**UNIT 4 Chapter 7 ABSOLUTE VALUE AND RECIPROCAL**

**7.1 Absolute Value**

A) Absolute Value – For a real number, a, the absolute values is written as and is a positive value

 a if a > 0

= 0 if a = 0 Ex. $\left|-7\right|=-\left(-7\right)=+7$

-(-a) if a < 0

Geometrically, we can think of an absolute value of a real number a, written , as **the distance from zero** on the real number line regardless of direction

Distance from

0 to 3 is

0

= 3 units

Distance from

0 to - 3 is

0

= 3 units

X

-10

-9

-8

-7

-6

-5

-4

-3

-2

-1

0

1

2

3

4

5

6

7

8

9

10

From zero the distance to +3 or – 3 is 3 units.

The sign on a number with regards to the number line *simply represents* ***direction*.**

B) Properties of absolute value

Let x and y be any real numbers then

1. 
2. 
3. 

**Examples**

1. Determine the absolute value of a number.
2. 
3. 
4. 
5. 
6. Compare and order absolute values. Write the real numbers in order from least to greatest (ascending order)
7. 

*Solution*

Let’s first evaluate each number

Now we will express all numbers in decimal form (easier to compare values)

Arrange from least to greatest

Now return to original form

C) Evaluating absolute value expressions

Evaluate the following:

i) $\left|5\right|-\left|-7\right|=$

ii) 

iii) $\left|-3(6-15)^{2}+22\right|$



**Practice:** Evaluate the following:

$$\left|-4\right|-\left|-3\right|$$

 $\left|-12+8\right|$

$$\left|12\left(-3\right)+5^{2}\right|$$

 $\left|-2(6-10)^{2}+20\right|$

Solutions:

a) 1 b) 4 c) 11 d) 12

**7.2 Absolute Value Functions**

**Absolute Value Function**

Let’s recall the definition of a **FUNCTION**

A) Function- A rule that assigns to every element ***x*** of a set a unique element ***y*.**

Often written as 

 

B) Absolute Value Function – A function that involves the absolute value of a variable

 

C) Compare linear functions with corresponding Absolute Value Functions:

* Complete the table of values below for $f\left(x\right)= x$ and $g\left(x\right)= \left|x\right| $

|  |  |  |
| --- | --- | --- |
| ***x*** | ***f(x)*** | ***g(x)*** |
| -3 | -3 |  |
| -2 | -2 |  |
| -1 | -1 |  |
| 0 | 0 |  |
| 1 | 1 |  |
| 2 | 2 |  |
| 3 | 3 |  |
|  |  |  |

* Use the coordinates to sketch the graphs of the functions:

f(x)

g(x)

 

* Compare and contrast the two graphs
* From the graph, explain why the absolute value relation is a function.



The x-intercepts are the same for both functions (0,0). This is an **invariant point**.

The graph shows how $y=\left|x\right|$ is related to the graph of *y=x*. Since $\left|x\right|$ cannot be negative, the part of the graph of *y = x* that is below the x-axis is reflected in the x-axis to become the line *y = -x* in the interval x < 0.

D) Piecewise Function Notation

A function is said to be piece wise if it is composed of **two or more** separate functions.

* Each separate function has its own specific domain.
* The combination of all these functions make up the overall (piecewise) functions.

In general, we can express the absolute value function as the piecewise function

  + $f(x)$

 $-f\left(x\right) $

**Practice.** Consider the absolute value function $y=\left|x-4\right|$

1. Determine the x and y intercepts.



1. Sketch the graph:
* plot the intercepts
* build a table of values

c. State the domain and range.

d. Express as a **piecewise** function.

E) Compare quadratic functions with corresponding Absolute Value Functions:

* Complete the table of values below for $f\left(x\right)=x^{2}-4$ and $h\left(x\right)=\left|x^{2}-4\right|$:

|  |  |  |
| --- | --- | --- |
| ***x*** | ***f(x)*** | ***h(x)*** |
| -3 |  |  |
| -2 |  |  |
| -1 |  |  |
| 0 |  |  |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |

* Use the coordinate grid below to sketch the two graphs:



h(x)

f(x)

* Compare and contrast the two graphs
* If you could sketch the graph of *h(x)* using two quadratic functions, what would they be?

