**Unit 6 CH 1 SEQUENCES & SERIES**

**1.1 Arithmetic Sequences**

**I. Sequences.**

− set of numbers in some order.

− each number in a sequence is called a **term**

**II. Notation**

t1 = a = 1st term

t2 = 2nd term.

tn = nth term or general term (any term).

The sequence 8 , 13, 18, **. . .**  is an infinite sequence as there is no end to the number of terms.

The sequence 8, 13, 18, **. . .** , 63 is a finite sequence, since the last term is specified.

**III. Arithmetic Sequence.**

− sequence in which the difference (common difference) between consecutive terms is a

constant.

Examples:

1. Which of these are arithmetic sequences? What is d? (Adding/subtracting next term)

a. 2, 6, 12, 20,  **. . .**

b. 9, 13, 17, 21, **. . .**

c. 3, 6, 12, 24, **. . .**

d. −3.8, −1.7, 0.4, 2.5 **. . .**

e. 11, 5, −1, −7, **. . .**

**IV. Developing a formula**

7, 10, 13, 16, 19, **. . .**

**(**7 + 3) (7 + 3 + 3) (7 + 3 + 3 + 3) (7 + 3 + 3 + 3 + 3)

 ( + d) ( + 2d) ( + 3d) ( + 4d)

common difference

t 13 = 7 + (13 − 1)(3) ***tn =  + (n − 1)d***

number of

term

1st term

term

Examples:

1. Use a formula **to find specific term**.

a. 11, 18, 25, **. . .** Find t22 *tn =  + (n − 1)d*

b. **. . .** Find t18 *tn =  + (n − 1)d*

c. −5, −1, 3, **. . .** Find t10 *tn =  + (n − 1)d*

d. 2x, 5x, 8x, **. . .** Find t62 *tn =  + (n − 1)d*

2. Find a **simplified form of tn** (general term) for the following.

a. 6, 13, 20, **. . .**  b. 2, −3, −8, **. . .**

3. Find **the number of terms** in each.

a. 6, 10, 14, **. . .** , 170

b. 8, 1, −6, −13, **. . .** , −146

c. 

4. Find the missing terms in each of the following.

a) 29 \_\_\_\_ \_\_\_\_ \_\_\_\_ \_\_\_\_ −6 b) \_\_\_\_ \_\_\_\_ \_\_\_\_ 18.3, 17.2

(How many terms?)

c) \_\_\_\_ 19 \_\_\_\_ \_\_\_\_ 7 (How many terms?...a little tricky, here)

5. Given *t*16 = −18 and d = 4 for an arithmetic sequence, find *t1*.

6. a) Draw a graph for the following

arithmetic sequence: 2, 5, 8, 11, 14  **. . .**

b) Find *tn*

c) What is the slope of this graph?

d) What is the difference between *tn*= 3n − 1 and y = 3x − 1?

Is this a continuous or a discrete graph?

7. Determine x so that 2x + 3, 5x − 1 and 7x + 4 form an arithmetic sequence.

8. Given  = 27 and  = −5 find  and d.

9. **Question 23 on page 19.**

The Diavik Diamond Mine is located on East Island in Lac de Gras East, Northwest

Territories. The diamonds that are extracted from the mine were brought to surface when

the kimberlite rock erupted 55 million years ago. In 2003, the first production year of the

mine, 3.8 million carats were produced. Suppose the life expectancy of the mine is 20 years,

and the number of diamond carats expected to be extracted from the mine in the 20th year

is 113.2 million carets. If the extraction of diamonds produces an arithmetic sequence,

Determine:

a) the common difference.

b) what this value means.

10. Determine the number of multiples of 7 between:

a. 14 and 210 inclusive

b. −51 and 348

11. = −5 and  = 10. find and d.

