**UNIT 5 Chapter 8 SYSTEMS OF EQUATIONS**

**Review of Systems of Equations**

Solving a system of equations is finding a common point of 2 or more equations.

In Math 10 Common, we learned how to solve these systems 4 ways:
 1. graphically (drawing on grid paper)
 2. graphically (using our calculator)
 3. algebraically (using the substitution method)
 4. algebraically (using the elimination method)

When we solved for the system graphically, we put the 2 equations into the calculator and solved using the intersect function. (2nd CALC #5 Intersect)

**Ex 1. Solve the following system of equations using your calculator:**

 0 = 2x – y + 1

 8 = 6x – 2y

 Graph both of the lines in your calculator.
 To do this, we need to solve both equations for y.

 y =

 y =

 This system gives us one solution, which is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

Recall:

If the lines have different slopes, there is **one solution.**

If the lines had the same slope, there would be **no solution**.

If the lines were the same, there would be an **infinite number of solutions**.

**8.1 Solving Systems of Equations Graphically**

**What is the solution for the following system of equations:**

 y = 3x – 4

 y = x2 – 4

To solve, put both equations into your calculator and graph.

Find the intersection points. **ANS**:



Read Example #2 on Pg. 428 in the textbook. Complete “Your Turn”.

Solution?\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

How many solutions can a linear–quadratic system have?

**Ex. 3 Quadratic – Quadratic system**

 y = x2 – x – 3

 y = –3x2 + 3x + 21

We can now use the intersect function on the calculator to find both intersection points.

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ and \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Draw a sketch of the system above.

Do all parabolas intersect? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Consider

 

**8.2**  **Solving Algebraically**

When we solved systems algebraically last year, we learned 2 methods: substitution and elimination. We are going to be using the same concepts when we have a quadratic.

Let’s review substitution first.

When we did substitution, we followed a series of steps.

1. Use one equation and solve for one of the variables. Make sure you solve for the easiest variable. Look for the variable that has a coefficient of 1 if possible. Don’t do extra steps if you don’t have to!

2. Once you have your variable solved for, you substitute it into the other equation. You now have an equation with only one variable. You now have to solve for the variable in the equation using reverse BEDMAS.

3. Use your solution for the variable to find the other variable by substituting the answer into either one of the equations.

4. Write the solution as a coordinate (x, y).

**Ex 1.** x + 4y = 6

 2x – 3y = 1

**Ex 2.** $3x+4y=-4$

$$x+2y=2$$

**Ex 3.** $2x-4y=7$

$$4x+y=5$$

Now, let’s review elimination. **Elimination** had a series of steps where we manipulated entire equations by a coefficient.

1. Look at the 2 equations and see if you can add/subtract the equations to eliminate one of the variables. If you can’t eliminate a variable with the equations as they are, you multiply entire equations by a coefficient to create an equation where elimination can happen.

2. Once you have added/subtracted the equations and have eliminated one of the variables, you solve for the remaining equation using reverse BEDMAS.

3. Use your solution to find the other variable by putting your solution into one of the other equations that has the remaining variable.

4. Write your solution as a coordinate.

**Ex 1.** 3x + 2y = 19

 5x – 2y = 5

**Ex 2.**  $3x-4y=7$

 $5x-6y=8$

**Ex 3.** $\frac{2}{3}x-\frac{1}{2}y=4$

$$\frac{1}{2}x+\frac{1}{4}y=\frac{5}{2}$$

With quadratics, the method is the same but we have to solve for the variable by factoring your equation.

**Ex 4.** y = 3x2 + 6x – 10

 y = 12x – 1

***Each method has its advantages in different situations and you need to be familiar with the strengths of both to decide which to use. NOTE: If you use elimination to solve a system with a QUADRATIC, YOU MUST ELIMINATE THE \_\_\_\_\_\_\_\_\_\_\_ VARIABLE.***

Try this one, using elimination.

