**5.1 WORKING WITH RADICALS**

**I. Radicals**

Radical – The root of a quantity

Where  is the radical sign

 r is the \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

 x is the\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

 n is the \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

 

Note –The index shows the type of root

 ie-  is a fourth root

* If there is no number in the index, the root is a **square** root
* If n = 2 we do not write a number for the index

ie-  is a square root, n = 2

**Entire Radical** - A radical  where r = 1, r ≠ 0

ie- 

**Mixed Radical**- A radical  where r ≠ 1, r ≠ 0

ie- 

**Like Radical**- When radicals have the *same radicand and the same index*

ie- 

n =

x =

n =

x =

Equal

n =

x =

Equal

n =

x =

**Adding and Subtracting Radicals**- we can only add or subtract like radicals



\*NOTE\* We may need to convert radicals to a different form (entire or mixed) before we can identify like radicals.

**Restrictions on Variables**

If a radical  represents a real number and has an even index (ie – n= 2, 4, 6, 8….) then the radicand “x” must be non-negative ( ie - )

**II. Properties of Radicals**

1. 
2. 

We can use the properties of radicals to

* Change radical forms (mixed entire)
* Simplify radicals

***A radical is in the simplest form if***

* ***The radical does not contain a fraction or any factor which may be removed***
* ***The denominator does not contain a radical***

**III. Examples**

1. **Converting Mixed Radicals to Entire Radicals**
2. 

We want to express 5 as a square root. The radicand “n” is equal to 2 so we must “square” 5 ()

1. 
2. 

\*Note\* For the Radicand in the original expression to be a real number, a ≥ 0 because the index is even (2).

1. 

We want to express 2b as a CUBE root, the radicand, n=3, so we must “cube” 2b…

Since n=3 is an odd number, we have no restrictions on our variable (b)

Try Practice #1, page4

1. **Express Entire Radicals as Mixed Radicals**

It is important to know your **perfect squares** and your **perfect cubes:**

**(Write these into your syllabus)**

 **Perfect Squares -** 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, etc.

 

 **Perfect Cubes** - 8, 27, 64, 125, 216 etc.

 

1. 

Solution

 because n=2, we look for the GREATEST perfect squares that will divide into 150

  (then simplify the radical)

 

1. 

Solution



Break up so we can express as a square

Use greatest perfect square factor

1. 
2. 

8 = 2 x 2 x 2

27 = 3 x 3 x 3

64 = 4 x 4 x 4

125 = 5 x 5 x 5

216 = 6 x 6 x 6

Solution

Because n=3 (cube root) we must look for perfect cubes



Complete Practice #2

**PRACTICE**

#1. Write each of the following as an entire or pure radical:

1. =\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ 6. =\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

2. =\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ 7. =\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

3. =\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ 8. =\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

4. =\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ 9. =\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

5. =\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ 10. =\_\_\_\_\_\_\_\_\_\_\_\_\_\_

11. =\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

12. =\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

13. =\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

14. =\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

#2. Write each of the following as a mixed radical in simplest form:

1. =\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ 8. =\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

2. =\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ 9. =\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

3. =\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ 10. =\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

4. =\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ 11. =\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

5. =\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ 12. =\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

6. =\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ 13. =\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

7. =\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ 14. =\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

15. =\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

16. =\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

17. =\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

18. =\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**#1 #2**





**C**. **Adding and Subtracting Radicals** – Simplify the following.

Remember we can only combine radicals through addition and subtraction, if the

**radicands and the indexes are equal** (sometimes we need to make some manipulations)

Note:  (add/subtract the **coefficients only**)

r = 2 (Indexes are equal)

a = 2 (Radicands are equal)

1. 
2. 









1. 

 \*\*\*All radicands are already simplified\*\*\*

 (rearranged to make it easier to find)





4. Simplify the following expressions – identify any restriction on the variables

1. 









 

**Practice:** Add and/or subtract. Express as exact values in simplest form (refer to page 2).

1. 

1. 

1. 
2. 
3. 
4. 

7. 

**D. Ordering Radicals**

Order each set of numbers from least to greatest (Ascending Order)

\*This can be done in various ways

 - Change to entire

 - Use calculator to get decimals

1.  ,  ,  ,  , 

 ,  ,  ,  , 

  ,  ,  ,  , 

  ,  ,  ,  , 

Smallest  ,  ,  ,  ,  Largest

**Practice**

Arrange in ascending order (Change to entire).

 1) $4(13)^{\frac{1}{2}} 8\sqrt{3} 14 \sqrt{202} 10\sqrt{2}$

 2)    9

**5.2 MULTIPLYING &DIVIDING RADICAL EXPRESSIONS**

**Part I – Multiplying and Dividing**

1. **Multiplying Radicals**

When multiplying radicals, we simply

* Multiply coefficients
* Multiply the radicands
* Simplify!!

