ into the trinomial

The 3x multiplies into the trinomial and then the –2 multiplies into the trinomial.

$$=3x^3+15x^2-21x-2x^2-10x+14$$
$$=3x^3+13x^2-31x+14$$

2. $4a(a+3)+2a(a-1)-a(2a+4)$

$4a^2+12a+2a^2-2a-2a^2-4a$

$4a^2+6a$

3. $(2x-1)(x+3)-[(x-4)(x-5)]$ square brackets.

$= 2x^2+5x-3-[x^2-9x+20)$

$= 2x^2+5x-3-x^2+9x-20$

$= \boxed{x^2+14x-23}$

4. $(x+3)(x+2)(x-1)$

$(x+3)(x^2+x-2)$

$x^3+x^2-2x+3x^2+3x-6$

x^3+4x^2+x-6

5. $-(x-3)^2-[(x+5)(x-5)]$ Insert square brackets

$-(x^2-6x+9)-(x^2-25)$ Keep the negative out of the squaring of (x–3) and out of FOILing (x+5)(x–5)

$-x^2+6x-9-x^2+25$ Distribute the -1 into the brackets.

$-2x^2+6x+16$ Collect like terms.

6. $3t^2 - (3 - 2t)^2 + 5(2t - 1)(2t + 1)$

$3t^2 - (9 - 12t + 4t^2) + 5(4t^2 - 1)$

$3t^2 - 9 + 12t - 4t^2 + 20t^2 - 5$

$19t^2 + 12t - 14$

7. $2(a + 3)(a - 3) + 3(a + 2)^2$

$2(a^2 - 9) + 3(a^2 + 4a + 4)$

$2a^2 - 18 + 3a^2 + 12a + 12$

$5a^2 + 12a - 6$

8. $(3x - 4y)^3$

$(3x - 4y)(9x^2 - 24xy + 16y^2)$

$27x^3 - 72x^2y + 48xy^2 - 36x^2y + 96xy^2 - 64y^3$

$27x^3 - 108x^2y + 144xy^2 - 64y^3$

9. $(2x + 3y - z)(x - 2y + 4z)$

Here we have a trinomial times a trinomial.
We can look at it in a couple of ways.

Area Model

	x	−2y	4z
2x	$2x^2$	−4xy	8xz
3y	3xy	$-6y^2$	12yz
−z	−xz	2yz	$-4z^2$

Collect like terms = $2x^2 - xy + 7xz - 6y^2 + 14yz - 4z^2$

Distributive Property

Distribute the 2x, 3y and the −z into the trinomial $(x - 2y + 4z)$

$(2x + 3y - z)(x - 2y + 4z)$

3.3 Common Factors of a Polynomial

Recall what a GCF is from section 3.1 When we remove/pull out a GCF, we are DIVIDING EACH term by the GCF

When dealing with polynomials, always look to see if there is a GCF for the polynomial.

A. Factor using GCF. Note: **Always look for GCF! Always!**

1. $5x^2 + 10x$

   $= 5x(x+2)$

2. $x^3 + 7x^2 - 3x$

   $x(x^2+7x-3)$

3. $8a^2b + 16a^2b^2 + 32a^2bc$

   $8a^2(b+2b^2+4bc)$

4. $x(a + b) + 7(a + b)$

   $(x+7)(a+b)$

5. $7a(x - 1) + 2(x - 1)$

   $(x-1)(7a+2)$

6. $3x^2(y + 5) + 2x(y + 5)$

   $(y+5)(3x^2+2x)$

7. $3x(5 - x) - 9(5 - x) + 8y(5 - x)$

   $(3x-9+8y)(5-x)$

B. Division of Monomial by Monomial

Ex: 1. $\dfrac{50x}{10x} = 5$

2. $\dfrac{20x^2}{5x} = $ $4x$

3. $\dfrac{20x^3y^4}{-5x^2y^2} = $ $-4xy^2$

4. $\dfrac{(3x^2y^2)(5xy)}{-3xy} = $ $\dfrac{15x^3y^3}{-3xy} = -5x^2y^2$

C. **Division of Polynomial by Monomial**

Rule: Take **EACH** term in the numerator (top) and divide it by the term in the denominator.