**Practice.**  Consider the absolute value function $f\left(x\right)=\left|x^{2}-x-2\right|$

a. Determine the x and y intercepts.

b. Sketch the graph.

c. State the domain and range

d. Express as a piecewise function.

**7.3 Absolute Value Equations**

A) **Absolute Value Equation**

* An absolute value equation is an equation that includes the absolute value of an expression that involves a variable.
* They can be solved algebraically or graphically.

When solving these equations, we treat absolute value symbols in the same manner as we treat brackets ( )

\*\*Note\*\* We can apply the definition of absolute values to expressions as well

Example: This could be: **+** (3x – 2)

  Or it could be: **-** (3x – 2)

Then can simplify this by removing the brackets:

 3x – 2

  - 3x + 2

So when solving an absolute value equation algebraically, we have **two cases to consider**:

**CASE I**

The expression inside the absolute value symbol is positive or zero

**CASE II**

The expression inside the absolute value symbol is negative

After solving our absolute value equation algebraically, we must examine our solution (roots). We must **verify** our roots through substitution into our original equation and conditions.

* Roots that do not satisfy our original equation are called \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_

\*\*Note\*\* Does the absolute value function of the form where  have a solution?

**Examples**

1. Solve 

Algebraically



Case I (+)

 Add 7 on both sides

 Divide by 2



Case II (-)

 Use distributive property

 Subtract 7 from both sides

 Divide by -2



We must verify our answers  and  by substituting into our original equation

 

For  For 

 

 

 

 

Yes L.H.S = R.H.S. Yes L.H.S = R.H.S.

Therefore, our solution is or 

ii) Solve an absolute value equation with **an extraneous solution**

Solve 

iii) Absolute value equation with **no solution**

Solve 

Solution

Algebraically



 STOP! Is this statement ever true? No!

Therefore, the solution for this equation is the empty set or  or NO SOLUTION

**Assignment: Pg. 389 # 2, 4, 5**

iv) Solve an absolute value **problem**

A manufacturer rejects 275g boxes of animal crackers when the actual mass of the box differs from the stated mass by more than 3.5 g. Model and solve an absolute value equation for the maximum and minimum mass, m, of animal cracker boxes for this manufacturer

Solution

**Graphically** (MATH) – NUM – 1: ABS(

Again, we can solve any absolute value equation graphically using our **calculator.**

There are two ways to do this:

*Option 1:*

Solve   ---means round to the nearest hundredth

Put into calculator - into  and into 

And then determine the intersection points using Intersection function

2nd Trace #5 Intersect Enter

Scroll cursor using the arrow keys until cursor is as close to intersection point as possible, then press Enter 3 times

Repeat process for all points of intersection.

Our answer is **ONLY** the x coordinates, therefore, x = -1.54 and x = 4.54

Option 2

 Get equation = 0



Put into calculator - into  and into (not necessary…it’s the x-axis)

Solutions are again the **points of intersection**, (-1.541381 , 0) and (4.5413813 , 0)

Our answer is again only the x values

Therefore, x = -1.54 and x = 4.54

**Practice (Graphically):**

These are done entirely on your calculator. This is helpful when you are dealing with 2 absolute value expressions in one equation.

#  Don’t forget to close your brackets!

Find the intersection of the two graphs. 2nd, Calc 5: Intersection

 1) 

 2) 

 3) 

 4) 

 5) ****

 6)

**In summary:**

The graph of an absolute value function 

We can analyze graphically by examining

* x intercepts (let and solve for x)
* y intercepts (let  or  and solve)
* maximum or minimum values
* domain (the set of all values for the independent variable “x” in our function)
* range (the set of all values for the dependent variable “y” in our function)

How the graph of relates to 

Domain- The domain for  is the same as the domain for 

Range- The range for will depend on the range for 

When comparing the graphs of  and we often have points that remain the same for both and . These are called Invariant Points.