**1.2 Arithmetic Series**

**Arithmetic series** is the **sum** of the terms of an arithmetic sequence.

In order to understand the formula, consider:

3 + 10 + 17 + 24 + 31 Find S5

S5 = 3 + 10 + 17 + 24 + 31

S5 = 31 + 24 + 17 + 10 + 3

2S5 = 34 + 34 + 34 + 34 + 34

2S5 = 5(34)

S5 = 

***n***  = number of terms;  = first term; ***tn*** = last term; ***d*** = common difference

**\*\* You use the first formula when you know the last term or have to find the last term **

Examples:

1. Find the following **sums**.

a) S34 for 14 + 20 + 26 + **. . .**

b)  for  **. . .**

**\*\*** c. 

2. Find the sum of −8 −4 + 0 + 4 + **. . .** + 72

3. Find the sum of 3 + 7 + 11 + **. . .** + 143.

4. Find the sum of all the multiples of 6 between 24 and 396 inclusive.

5. Given t1 = 17 and t38 = 128, find S53.

6. A stack of logs has 3 in the 1st row, 7 in the 2nd row, 11 in 3rd row and so on. If there are

271 logs in the last row, how many rows? logs?

7. The sum of the first 12 terms of an arithmetic sequence is 228. The common difference

is 4. Find the first 3 terms.

8. In an arithmetic series t1 = 11, tn =119 and Find the value of *n.*

9. In an arithmetic series  and . Find .

10. If in an arithmetic sequence, and , find and d.

11. In an arithmetic series where and , what is  and d?

12. In an arithmetic series, and , find  and d.

13. **Question 9 on page 28.**

A training program requires a pilot to fly circuits of an airfield. Each day, the pilot flies

three more circuits than the previous day. On the fifth day, the pilot flew 14 circuits. How

many circuits did the pilot fly:

a. on the first day?

b. in total by the end of the fifth day?

c. in total by the end of the nth day?

**1.3 Geometric Sequences**

**Geometric sequences:**  sequence of numbers where the **ratio** I **(**common ratio) between consecutive terms is a constant.

Eg. 5, 15, 45, 135, **. . .** , , ; r = 3

r = , etc

Look at 5, 10, 20, 40, **. . .**

5, , **. . .**

5, , **. . .**

t4 = 5(2)4 − 1 or 5(2)3

Therefore ****

***tn*** = term;  ***t1*** = first term; ***r*** = common ratio; ***n*** = number of the term

Examples:

1. Which are geometric sequences? Find , *r* and  for those that are.

1. 2, 4, 6, **. . .**

b. 4, 8, 16, 32, **. . .**

c. 5, −15, 45, **. . .**

d.  **. . .**

e.  **. . .**

2. Find the indicated term in each of the following geometric sequences.

a) *t6* for4, −12, 36 **. . .** b)  c) 

3. Find the missing terms in each of these geometric sequences:

a. \_\_\_\_ , \_\_\_\_ , \_\_\_\_, 56, 112, 224. B. 20, \_\_\_\_, \_\_\_\_, \_\_\_\_, 320

4. In a geometric sequence t2 = 48 and t5 = 162. Find a and r.

Let 48 be the first term 48 \_\_\_\_ \_\_\_\_ 162. Then 162 becomes the fourth term.

5. The value of a house **increases by 4%** each year. The value of a house now is $500 000.

a) What is the value of the house after: i. 1 year? Ii. 2 years?

b) Is this a geometric sequence? What is r?

c) What is the value of the house after 8 years (nearest dollar)?

d) After how many years will the house be worth at least $900 000?

6. **Question 10 on page 40.**

The colour of some clothing fades over time when washed. Suppose a pair of jeans fades by

5% with each washing.

1. What percent of the colour remains after one washing?

b) If *t1* = 100 (100%), what are the first four terms.

c) What is the value of **r** for your geometric sequence?

d) What percent of the colour remains after 10 washings?

e) How many washings would it take so that approximately 25% of the original colour

remains in the jeans?

**1.4 Geometric Series**

**Geometric Series** − is the **sum** of the terms of a geometric series.

Formulas for finding the sum of a geometric series are:



 = sum of n terms;  = first term; ***r***  = common ratio;  = the last term

Use the **second formula** when you know the **last term** or if you have to find the **last term** ().

Examples:

1. Which of the following series are geometric? Determine indicated sum for those that are. Round to the nearest hundredth, if necessary.

a) 5 − 30 + 180 − 1080 **. . .** *S7*

b) 3 + 5 + 7 + 9 + **. . .** *S13*

c) 4 + 6 + 9 + 13.5 + **. . .** *S9*

d) 27 + 18 + 12 + 8 + **. . .** *S8*

(exact)

2. Determine sums of the following geometric series.

a) 5 + 15 + 45 + **. . .** + 10 935

b)  (nearest hundredth)

3. In a geometric series *Sn* = 605, r = 3 and *tn* = 405. Find *t1*.

Since we know the last term (*tn)*, we can use the second formula).