$\frac{3}{2} + \frac{6}{2} = \frac{3+6}{2}$ Ex: 1. $\frac{4t^2 + 8t}{4t} =$ Think $\frac{4t^2}{4t} + \frac{8t}{4t} = \boxed{t + 2}$

2. $\frac{10m^3 + 5m^2 + 15m}{5m} =$ $\boxed{2m^2 + m + 3}$ or $\frac{(\cancel{5m})(2m^2 + m + 3)}{(\cancel{5m})}$

3. $\frac{6r^4 - 3r^3 + 9r^2}{3r^2} =$ $\boxed{2r^2 - r + 3}$

4. $\frac{8x^2 + 16x + 24}{8} =$ $\boxed{x^2 + 2x + 3}$

R Assign: 155 / 7 bdf
8-10 ace GCF
12a
14 ac
16 ace
18

3.5 Trinomial Factorization (ax^2+bx+c), a=1

Recall: $(x + 4)(x + 3) = x^2 + 7x + 12$

– The middle term's coefficient is the sum of the last terms of each binomial.
– The end term is the product of the last terms of each binomial.

Ex: 1) $x^2 + 8x + 16$
$= (x+4)(x+4)$ Perfect square!
$= (x+4)^2$

All positive
1 6
$\boxed{2\ 3}$

2) x^2+5x+6
$=(x+?)(x+?)$
$(x+3)(x+2)$

Find two binomials that are factors of this trinomial. The ? must be replaced by numbers that are factors of 6 and ALSO add up to 6. Try 2 and 3.

Pos / Neg. -8
1 15
$\boxed{-3\ -5}$

3) $x^4-8x+15$
$(x^2-3)(x^2-5)$

Watch for exponent on the x.
Watch the signs.

Practice sheet on SB - White boards

4) $x^{10} + 3x^5 - 28$
$(x^5+7)(x^5-4)$

First try $x^2+3x-28$ $\left.\begin{matrix}+7\\-4\end{matrix}\right\}+3$
$(x+7)(x-4)$

5) $x^2 - 10xy + 21y^2$
$(x-7y)(x-3y)$

First Try $(x^2-10x+21)$ $\begin{matrix}-7\\-3\end{matrix}$
$(x-7)(x-3)$

6) $x^{10} - 7x^5y^3 - 30y^6$
$(x^5-10y^3)(x^5+3y^3)$

Try
$x^2-7x-30$ $\begin{matrix}-10\\+3\end{matrix}$
$(x-10)(x+3)$

$x^{10}+3x^5y^3-10x^5y^3-30y^6$
$x^{10}-7x^5y^3-30y^6$

GCF 7) $2x^2 + 10x - 28$

$2(x^2 + 5x - 14)$ $+7$ -2

$\boxed{2(x+7)(x-2)}$

$2(x^2 + 5x - 14)$

✓ $2x^2 + 10x - 28$

GCF 8) $x^3 - 2x^2y^2 - 8xy^4$

$x(x^2 - 2xy^2 - 8y^4)$ -4 $+2$

$\boxed{x(x-4y^2)(x+2y^2)}$

$x(x^2 + 2xy^2 - 4xy^2 - 8y^4$

$x(x^2 - 2xy^2 - 8y^4)$

$= x^3 - 2x^2y^2 - 8xy^4$ ✓

Remember:

1. Both terms positive (middle and last), then both parts are positive.
   $x^2 + 12x + 32 = (x+8)(x+4)$

2. Last term positive, middle term negative, then both parts are negative.
   $x^2 - 12x + 32 = (x-8)(x-4)$

3. Last term is negative, then one part is negative and the other is positive.
   $x^2 - 4x - 32 = (x-8)(x+4)$
   $x^2 + 4x - 32 = (x+8)(x-4)$

4. The middle term contains the variables that are found in the first and last terms. The exponents are half of what they are in the first and last terms.

3.6 Factoring trinomials ($ax^2 + bx + c$, $a \neq 1$)

There are different methods to factoring these types of trinomials.
Remember to look for a GCF EVERY time...it's the first thing you should do.
Note that there are polynomials that are not factorable. Use decomposition : 2 Methods

1. $2x^2 + 13x + 15$

$2x^2 + 10x + 3x + 15$

$2x(x+5) + 3(x+5)$

$(2x+3)(x+5)$

$2 \times 15 = 30$
Find Factors of 30 - that give you 13 → 10, 3
Split middle term
GCF : Group.