*Let’s review some of the functions on our calculator*

Anything in a  denotes a key on our calculator

* Absolute Value Symbol -  Num #1: ABS ( 
* To input a function into our calculator that is to be graphed, be sure the function is in y = form or f(x) = form

 type function into 

 Type 2nd function into , etc…

* Adjust window settings 

Default window settings (the window settings that our calculator reverts to when memory is cleared)



scale

scale

max

max

min

min

* Table of Values 
* This function allows us to determine
	+ - y value given x value #1 : value
		- any zeros #2 : zeros
		- minimum of graph #3 : min
		- maximum of graph #4 : max
		- intersection points of graphs #5 : intersection

\*\*Be sure to know how to use your calculator, it is a very powerful tool (if used properly)\*

**7.4 Reciprocal Functions REPLACE WITH ABS VALUE WORKBOOK SECTION**

When the reciprocal function is 

\*\*\*Note\*\*\* 

Inverse

Function

Reciprocal

Function

What about the points?

Original Reciprocal

 

  So the value stays the same and we take the reciprocal of the y value

What about the graphs?

|  |  |
| --- | --- |
| Original Function  | Reciprocal Function  |
|  (value of ) | Value is undefined and vertical asymptote exists |
| Value of  | Values is  |
| Function > 0 (positive) | Function > 0 (positive) |
| Function < 0 (negative) | Function < 0 (negative) |
| Value of function ()**increases** over a certain interval | Value of function () **decreases**  over the same interval |
| Value of Function ()**decreases** over a certain interval | Value of function () **increases** over the same interval |
| Linear Graph  | Invariant points occur when   |
| Quadradic GraphIe.  | Invariant points when Ie. are invariant points \*\*\*\*\*\*\*\*\* |

1. **Asymptotes**

An asymptote is a line that a curve approaches but never touches.

We have various types of asymptotes, two of which are:

1. *Vertical Asymptote*

For reciprocal functions, the vertical asymptotes occur at the non permissible values of the function.

The line *x=a* is a vertical asymptote if the curve approaches the line more and more closely as

 x approaches a.

The curve (graph) will never cross a vertical asymptote.

Why?

Because the vertical asymptotes occur at x=a which are all

undefined values for the function.

The vertical asymptotes for will occur at the

x intercepts of 

Why?

Because the coordinates of x intercepts are for 

And when  our new point(s) for the reciprocal transformation 

are going to be  is undefined therefore, vertical asymptote

1. *Horizontal Asymptote*

A horizontal asymptote describes the behavior of our graph for very large and very small values of x.

The line is a horizontal asymptote if the values of the function approach b for very large positive or negative values of x.

1. **Invariant Points**

When comparing an original graph to a transformation (a new graph)

Ie.  and the invariant points are points that do not change. A point that remains unchanged after a transformation has occurred.

Invariant points for a reciprocal functions (linear and quadratic) occur when  so to determine the x values of the invariant points of we simply solve 

**A further comparison of  and **

We will use the simple comparison of  and 

|  |  |  |
| --- | --- | --- |
| **Characteristic** |  |  |
| **Domain** |  |  |
| **Range** |  |  |
| **End Behaviour** | As x becomes a very large positive value, y also becomes a very large positive valueAs x becomes very small, y also becomes very small | As x becomes very large, y approaches zero from aboveAs x becomes very small, y approaches zero from belowHorizontal asymptote at  |
| **Behavior at x = 0** |  | UndefinedTherefore, vertical asymptote at  |
| **Invariant Points** |  |

X

Y

-10

-9

-8

-7

-6

-5

-4

-3

-2

-1

1

2

3

4

5

6

7

8

9

10

-10

-9

-8

-7

-6

-5

-4

-3

-2

-1

1

2

3

4

5

6

7

8

9

10

0

y = x

X

Y

-10

-9

-8

-7

-6

-5

-4

-3

-2

-1

1

2

3

4

5

6

7

8

9

10

-10

-9

-8

-7

-6

-5

-4

-3

-2

-1

1

2

3

4

5

6

7

8

9

10

0

y = 1/x

**Examples**

1. Graph the reciprocal of a linear function.

Consider ****

1. Determine its reciprocal function ****
2. Determine the equation of the vertical asymptote of the reciprocal function
3. Determine any invariant points
4. Graph the function  and its reciprocal function ****

Solution

1.A. The reciprocal of ****simply ****

B. Vertical asymptotes occur where function is undefined (non-permissible values) N.P.V.’s or at x intercepts of 

To determine N.P.V.’s we set our denominator equal to zero and solve for x



1. Invariant points occur where 

 

