**Unit 1 Chapter 3: Quadratic Functions**

**Math 20-1 UNIT 1, CH 3: QUADRATIC FUNCTIONS**

**3.1 Investigating Quadratic Functions in Vertex Form**

Open your textbooks p. 143 and with a partner complete the *Investigate Graphs of Quadratic Functions in Vertex Form* questions.

**A. The graph of quadratic function is a \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.**

*1) Look at the following graphs:*



Direction of opening:

A:

Minimum or maximum value: (coordinate)

A:

**A**

**B**

B:

Axis of symmetry (equation):

A:

B:

*2) What conclusions can be made about the graph?*

A graph that has a maximum value:

A graph that has a minimum value:

To write the equation for the axis of symmetry we\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

*In order to describe a quadratic function, we need to know some key elements:*

* \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
* \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

These elements are easy to identify given a graph. Too easy…let’s move onto equations!

**B. Vertex form:** $f\left(x\right)= a\left(x-p\right)^{2}+q$

Using *technology* graph:

(Use regular window settings: x: [-10, 10, 1] y: [-10, 10, 1] (ZOOM 6: ) )

a) **y = -(x – 4)² - 3**

*Looking at your graph, determine the following:*

1. Coordinates of the vertex:
2. Equation of the axis of symmetry:
3. Maximum or minimum value:
4. Domain and range:
5. Coordinates of the y-intercept:

*What connections can you make between the answer we found and the equation for the graph?*

* Vertex:
* Axis of symmetry: x =
* Maximum/ minimum & value:

b) Given $y=\frac{1}{2}\left(x-2\right)^{2}-4$, find:

1. Coordinates of the vertex:
2. Equation of the axis of symmetry:
3. Maximum or minimum value:
4. Domain and range:

c) Given vertex (-3, -7), a = 2, parabola opens down, what is the equation?

**3.1 Investigating Quadratic Functions in Vertex Form Part II**

1) Given the equation $y=2\left(x+1\right)^{2}-3$**,**

* Explain how this graph will differ from the basic equation $y=x²$
* If we wanted to manually graph $y=2\left(x+1\right)^{2}-3$, what steps could we use?
* How does the value of ***a*** affect the graph?

2) $y=x^{2}$ is the original (basic) parabola. What transformations have occurred to give:



 $y=2x^{2}$

$$y=2(x+1)^{2}-3$$

3) Determining a Quadratic Function in Vertex Form Given Its Graph:



Use the form: $f\left(x\right)= a\left(x-p\right)^{2}+q$ as our base.

1. What is the most important piece of information about the parabola?

*Vertex:*

1. Where does this information fit into the equation?

1. What piece of information is missing?
2. How can we find this ‘a’ value?

Find the equation of the following graph:

4) Determining the number of x-intercepts using ***a*** and ***q***:

Determine the number of x-intercepts a quadratic function has by examining:

* The value of ***a*** to determine if the graph **opens upward** or **downward**
* The value of ***q*** to determine if the vertex is **above**, **below**, or **on** the x-axis

a) $f\left(x\right)= 0.5x²-7$

|  |  |  |  |
| --- | --- | --- | --- |
| Value of a | Value of q | Sketch the Graph | Number of x-intercepts |
| a > 0opens upward | q < 0vertex is  |  |  |

b. $f\left(x\right)= -2(x+1)²$

|  |  |  |  |
| --- | --- | --- | --- |
| Value of a | Value of q | Sketch the Graph | Number of x-intercepts |
| a < 0opens  | q = 0vertex is  |  |  |

c. $f\left(x\right)= -\frac{1}{6}\left(x-5\right)^{2}-11$

|  |  |  |  |
| --- | --- | --- | --- |
| Value of a | Value of q | Sketch the Graph | Number of x-intercepts |
| a < 0opens  | q < 0vertex is  |  |  |

5) Model Problems Using Quadratic Functions in Vertex Form

1. Suppose a parabolic archway has a width of 280 cm and a height of 216 cm at its highest point above the floor.



a) Write a quadratic function in vertex form that models the shape of this archway.

**

 b) Determine the height of the archway at a point that is 50cm from its outer edge.

**3.2 INVESTIGATING QUADRATIC FUNCTIONS IN STANDARD FORM**

 $f\left(x\right)=a(x-p)^{2}+q$ *vs* $f\left(x\right)=ax^{2}+bx+c$

1) Graph $y=-1x^{2}+4x+5$.

 a) Does the graph have any symmetry? If so explain the symmetry.

 b) What is the maximum or minimum value?

2) Graph the function $y=-x^{2}+4x+c$, substitute the values 10, 0, and -5 in for ***c***, pay particular attention on how the graphs change and describe any patterns.

3) Using the function $y=ax^{2}+4x+5$, substitute -4, -2, 1 and 2 in for ***a***, pay particular attention on how the graphs change and describe any patterns.

4) Can a=0? Why do you think that is?