Method 2

$2x^2 + 13x + 15$

$2x$	+	3	+3
$1x$	+	5	+10

$(2x+3)(x+5)$

2. $5x^2 + 16x + 3$

$5 \times 3 = 15$
15, 1

$5x^2 + 15x + x + 3$

$5x(x+3) + 1(x+3)$

$(5x+1)(x+3)$

$5x^2 + 16x + 3$

$5x$	+	1	+1
$1x$	+	3	+15

$(5x+1)(x+3)$

3. $12x^4 - 16x^2 + 5$

60
10, 6

$12x^4 - 6x^2 - 10x^2 + 5$

$6x^2(2x^2-1) - 5(2x^2-1)$

$(6x^2-5)(2x^2-1)$

$12x^4 - 16x^2 + 5$

$6x^2$	-	5	-10
$2x^2$	-	1	-6

$(6x^2-5)(2x^2-1)$

4. $8x^{20} + 10xy - 3y^2$

$4x^{10}$	+	$3y$	+6
$2x^{10}$	+	$1y$	+4

8	3	3,
1	1	8

5. $10x^2 - 44xy - 30y^2$

$2(5x^2 - 22xy \quad 15y^2)$

$2(5x + 3y)(x - 5y)$

6. $2x^2 + 11x + 12$

$2x + 3 \quad +3$

$1x + 4 \quad +8$

$(2x+3)(x+4)$

24

8, 3

$2x^2 + 8x + 3x + 12$

$2x(x+4) + 3(x+4)$

$(2x+3)(x+4)$

Lizzie method

7. $6m^2 + 13m - 5$

30

$3m - 1 \quad -2$

$2m + 5 \quad +15$

$(3m-1)(2m+5)$

8. $4x^2 - 5xy - 6y^2$ 24 8 3

$4x + 3y \quad +3$

$1x - 2y \quad -8$

$(4x+3y)(x-2y)$

$4x^2 - 8xy + 3xy - 6y^2$

$4x(x-2y) + 3y(x-2y)$

$(4x+3y)(x-2y)$

9. $6x^2 - 5x - 4$

$3x - 4 \quad -8$

$2x + 1 \quad +3$

$(3x-4)(2x+1)$

3.8 Factoring Special Trinomials

I. Factoring a difference of squares. Let's go back to $(3x+2y)(3x-2y)$. If we remember, multiplying conjugates was quite simple. Factoring these binomials is quite simple as well. $(x+y)(x-y)=(x^2-y^2)$ There are three things to notice about the answer:

1. The answer is a **binomial.**
2. The answer has **subtraction.**
3. Both terms are **perfect squares.**

Remember, if you notice the three things mentioned above, you have what's called **A Difference of Squares.**

Let's Factor

Ex: 1. $x^2 - 16$

$= (x-4)(x+4)$

2. $x^2 - 25y^2$

$(x-5y)(x+5y)$

3. $49x^2 - 64$

Each answer has a (−) and a (+)

4. $x^4 - 81$

$(x^2+9)(x^2-9)$

$(x^2+9)(x-3)(x+3)$

5. $x^2y^3 - y$ (GCF)

$y(x^2y^2-1)$

$y(xy-1)(xy+1)$

6. $3y^6 - 12d^2$

$3(y^6-4d^2)$

$3(y^3-2d)(y^3+2d)$

7. $x^{20} - 16$

$(x^{10}-4)(x^{10}+4)$

$(x^5-2)(x^5+2)(x^{10}+4)$

II. Factoring Perfect squares.

Recall that $(a + 3)^2 = a^2 + 6a + 9$

We learned that we could FOIL the binomial, or we could square the first term, square the last term, multiply the two terms and double the product.

Thus, in general $(a+b)^2 = a^2 + 2ab + b^2$

So, when we have a trinomial of the form $a^2 + 2ab + b^2$, it is called a **Perfect Square Trinomial** and can easily be factored back to a single binomial squared.

Ex: 1. $a^2 + 8a + 16 \quad = (a + 4)^2$

2. $9a^2 + 120a + 400 \quad = (3a + 20)^2$

3. $81a^2 - 72ab^3 + 16b^6 \quad = (9a - 4b^3)^2$

Notice in the first three questions how the first and last terms were perfect squares. That's a requirement for **Perfect Square Trinomials.**

4. $28a^2 + 28a + 7$ GCF?

$7(4a^2 + 4a + 1)$

$7(2a + 1)^2$

Recall:

$(2a+1)(2a+1)$.

$4a^2 + 2a + 2a + 1$

5. $15ax^2 + 90ax + 135a$

$15a(x^2 + 6x + 9)$

$15a(x+3)^2$

6. $1 - 12a + 36a^2$ This question can be left as is and factored or rearranged in descending order of x, then factored.