4. 2 + 10 + 50 + **. . .** = 39 062. How many terms are in this series?

5. 9, 15, 3x + 7 **. . .**  form a geometric sequence. Find the value of x.

6. In a geometric series, r = −2, *S7* = 258. Find *t1*.

7. **Question 13 on page 55.**

An advertising company designs a campaign to introduce a new product to a metropolitan

area. The company determines that 1000 people are aware of the product at the beginning

of the campaign. The number of new people aware increases by 40% every 10 days during

the advertising campaign. Determine the total number of people who will be aware of the

product after 100 days.

**1.5 Infinite Geometric Series**

2 + 6 + 18 + 54 + **. . .**

   

Its sequence of partial sums is 2, 8, 26, 80, **. . .**

It is rather obvious that the sum of this geometric series gets larger and larger as the number of

terms increases. The sequence of partial sums does not approach a specific value. The

geometric series is therefore **divergent**.

****       

Its sequence of partial sums is 

As the number of terms increases the sequence of partial sums approaches a fixed value of 4.

This geometric series is **convergent** and its sum is **4**.

A geometric series will be convergent whenever **−1 < r < 1**

When **n** is infinite and **−1 < r < 1** then 

 *rn* = 0 for very large values of n.

 0.9200000 = 0 on calculator

Examples:

1. State whether the following infinite geometric series are divergent or convergent.

a. 

b. 2 + 2.4 + 2.88 + **. . .**

c. 54 − 36 + 24 + **. . .**

2. Determine the sum of these geometric series.

a. 

b. 

c. Show that,  = 0.72 + 0.0072 + 0.000072 + **. . .**

3.  Find *t1*. 4.  Find r.

5.  and  = 15. Find r.

6. 2 + 10x + 50x2 + **. . .** has a sum of 8. What is the value of x? r?

7. **Question 8 on page 63.**

In its first month, an oil well near Virden, Manitoba produced 24 000 barrels of crude.

Every month after that, it produced 94% of the previous month's production.

a. If this trend continues, what would be the lifetime production of this well?

b. What assumptions are you making? Is your assumption reasonable?

**Math 20-1**

**Chapter 1: Sequences & Series Review**

1. Which sequence has a common difference of −5?

a. −18, −23, −24 **. . .** b. 18, 23, 28 **. . .** c. −3, 7, −11 **. . .** d. 17, 12, 7 **. . .**

2. Which of the following is a geometric sequence?

a. 4, 11, 18, 25, **. . .** b. 3, −6, −12, 24, **. . .**

c.  d. 2y, 10y4, 50y8, 250y13, **. . .**

3. For the arithmetic sequence 7, 4, 1,  **. . .** find:

a. a simplified form of *tn* b.  c. 

4. In an arithmetic sequence *t14* = 96 and d = 5. Find *t1*.

5. Find the sum of the multiples of 5 between 13 and 538?

6. In an arithmetic series the first term is 25x2 and the last term is 85x2. If the sum of this

series is 715x2, find n, the number of terms.

7. In an arithmetic series  = 17,  = 128. Find .

8.  is a geometric series. Find:

a. r b. *t23* (±0.01) c. *S17* (±0.01)

9. Find the missing terms in each of the following geometric sequences.

a. \_\_\_\_\_, \_\_\_\_\_, 36, −108, 324 b. \_\_\_\_\_, 50, \_\_\_\_\_, \_\_\_\_\_, 25.6

10. Find the sum of 5 + 10 + 20 + **. . .** + 10 240 to the nearest tenth.

11. A photocopier was set to increase the dimensions of a drawing by 15%. If the increase was repeated until the final dimensions were at least 3 times as large as the original, how many times was the increase carried out?

12. Mrs. Plouffe makes a deal with her class of 28 students. She will give each student $100 on the

first day and increase this by $100 (for each student) each day for a 30-day period. In return the

class will give Mrs. Plouffe 1¢ on the first day, 2¢ on the second day, 4¢ on the third day and so

on for the same 30-day period. Who has made the better deal? Explain.

13. If  is written as an infinite geometric series what is:

a. *t1* b. r c.  as a fraction

14. Find the sum of  (exact)

15. If the sum of an infinite geometric series is −30 and , find *t1*.

16. An oil well produces 37 000 barrels of oil during its first week of operation. Its production decreases by 5% each week. If this trend continues what is the expected lifetime production of this well?

