**UNIT 6 Chapter 7 Systems of Linear Equations**

**7.1**

**Systems of Equations**

• \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

 \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

• usually the number of equations equals the number of variables

**Solution to a System of Equations**

• \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Recall linear equations:



 

  ( one solution)

Whereas  has 2 variables having many solutions {(1,5), (2,4), ( 3,3)} are just a few. *How can we represent all of the solutions? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_*

Example:

 1. Which of the following is a solution to the system y = 3x + 5 and 6x − 5y = −43?

 a. (1, 8) b. (−5, 3) c. (−2, 11) d. (2, 11) Answer : \_\_\_\_\_\_\_\_\_\_\_\_

 For a solution to be valid, **it must satisfy both equations simultaneously**.

 i.e.  works in the first equation but not the second.

**Solving Systems**

We can solve systems

* algebraically by **substitution** or **elimination**
* **graphically**

**7.2 Solve Systems Graphically (no calculator)**

**Solving Systems (Linear) Graphically**

 1. Graph each line.

 2. Find point of intersection from graph.

Refer to # 1,2,3 Text p 405,406 for further examples.

 1. y = 2x − 5

 2x + 3y = 9



 2. y = 8 – x

 14 = 3x – 2y



 3. y = 5x – 3

 3x + y + 11 = 0

 4. y = 

 6y + 2x = −1

 5. 10x − 4y = 8

 y = 

**7.3 Solving Systems Graphically (use calculator)**

Steps

1. type each equation into y = on you calculator

 equation 1 into y1 =

 equation 2 into y2 =

2. ensure your window is such that you can see the two lines cross.

3. hit 2nd Calc (Trace)

4. hit 5 intersect

5. hit enter 3 times. You now have the point of intersection. (1st curve, 2nd curve matters only if you

 have more than 2 equations).

Examples:

**NOTE:** If the point of intersection is above or below the window, it is not necessary to adjust the window. If the point of intersection is right or left of the window it must be adjusted. Round to the nearest tenth.

 1. x+ y = 2

2x – y = 7

 2. y = 2x –5

 x + y = 8

 3. 2x + y = 3

 3x + 2y = 11

 4. 3x +5y = 5

 2x + y = 22

**7.4 Substitution Method**

The goal is to get one of the equations to either  or .

You then substitute that equation into the other equation and solve for the remaining variable.

Examples:

 1. y = 5x − 1

 3x + y = 15

 The first equation is in y = form. *Substitute equation 1 into equation 2.*

 

 Once you have x, solve for y.



 **The solution to the system is (2,9)**

 2. x = 3y − 7

 3x + 2y = −10

 3. 3x − y = 7

 12x = 4y + 3

**ANS:** 

*When you work down to constant (other than 0), you have parallel lines. No solution.*

 4. p = m + 5

 4m − 6p + 30 = 0

 5. 

 x + 

**ANS:** (8, −10)

 6. 

 

**7.5 Elimination Method**

The goal is to get the additive inverse for one variable in the system.

**Decimals are acceptable (not ideal) but avoid fractions!**

Examples:

 1. 2x − y = 10

 5x + y = 11

 The negative y in equation 1 is opposite the positive y in equation 2.

 Add the two equations together.

 

 

Once you have x, insert into either original equation to get y.

 2(3) − y = 10

 6 − y = 10

 −y = 4

 y = −4

 **ANS:** 

 2. 4x − 3y + 11 = 0

 5x + 6y = −4

3. 2c + 5d = 17

 7c = 3d + 2

 4. 2x − 5y − 11 = 0

 −4x + 10y − 3 = 0

 5. 

 3(x − 4) − (7 − 4y) = 8

6. 0.3x − 0.5y = 1.2

 0.7x − 0.2y = −0.1

*Multiply each equation by 10 to get rid of the decimals. You can work with the equations as they are but integers are easier to work with.*

Additional Examples:

 1. 5(m − 3) + 2(n + 4) = 10 2. 4x − 5 = 2y

 3(m + 4) − 4(n + 3) = −21 1 = 5y − 10x

 3.  

**7.6 Properties of Systems of Linear Systems**

We know that two lines have three potential scenarios:

1. They cross once and have one solution.

2. They are parallel, thus never cross and thus have no solution.

3. They are the same line, thus they cross an infinite number of times.

Each of these scenarios has a name. Let's look at their names.

1. **One Solution – Independent System—intersect , only one solution**

 y = 2x–5

 2x + 3y = 9 **ANS: x = 3, y = 1 ( 3, 1)**

Independent System, one solution

1. **No Solution– { } – Inconsistent System ( Parallel lines)**

y =** ANS:** No Solutions { }

6y + 2x = – 1 parallel lines

1. **Infinite Number of Solutions  Same line – Dependent (also known as Coincident)**

 10x – 4y = 8 **ANS:**

y = ** Same line , infinite number of solutions**

**Summary:**

Look at slopes and y – intercept to determine if systems are independent, inconsistent or dependent.

Independent: If slopes are \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_🡪 one solution

Dependent: If slopes are \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ and y – intercepts are the same, same line 🡪 infinite number of solutions

Inconsistent: If slopes are **equal** but y –intercepts are\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_, parallel lines 🡪 No solutions

Examples:

1. y = 3x – 6 3. y = 

 y =  y = 

Independent Inconsistent

1. y = – 2x + 7

 y = – 2x + 7

 Dependent

**Try :**

1. y = x– 3 ( nearest hundredth) **ANS**

y = 8x +2

1. 2x + y = –10 **ANS:**

4x + y – 30 = 0

**PROBLEM APPLICATION EXAMPLES**

4 Steps:

1)

2)

3)

4)

**Example 1**

Betty and Rick are off to the movies with some friends and decide to take the bus.