Option 1 - Leave as is.

$1 - 12a + 36a^2$

$= (1 - 6a)^2$

Option 2 - Rewrite

$36a^2 - 12a + 1$

$= (6a - 1)^2$

These two answers appear different yet they are the same. One answer is negative while the other answer is positive. Think about it, $\sqrt{36} = 6$ *and* -6. Thus, if your answer is squared, then the number being squared could be positive or negative.

Creating Factorable Trinomials.

We know that when we factor ax^2+bx+c, if a = 1 then we need two numbers that multiply to c and add up to b.

$x^2+11x+30=(x+5)(x+6)$ 5 times 6 = 30 and 5+6 = 11

Question: Fill in the box to create factorable trinomials.

a. $x^2+\square x+6$ We need two numbers that multiply to 6. (1&6) or (2&3)

If we use 1&6, then our answer is x^2+7x+6
If we use 2&3, then our answer is x^2+5x+6

But wait! Doesn't $-1\times-6=6$? Or $-2\times-3=6$? Yes! So... we also have....

If we use -1&-6, then our answer is x^2-7x+6
If we use -2&-3, then our answer is x^2-5x+6

Final Answer could be, 7 or 5 or -7 or -5. Any of the four will make the trinomial factorable.

b. $x^2+\square x-14$ We need two numbers that multiply to 6. (1&-14) or (-1&14) or (2,-7) or (-2&7)

If we use 1&-14, then our answer is $x^2-13x-14$
If we use -1&14, then our answer is $x^2+13x-14$
If we use 2&-7, then our answer is $x^2-5x-14$
If we use -2&7, then our answer is $x^2+5x-14$

Final Answer could be, 13 or 5 or -13 or -5. Any of the four will make the trinomial factorable.

c. $x^2+4x+\square$ Now we need two numbers that add to 4.
$(1+3)(2+2)(-1+5)(-2+6)$... There are an infinite number!

So choose a pair. Let's try 1 and 3.

$(x+1)(x+3)=x^2+4x+3$
Or we could try -1 and 5.
$(x-1)(x+5)=x^2+4x+4$

So choose a pair of numbers that add to 4 and FOIL away!

d. $x^2 - 5x + \square$

Now we need two numbers that add to –5.
Again, infinite possibilities.

Let's try –8 and 3.

$(x-8)(x+3) = x^2 - 5x - 24$

Some Tricky Factorization

Factor the following:

1. $x^4 - 81$

$= (x^2+9)(x^2-9)$

$= (x^2+9)(x-3)(x+3)$

A difference of squares question.
The (x^2-9) can be factored again!

Rule = factor until you can factor no more.

2. $x^4 - x^2 - 12$

Need two numbers that multiply to –12 and add to –1

$= (x^2-4)(x^2+3)$

First binomial is a difference of squares.

$= (x-2)(x+2)(x^2+3)$

3. $x^4 - 120x^2 - 121$

Need two numbers that multiply to –12 and add to –1

$= (x^2+1)(x^2-121)$

Second binomial is a difference of squares.

$= (x^2+1)(x-11)(x+11)$

Assignment

Page 147 #7, 8	Square and Cube Roots
Page 187 #20a	Volume of Cube
Page 156 #17	Neat Factoring Question
Page 167 #19, 20	Determining b and c values to make trinomials factorable
Page 195 #20	Two step factorization

Practicing all types of Factoring.

YouTube video of what to look for when factoring.
http://www.youtube.com/watch?v=KDoHYS4dX1s

I. All types of factoring.

Ex:

1. $x^2 - 7x - 18$

$$= (x-9)(x+2)$$

2. $6m^2 + 19mn + 10n^2$

$$= (3m+2n)(2m+5n)$$

3. $2x^2 - 8$

$$= 2(x^2 - 4)$$
$$= 2(x-2)(x+2)$$

4. $x^3 - 4x^2 + 4x$

$$= x(x^2 - 4x + 4)$$
$$= x(x-2)^2$$

5. $x(m-2) - 4(m-2)$

$$= (m-2)(x-4)$$

6. $3ab - 9ab^2 + 6a^2b$

$$= 3ab(1 - 3b + 2a)$$

7. $2y^2 + 7y - 4$

$$= (2y-1)(y+4)$$

8. $x^4 - 7xy + 12y^4$

$$= (x^2 - 4y^2)(x^2 - 3y^2)$$
$$= (x-2y)(x+2y)(x^2 - 3y^2)$$