Solve



D) Graph of and ****



* Graph ****and **** on calculator
* Use the table ie. 2nd Graph to show that the y-co ordinates of ****simply the reciprocals of the y-coordinates of ****
* Also use Table to show that x intercepts of ****become asymptotes on **** (table doesn’t work use intersection method)
* Invariant points occur where

** **

* Domain  
* Range  
1. Sketch **the reciprocal** of a quadratic function, $y=x^{2}-4$

**Absolute Value Review**

Section 7.1 Absolute Value

1. In your own words give a definition of absolute value.
2. Evaluate
	1. 
	2. 
	3. 
	4. 
	5. 
	6. 
	7. 
	8. 
	9. 
3. Evaluate
	1. 
	2. $\left|\left|3^{2}-2^{4}\right|+\left|-11-5\right|-2\right|$
	3. 

Section 7.2 Absolute Value Functions

1. Consider the function and 
	1. Sketch the graph of each function on the same coordinate grid
	2. Determine the Domain and Range for both f(x) and g(x)
	3. How do the graphs of f(x) and g(x) differ/same?
2. Write the following absolute value functions as piecewise functions
	1. 
	2. 
	3. 
	4. 
3. Do the graphs of all functions change when we take the absolute value of those functions?
4. The cross section of the sloping roof of a house is represented on a coordinate grid so that the points representing the bottom of the roof lie on the x-axis. The equation of the function describing the cross section is



Where h(x) meters is the height of the roof and x meters is the horizontal distance from the centre of the roof. What is the width of the bottom of the roof? What is the maximum height of the roof above the x-axis?

Section 7.3 Absolute Value Equations

1. Solve each absolute value equation algebraically, or graphically if there are 2 absolute value expressions (your choice).
	1. 
	2. 
	3. 
	4. 
	5. 
	6. 
	7. 
	8. 
2. Solve each absolute value equation algebraically, remembering to check for extraneous solutions.
	1. 
	2. 
	3. 
	4. 
	5. 
	6. 
	7. 
	8. 
	9. 
	10. 

10. The distance of the earth from the sun changes at different times of the year. The maximum and minimum distances of the earth from the sun can be represented by the equation



Where d is measured in millions of kilometers. Solve the equation to find the maximum and minimum distances of the earth from the sun.

11. An equation that describes the maximum and minimum temperatures at which a chemical compound is a liquid under normal conditions is



Where T is the temperature in degrees Celsius. Identify the chemical compound.

Section 7.4 Reciprocal Functions

1. Sketch the graphs of and on the same set of axes. Label the asymptotes, the invariant points and the intercepts.
2. 

b. 

Answers

$$\frac{5}{4}$$



b.21 c.18

4. A.

X

Y

-10

-9

-8

-7

-6

-5

-4

-3

-2

-1

1

2

3

4

5

6

7

8

9

10

-10

-9

-8

-7

-6

-5

-4

-3

-2

-1

1

2

3

4

5

6

7

8

9

10

0

Created with a trial version of Advanced Grapher - http://www.alentum.com/agrapher/

B. Domain and Range



C. The graph of g(x) is

* The graph of f(x) when 
* The graph of f(x) when 
* A reflection of f(x) about the x-axis when 

 

-4x + 1

-2x-1

6. The graphs of functions only change when we apply absolute value. If any part of the original function graph is below the x-axis, the part of the graph that is below the x-axis will then reflect over the x-axis.

7. The width of the roof is 8m. The height of the roof, above the x-axis is 4m

 



10. Maximum distance of the earth from the sun in millions of Kms is d =152

 Our minimum distance of the earth from the sun in millions of Kms is d=147

11. Water is the chemical compound

12.a. Asymptotes

 y = f(x) – None

 y = 1/f(x)– Horizontal y = 0 Vertical x = 2/3

 Invariant points occur at ( - 2 , 1) and ( -8/3 , -1)

 Intercepts y = f(x) – x intercept – (-7/3 , 0) y intercept – (0 , 7)

 y = g(x) – x intercept – (none) y intercept – (0 , 1/7)

 b. Asymptotes

 y = f(x) – None

 y = 1/f(x)– Horizontal y = 0 Vertical x = 8/5

 Invariant points occur at (4/5 , 1) and ( 12/5 , -1)

Intercepts y = f(x) – x intercept – (8/5 , 0) y intercept – (0 , 2)

 y = g(x) – x intercept – (none) y intercept – (0 , 1/2)