Betty buys 5 admission tickets and 3 bus tickets. She pays $65.00. Rick buys 2 admission tickets and 1 bus ticket. He only pays $25.00. Set up a system and determine the price of each ticket.

**Example 2**

Montey purchased $ 20,000 in two types of savings bonds. One bond earned him 8 % annually and the other bond earned 5% annually. The total interest earned was $ 1360. How much money was invested at each rate?

**Problem Solving**

Examples:

 1. The perimeter of a rectangle is 70 m. 5 times the width is 7 more than twice the length. Find the dimensions.

 2. The population of Smoky Lake is 5 times the population of Bellis. Their total population is 1266. Find the population of each.

 3. John invests $30 000; some at 4% and the remainder at 7%. If he earned $1 560, how much did he invest at each rate?

 4. Tom has $330 in $5 and $20 bills. He has 24 bills in total. How many of each?

 5. Bill took 8 hours to travel 93 km. Part of the trip he biked at 15 km/h and the remainder he walked at 6 km/h. How far did he bike? walk?

**Problems and Review**

**1.** Which statement below is **false** for this linear system?

3*x* – 4*y* = –9.5 ➀

 ➁

**A.** If you multiply equation ➁ by 8, then add the new equation to equation ➀,
you can eliminate *y*.

**B.** The system has one solution because the slopes of the lines are different.

**C.** If you replace equation ➁ with 4*x* – *y* = –4, the new system will have the same solution as the original system.

 **D.** The solution of the linear system is: (2, –0.5)

**2.** Which system has **exactly one** solution? Thus, which system is **independent**?

 **A.** *y* = –4*x* – 2 ` **B.**6*x* – 3*y* = –1

 *y* = –4*x* + 5 –2*x* + *y* = 4

 **C.**  **D.** *y* = 3*x* – 2

  *y* = 3*x* + 2

**3.** Solve each linear system.

**a)** –3*x* – 6*y* = 9 **b)** 3*x* – 4*y* = 13 **c)** 

2*x* + 2*y* = –4 5*x* + 3*y* = 12 

**4.** Given the linear equation 4*x* – 2*y* = –4, write another linear equation that will form a linear system with each number of solutions. Explain what you did.

**a)** exactly one solution **b)** no solution **c)** infinite solutions

**5. a)** Write a linear system to model this situation:

In Claire’s school, 41 of the 80 grade 10 students were not born in Canada. Sixty percent of the boys and 40% of the girls in grade 10 were not born in Canada.

**b)** Solve this related problem: How many boys and how many girls are in grade 10? Explain what you did.

**6.** A gift shop sold hand-made moccasins. One order of 4 pairs of children’s moccasins and 3 pairs of women’s moccasins cost $244.65. Another order of 2 pairs of children’s moccasins and 4 pairs of women’s moccasins cost $229.70.
**a)** Write a linear system to model this situation.
**b)** Solve this related problem: What is the cost for a pair of each type of moccasin?

**ANSWERS**

**1.** D **2.** C

**3. a)** *x* = –1, *y* = –1 **b)** *x* = 3, *y* = –1

 **c)** *x* = , *y* =  Explanation is below:

I multiplied equation ➀ by 12 and equation ➁ by 6.

That left me with 6*x* – 4*y* = 5 ➂ and 5*x* + 3*y* = 1 ➃.

I multiplied equation ➂ by 3 and equation ➃ by 4 to get 18*x* – 12*y* = 15 and

20*x* + 12*y* = 4. I then added these equations to get 38*x* = 19, or *x* = .

I substituted this value for *x* in equation ➃ and solved for *y*.

I got *y* = . I then verified my solution.

**4.** Equations may vary.

 **a)** I wrote 4*x* – 2*y* = –4 in slope-intercept form to identify its slope: *y* = 2*x* + 2

The slope of the line is 2. For a linear system with one solution, the lines must have

different slopes. So, I let the second line have slope –3. Its equation is: *y* = –3*x* + 2

**b)** For a linear system with no solution, the lines must be parallel. The lines must have equal slopes but different *y*-intercepts. The equation *y* = 2*x* + 2 has slope 2 and
*y*-intercept 2. The second line must also have slope 2. Let its *y*-intercept be –2.
Its equation is: *y* = 2*x* – 2

**c)** For a linear system with infinite solutions, the lines must be coincident. The equations must be equivalent. To determine an equivalent equation, I multiplied the given equation by 2: 2(4*x*)– 2(2*y*)= 2(–4)
Its equation is: 8*x* – 4*y* = –8

**5. a)** Let*b* represent the number of boys in grade 10 and *g* represent the number of girls in grade 10. The linear system that models the situation is:
*b* + *g* = 80 ➀

0.60*b* + 0.40*g* = 41 ➁

**b)** There are 45 boys and 35 girls in grade 10. I eliminated *b* by multiplying equation ➀ by 0.6 and subtracting the new equation from equation ➁. I got 0.2*g* = 7, or *g* = 35. I then substituted this value into equation ➀ and solved for *b*. I got *b* = 45. I then verified my solution.

**6.** **a)** Let *c* dollars represent the cost of a pair of children’s moccasins and let *w* dollars represent the cost of a pair of women’s moccasins.

4*c* + 3*w* = 244.65

2*c* + 4*w* = 229.70

**b)** $28.95 for children’s moccasins and $42.95 for women’s moccasins