6x2 – x – y = –1

4x2 – 4x – y = –6

More Examples

1.
  **ANS**:

2.
  **ANS**:

3.
  **ANS**:

4.
  **ANS**:

5.
  **ANS**:

6.
  **ANS**:

7.
  **ANS**:

8. Round to the nearest hundredth. **Requires quadratic formula to solve.**
  **ANS**:

**Word Problems**

Now that we have the mechanical skills, we need to be able to use them to solve problems.

**Ex 1.** Skeet shooting requires a shooter to hit a projectile while in flight. The target is launched on a path defined by h(t) = –2t2 + 20t, where h(t) is the height of the projectile in meters and t is the time of the projectile in seconds, and the gun is shot along the line h(t) = 14t. After how many seconds does the shooter hit the target? How high is the projectile when it is hit?

**Read Example 5 Pg. 449 in the text.**

**Practice:**

a) A ball is hit in a baseball game and models the path **h = –0.1x2 + 2x.** The outfielder jumps to catch the ball and her path is modeled by the equation **h = –x2 + 39x – 378.** Let h be the height of the ball in metres above the ground and x by the distance the ball has travelled from home plate. What does the point of intersection represent in this situation?

**UNIT 5 Chapter 9 LINEAR & QUADRATIC INEQUALITIES**

**9.1 Linear Inequalities in Two Variables**

Reviewing **single** variable inequalities:

Instead of having an exact solution by using an equal sign, we can have a **range of solutions** that satisfy an inequality by using inequalities.

Inequalities are:

 < less than ≤ less than or equal to

 > greater than ≥ greater than or equal to

These are used in the same way as the equal sign with algebra with one exception:

 🢡**When multiplying or dividing by a negative number, the inequality sign flips.**

 This happens because when we change a number from positive to negative through multiplication or division, we jump to the opposite side of the number line.

**Ex 1.** 3x – 16 < 5x + 2

If we choose to move the x’s to the left side of the equation, we need to be aware of the change in the sign. If we move the x’s to the right side of the equation to keep x positive, we don’t have to worry about it.

 3x – 16 < 5x + 2 3x – 16 < 5x + 2

 –5x + 16 –5x + 16 –3x – 2 –3x – 2

 –2x < 18 –18 < 2x

 –2 –2 2 2

 x > –9 –9 < x

 We get the same solution either way.

This solution is also represented by a graph.

Since – 9 is not part of the solution (the x is not equal to –9), we represent this by having an open circle around the –9. If the inequality sign had the equals to, we would close the circle by filling it in.

 –10 –9 –8

Graphing Linear Inequalities with **2** variables

We know how to graph a line in y = mx + b form. To graph an inequality, we need to give a **range of solutions** **by shading** on one side of the line we create. So, you need to think about what the inequality means…

Look at the following graph.

We can see that **the border** between the shaded part and the non–shaded part is a line.

 y–intercept: –3 (b) slope: 1 (pick two points to determine the slope)

 So, the equation of the line is **y = x – 3.**

Now, the shaded part above the line is the solution that we want.

 How do we represent this? Is this y ≤ x – 3 or y ≥ x – 3?

Take a **test point** in the shaded area. Select an easy point to use like (0,0).
Which is correct? 0 ≤ 0 – 3 or 0 ≥ 0 – 3

 0 ≤ –3 No! 0 ≥ –3 Yes!

 So, we can now see that the inequality represented by the graph is **y ≥ x – 3.**

 (Noticed how we shaded above the line and y ≥)

What is the difference between ≤ and < ?

 ≤ or ≥ will be a solid line to show the line is part of the solution.

< or > will be a dotted line to show the line is a boundary where the actual line is **not** part of the solution.

**Ex.** **1** Graph the line y < –2x + 3

1) Graph the line y = –2x + 3. Solid or dotted?

2) Use a test point to determine the side to shade.



**Ex. 2** 5x – 20y < 0



Read Example 3 on page 469.
What is the equation of the graph in “your turn”?

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

More examples:

1.  2. 