5) Using the function$ y=-x^{2}+bx+5$, substitute 2, 0, -2 and -4 in for ***b***, pay particular attention on how the graphs changes and describe any patterns.

6) How can the ***b*** value be 0 but not the ***a*** value?

The standard form of a quadratic function is $f\left(x\right)=ax^{2}+bx+c$**,** where a ≠ 0.

* ***a*** determines \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
* ***b*** influences \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
* ***c*** determines \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

*Ex.1 For each quadratic function, identify the following:*

* direction of opening
* coordinates of the vertex
* max or min value
* equation of the axis of symmetry
* x and y intercepts
* domain and range

a)

* direction of opening
* coordinates of the vertex
* max or min value
* equation of the axis of symmetry
* x and y intercepts
* domain and range

b)

* direction of opening
* coordinates of the vertex
* max or min value
* equation of the axis of symmetry
* x and y intercepts
* domain and range

**B.** Analyzing a Quadratic Function

A diver jumps from a 3-m springboard with an initial velocity of 6.8m/s. Her height, *h*, in metres, above the water *t* seconds after leaving the diving board can be modeled by the function:

 $h\left(t\right)=-4.9t²+6.8t+3$

a) Label some important points on the graph of the function, rounding values to the nearest hundredth.



b) What does the y-intercept represent?

c) What maximum height does the diver reach? When does she reach that height?

d) How long does it take before the diver hits the water?

e) What domain and range are appropriate in this situation?

f) What is the height of the diver 0.6s after leaving the board?

Assignment up to #15

**C.** Writing Quadratic Functions to Model a Situation

*Farmer Boswell wants to build a rectangular pigpen using* ***96 meters*** *of fencing.*

a) Write a quadratic function in standard form to represent the area of the pigpen.

b) What are the coordinates of the vertex? What does the vertex represent in this situation?

c) Label the graph for the function



d) Determine the domain and the range for the situation.

e) Identify any assumptions you made in modeling this situation mathematically.

f) Complete #15 and 16 on page 177

**3.3 COMPLETING THE SQUARE**

Vertex form tells us:

Standard form tells us:

To convert vertex form $f\left(x\right)= a\left(x-p\right)^{2}+q$ to standard form $y=ax²+bx+c$ we would need to:

Ex. $y=\left(x-4\right)^{2}-11$

However to go from standard form to vertex form we need to do more intensive math:

a. $f\left(x\right)=x²+6x+5$

**Method 1**: Using Algebra Tiles

Select the tiles that coordinate with the equation:



Using the x² tile and x-tiles, create an incomplete square to represent the first two terms. Leave the single tiles aside for now:



To complete the square, fill in the incomplete area by adding nine zero pairs, the nine positive tiles complete the square and the nine negative tiles maintain equality:



Simplify the expression by removing zero pairs:



You can now express the complete the square form by reading the tiles:



$$y=\left(x+3\right)^{2}-4$$

b) $y=x²+8x-7$

Using Algebra tiles:

**Method 2**: Algebraically

1. group the first 2 terms: (re-write them ……………leave a space) re-write coefficient
2. inside the brackets, **add and subtract** the square of ½ of the coefficient of the x-term (adding 0)
3. group the perfect square trinomial (use brackets and bump out extra number)
4. rewrite the perfect square trinomial as the square of a binomial
5. simplify the constant terms

Ex. 1 $y=x²-8x+5$

Group the first two terms

 1) $y=\left(x^{2}-8x \right)+5$

8/2 = 4

42 = 16

Add and subtract the square of half the coefficient of the x-term

 2) $y=\left(x^{2}-8x+16-16\right)+5$

Group the perfect square trinomial

 3) $y=\left(x^{2}-8x+16\right)-16+5$

4 & 5) $y=\left(x-4\right)^{2}-11$

Rewrite as the square of a binomial

The standard and vertex forms both represent the same function. **The vertex form allows to gain more information about the graph.**



Ex. 2 Convert to vertex form algebraically:

1. group the first 2 terms
2. **add and subtract** the square of ½ of the coefficient of the x-term
3. group the perfect square trinomial
4. rewrite the perfect square trinomial
5. simplify the constant terms
6. $f\left(x\right)=x^{2}-12x+5$
7. $y=-2x^{2}-8x+11$

1. $y=-3x²-18x-24$

**3.3 COMPLETING THE SQUARE, part II**

Convert $y=-3x²-27x+13$ to vertex form:

How can we confirm that these two forms are the same?

Ex. Convert to vertex form $y=3x²+30x+41$

**Writing a Quadratic Model Function:**

A sporting goods store sells reusable sports water bottles for $8. At this price their weekly sales are approximately 100 items. Research says that for every $2 increase in price, the manager can expect the store to sell five fewer water bottles.

a) Represent this situation with a quadratic function.

$$R=$$

b) Complete the square in order to see the vertex. (Convert to vertex form.)

c) Draw a sketch and label the axis and vertex.

d) **Determine the maximum revenue** the manager can expect based on these estimates. What **selling** **price** will give that maximum revenue?