\*Note: We can only multiply radicals with the same index

In general we have

 (outside x outside, inside x inside)

Where,  (k is a Natural Number)

  (a, b, m, n are real numbers)

DON’T FORGET!!!! If k is even then  (we can’t take an even root of a negative number using real numbers)

**Practice:**  Simplify (Multiplication). Express as exact values **in simplest form**.

1) $\sqrt{3}×\sqrt{5}$ 2) $\sqrt{3}×\sqrt{8}$ 3) $5\sqrt{2}×7\sqrt{3}$

4) $\sqrt{5}×2\sqrt{15}×\sqrt{21}$ *Think:* $\sqrt{5}×2\sqrt{5}\sqrt{3}×\sqrt{7}\sqrt{3}$ (at times, it’s easier to simplify early)

5) $-2\sqrt{2}×-3\sqrt{8}$ 6)  7) 

8)  9. 

Multiplication (Monomial × Polynomial & Binomial × Binomial)

 Multiplying with binomials.

1.  2.  3. 

4.  5.  6. 

7.  8. 

9.  10. 

11. 

1. **Dividing Radicals**

When dividing radicals, we simply

* Divide the coefficients
* Divide the radicands

Again….we can only divide radicals with the same index

In general, we have

  and 

 

If k is even then 

Restriction now because we cannot divide by zero

Simplify the following expressions:

$$\frac{\sqrt{68x^{2}}}{\sqrt{2x}}=\sqrt{\frac{68x^{2}}{2x}}=\sqrt{34x},x\geq 0$$

1) $\frac{12\sqrt{6}}{6\sqrt{2}}$ 2) $\frac{24\sqrt{54}}{6\sqrt{6}}$ 3) $\frac{5\sqrt{20}}{10}$

4)  5)  6)  7) 

8)  9)  10) 

\*\*Remember we can always check to see if our expressions are equal by using decimal approximations (our calculators)

**Part II Rationalizing Denominators**

When working with radicals we must remember to **always simplify** all radicals – this means that

* The coefficient (if a fraction) must be reduced
* If the denominator has a radical we must rationalize (get rid of radical)

**Rationalize** - To convert to a rational number without changing the value of the expression

\*\*Note\*\*

In math we have “tricks” we can always do

1. Add zero to an expression
2. Multiply an expression by + 1

Neither of these changes the value of the expression, only the “look” of the expression

***Rationalizing 2 types of expressions:***

**Type #1** An expression with a **monomial** denominator that contains a radical, we simply …

-multiply the numerator and denominator by the radical term (only) from the denominator



Equal to 1

*We are simply multiplying our expression by 1 in the form of a root over the same root.*

Examples:

 $\frac{\sqrt{2}}{3\sqrt{6}}$



**Type #2** An expression with a **binomial denominator** that contains a square root, we simply…

-multiply **both** the numerator and denominator by the CONJUGATE of the denominator

i.e.  are conjugates (Similar to difference of squares, where middle terms = 0)

***Rationalize:***



\*\* Note:  is the conjugate of \*\*



Again, we are only multiplying by “1”, so we are not changing the value, only the “look”.

**Practice**

 1)  2)  3)  4) 

 5)  (Two methods) 6) 

7)  8. 

Rationalizing **Binomial** Denominators

 1.  2. 

 3. (answer simplifies) 4. 

 5.  6. 

**5.3 RADICAL EQUATIONS**

Objective

To be able to:

* Solve equations involving square roots
* Determine the roots of a radical equation algebraically
* Identify any restrictions on the values for the variables
* Model and solve problem using radical equations

**I. Radical Equations**

Radical Equation – An equation with radicals that have variables in the radicands

i.e. 

 Variable in radicand Radical Equation

 

 Constant in radicand and no variable in radicand NOT a radical equation

Root- A root is the solution(s) to an equation

**When solving a radical equation we must remember to:**

1. Identify and state any restrictions on the variable
2. Identify all roots
3. Identify any extraneous roots

**Extraneous Root**- A number obtained in solving an equation that does not satisfy the initial restrictions on the variable.

Recall-

* To Eliminate a Square Root we simply raise both sides of the equation to the Exponent 2
* When multiplying or dividing both sides of an inequality by a negative number we must reverse our inequality symbol direction (Example #4 on page 21)
* To identify if a root is extraneous we simply substitute the value into the original equation and check for equality
* What would cause a restriction on a value for variables?
	+ We cannot divide by zero (Denominators 0)
	+ Radicands must be non-negative () if our index is even

**II. Examples** Here are a number of examples for you to examine or try.

**Radical Equations**

Examples (Algebraically):

 1.  **ANS**: 6 2.  **ANS**: 

 

 3.  **ANS**: 10 4.   **ANS**: 

 

**Examples (Graphically):**

 Get everything to one side of equation and then type into y =. Solve for x-intercepts (zeros)

 OR Enter both sides (Y1 and Y2) and find intercepts.