One variable :

3. 6x – 2 < 22 4. −2x – 13 ≤ –19

**9.2 Quadratic inequalities in one variable** (Interval analysis)

Steps: (use test intervals/sign analysis)

 1. Find zeros (x-intercepts) These are **critical numbers** to make up your intervals.

 2. Choose any number from each interval and determine whether it makes the function **positive** or **negative.** The results represents the sign of the entire interval.

Examples:

 1. **x2 − 2x < 24** We first put everything on one side.

 x2 – 2x – 24 < 0 Next, we factor to find the **critical points**.

 (x − 6)(x + 4) < 0 This means we want where the function is NEGATIVE.

**Steps to solve problem:**

1. Factor the equation if needed, to get the equation’s factors.

2. Plot the equation’s key points on a number line. For this equation, plot –4, and 6.

3. Choose a point bigger than 6 (i.e. 7) and insert into the factored equation. All components are positive so the function is positive when x > 6.

4. Choose a point between –4 and 6 (i.e. 0) and insert in to the factored equation. The function will be negative so the function is negative when –4 < x < 6.

5. Repeat one last time with a number smaller than –4 (i.e. –5)



6

–4

 Test Point: –5 Test Point: 0 Test Point: 7

 (–5–6)(–5+4) (0–6)(0+4) (7–6)(7+4)

 ( – ) ( – ) ( – ) ( + ) ( + ) ( + )

 + – +

**ANS**: –4 < x < 6 (The function is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ between these x-values.)

If we want to check our answers, we can graph the function and see where it is below the x–axis.



We can see that it intersects the x–axis at –4 and 6 and is below the x–axis for that interval. Therefore, we are correct!

**Ex.** 5x2 – x + 3 > 3x2 – 7x + 11

Read example 3 on page 482 and complete “your turn”.

**9.3**  **Quadratic inequalities in 2 variables**

With linear inequalities we used test points and understanding of the inequality signs to determine which side of the graph was shaded. We are going to use the same techniques when solving for quadratic inequalities.

**Ex**. y ≥ –2x2 + 8 Factors are -2(x – 2)(x + 2)

 We know that this graph has a y – intercept of (0,8) and x – intercepts of (±2, 0)

 So we plot what we know and create our graph. Solid line because of >



Now, we can choose **test points** to determine if we shade above or below the graph.

To be sure, we can choose 1 point on the other side of the graph.

Example 2: **Given a graph**, create the inequality.



First, I would find the formula of the quadratic using the x and y –intercepts. If the x and y-intercepts are not ‘clear’, look for the vertex and use: y = **a**( x - p)2 + q

 x – intercepts happen at \_\_\_\_\_, \_\_\_\_\_, therefore y = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

 y – intercept happens at **(0, –6)** Use this to determine the value of ‘**a**’.

Write the equation of the line (curve) in general form:\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Now, which way should I write my inequality? Use the test point (0, 0) which is in the shaded area. Test both options:

Therefore, your equation to represent this inequality is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

Do “Your Turn” on page 492.

**Word Problems**

Now we have to look for what we are being asked to find in order to properly use our inequalities. Key words for inequalities are:

 Up to how much No more than

 At least Is at or above

 Up to Greater than or equal to

**Ex.** 1 The length of a rectangle is 2 cm greater than its width. The area of the rectangle is **at**  **least** 20 cm2. What are possible dimensions of the rectangle to the nearest centimetre?

 Define the variables:

 At least means we use which symbol?

 What is the inequality?

 Use your calculator to make a good sketch. Indicate the **intervals**.

**Ex.** 2 Two numbers are related in this way; three plus 2 times the square of one number is greater than one–half the other number. Graph an inequality that represents this relationship. Use the graph to identify three pairs of numbers that satisfy this relationship.

**Ex.** 3 The cross section of a pedestrian tunnel under a road is parabolic and is modeled by the equation y = –0.3x2 + 1.8x, where y meters is the height of the tunnel at a distance of x meters measured horizontally from one edge of the path under the tunnel. In 2011, the tallest living person was about 2.465 m tall. Could he walk through the tunnel without having to bend over?

0

4

6

2

-2

*y*

8

6

8

2

-2

*x*

4