**ANSWERS:**

1. d 2. c 3. a. −3n + 10 b. −95 c. −938 4. 31 5. 29 203 6. 13 7. 5035

8. a.  b. 504.54 c. 356.49 9. a. 4, −12 b. 62.5, 40, 32 10. 20 475 11. 8 12. Mrs. Plouffe; Mrs. Plouffe would have to give $1 302 000 and the students would have to give $10 737 418.23.

13. a. 0.054 b. 0.01 c.  14. 54/5 15. −42 16. 740 000 barrels

**Textbook Review**

**Chapter 1 Review**: pp. 66 – 68

**Chapter 1 Practice Test**: pp. 69-70

**Math 20-1**

**Chapter 1: Sequences & Series Review (continued)**

1. How many integers are divisible by 3 between 27 and 1953 inclusive? (ANS: 643)

2. The sum of the first two terms of an arithmetic sequence is 9. The sum of the first six terms is 63. Determine the value of the first term *t1* and the common difference *d.* (ANS: t1 = 3, d = 3)

3. The sum of the first two terms of an arithmetic sequence is -14. The sum of the first seven terms is -154. Determine the value of the first term *t1* and the common difference *d.* (ANS: t1 = -4, d = -6)

4. A bacteria colony starts with a population of 2000, and increases at a rate of 12% per hour.

a.Determine the number of bacteria after 15 hours. (Round to the nearest whole number.) (ANS: 10 947 bacteria)

b. How many hours will pass before the bacteria population first reaches 1 000 000?

(ANS: 55 hours)

5. A car depreciates in value by 20% per year. If the car is now worth $40 000, how many years will pass before the car first becomes worth less than $10 000? (ANS: 7 years)

Series and Sequences Review Notes:

**Arithmetic Sequence** – A sequence in which the next term is formed by adding a constant (+/-) to the previous term.

**Geometric Sequence –** A sequence in which the next term is formed by multiplying the previous term by a constant.

A sequence can be regarded as a **function** relating the set of natural numbers to the terms of the sequence.

-**Domain** of the function is the set of natural numbers

-**Range** of the function is the set of terms of the sequence.

When graphed as a function, Arithmetic Sequences are **Linear** while Geometric Sequences are **Non-linear.**

The formula for the general term of an Arithmetic sequence is:

Where is the general term of the arithmetic sequence

is the first term

*d* is the common difference

*n* is the position of the term in the sequence

**Arithmetic Means –** the terms placed between two non-consecutive terms of an arithmetic sequence. Ex. 7, **14**, **21**, 28.

When the terms of an arithmetic sequence are added, the result is known as an **arithmetic series.**

The symbol is used to represent the sum of *n* terms of an arithmetic sequence.

The **formula** for the sum of n terms of an **arithmetic series** is.

If the common difference (*d*) is not known then the formula for the sum of n terms of an **arithmetic series** is…

You can use the terms from an arithmetic series to find the terms from a corresponding arithmetic sequence.

, *n*

A sequence where each term is obtained by **multiplying** the preceding term by a constant is called a **geometric sequence.**

The number you multiply to a term to get to the next term in a geometric sequence is called the **common ratio**. You can find it by using the formula…

, *n*

The formula for the general term of a geometric sequence is

\*note that a can be replaces with , they mean the same!!!

The terms placed between two non-consecutive terms of a geometric sequence are called **geometric means.** Example. 5, **10, 20,** 40

When the terms of a geometric sequence are added the result is known as a geometric series.

The formula’s for the sum of n terms of a geometric series are:

or

Where is the sum of the geometric series

*r*  is the common ratio of the series

*a* (or) is the first term of the series

An Infinite Series is **convergent** (i.e. the series approaches a particular value which we say is the sum of that series) **if the common ratio, *r*, is between -1 and 1.**

An Infinite Series is **divergent** (i.e. the series does not approach a particular value) **if the common ratio, *r*, is greater than 1 or less than -1.**

If *r=,*  the series does not converge to a particular value.

**Series and Sequences Formula Sheet**

**Arithmetic Sequences and Series**

**Geometric Sequences and Series**

, r ≠ 1

, r ≠ 1

,