1.  **ANS**: 5.47

**OTHER EXAMPLES - check**

 1.  **ANS**: \_\_\_\_\_\_\_\_\_ 2.  **ANS**: 

 3.  **ANS**: \_\_\_\_\_\_\_\_ 4.  **ANS**: \_\_\_\_\_\_\_\_

 5.  **ANS**: \_\_\_\_\_\_\_\_\_\_

**Stating restrictions:**

1.  a) State the restrictions on *x*

b) Solve the radical equation

Solution:

1. Remember that for the radical to be a real number our radicand must be non-negative ()

Radicand in the radical equation  is

 Now solve for “x”

 Add 2 to both sides

 Divide by 3

As long as , our radical will be a real number.

1. Solve 
2. Identify any restrictions and solve 

**RECALL:**

We can solve a quadratic equation many ways:

-Factor (if possible

-Complete the square

-Quadratic formula 

**Radical equation with an extraneous root:**

1.  Solve and state restrictions
2. State restrictions and solve 

1. **Solving more complex equations with two radicals**

Solve 

Solution

 Isolate 1 square root

 Square both sides

 Foil

Simplify



Isolate square root

 Square both sides

 Simplify

 Equate to zero

 Solve using preferred method

 Factor

 Set each factor = 0. Solve for x

x = 5 and x = 1 are roots (solutions) to , 

Both x = 5 and x = 1 satisfy the restriction 

Now, check for extraneous roots:

**Practice** : Solve: $7+\sqrt{3x}=\sqrt{5x+4}+5$

Solve: $\sqrt{3+x}+\sqrt{2x-1}=5$

**Solve problems involving radical equations.**

**Ex 1** What is the speed, in metres per second, of a 0.4kg football that has 28.8 J of kinetic energy? Use the kinetic energy formula, $E\_{k}=\frac{1}{2}mv^{2}$, where $E\_{k}$ represents the kinetic energy, in joules; *m* represents mass, in kilograms; and *v* represents speed, in metres per second.

$E\_{k}=\frac{1}{2}mv^{2}$,

**Ex 2** The profit, *P*, in dollars, of a business can be expressed as $P=-n^{2}+200n$, where *n* represents the number of employees.

1. What is the maximum profit? How many employees are required for this value?
2. Rewrite the equation from a) by isolating *n.*
3. What are the domain and range for the original function?

**Math 20-1 Chapter 5 – Radical Expressions and Equations Review**

Section 5.1 Working with Radicals

1. Convert each mixed radical to an entire radical
	1. 
	2. 
	3. 
	4. 
	5. 
2. Convert each entire radical to a mixed radical in simplest form
	1. 
	2. 
	3. 
	4. 
	5. 
3. Simplify
	1. 
	2. 
	3. 
4. Simplify and state any restrictions
	1. 
	2. 
5. Order the following numbers from least to greatest (ascending order)



Section 5.2 Multiplying and dividing radical expressions

1. Multiply and simplify
	1. 
	2. 
	3. 
2. Simplify and identify any restrictions
	1. 
	2. 
	3. 
3. Rationalize each denominator
	1. 
	2. 
	3. 
4. Simplify
	1. 
	2. 
5. Rationalize each denominator – state any restrictions
	1. 
	2. 
	3. 

Section 5.3 Radical equations

1. Identify the values of x for which the radicals are defined. Solve for x
	1. 
	2. 
	3. 
	4. 
2. Solve each radical equation. State any restrictions on the variables
	1. 
	2. 
	3. 
	4. 

Word Problems

1. The period, P, of a pendulum is the time in seconds, that it takes to complete one back and forth swing. The period is related to the length, , in cm of the pendulum by the equation



* 1. What is the period of a pendulum with a length of 25cm
	2. What is the length, in meters, of a pendulum with a period of 4 seconds
1. The approximate speed of a car before it brakes suddenly and skids is a function of the length of the tire marks it leaves on the road. For a dry road, the equation of the function is



Where d is the length of the tire marks, in meters and s(d) is the speed of the car, in kilometers per hour, before the brakes are applied.

1. If a car travelling at 100km/h suddenly brakes, what is the length of the tire marks? (nearest meter)
2. If the length of the tire marks is 80m, what was the speed of the car before the brakes were applied suddenly? (nearest km/h)
3. Tsunami Waves

The speed of a tsunami wave in the ocean is related to the depth of the water by the equation

 

Where, s, is the speed of the wave in meters per second and d is the depth of the water in meters. What is the depth of the water, to the nearest meter, if the speed of a tsunami wave is 10m/s?

1. The geometric mean of two numbers is the square root of their product. The geometric mean of 2 and 8 is , which is or 4.
	1. Find the geometric mean of 9 and 16
	2. The geometric mean of 5 and another number is 15. Find the other number
2. The square root of three less than a number is 12. What is the number?
3. Near the surface of the earth, the velocity of sound in air, V kilometers per hour, is approximately related to the temperature, T degrees Celsius, by following the equation,



* 1. On the hottest day ever recorded in Canada, the air temperature reached 45°C at Midale and Yellowgrass, Saskatchewan. Find the velocity of sound in air at this temperature, to the nearest 10km/h.
	2. At what temperature, to the nearest degree Celsius, does sound have a velocity of 1200km/hr in air?

Answers



d. Y > 24