**UNIT 3 Chapter 6 – RATIONAL EXPRESSIONS & EQUATIONS**

**6.1 Rational Expressions:**

− any expression that can be written in the form  where a and b are polynomials

and b ≠ 0.

eg. , 

**I. Non–permissible values (NPV’s)**

− value(s) of the variable(s) that would make the denominator zero. The expression would

be undefined.

*….Why can we not divide by 0 (zero)?*

Let’s first define division as the inverse of multiplication

For example,  We can think of 2 as the value of “x” such that

Well let’s now divide 6 by 0.  (we are not sure what this is equal to)

So in terms of multiplication… 

What number multiplied by 0 (zero) = 6?

We know that any real number multiplied by 0 (zero) = 0 (zero)

So we have no possible number for y.

**Practice: Determine the NPVs.** (Factor denominator whenever possible.)

**1.**  **2.**  **3.** 

**4.**  **5.** 

**6.**  **7.** 

**8.** 

**9.** 

**10.** 

**11.** 

**12.** 

**More practice: List non–permissible values.**

1.  2. 

3.  4. 

5.  6. 

7.  8. 

9.  10. 

### ANSWERS

**1.**  **2.** y  0, x  −3 **3.** 3, 7 **4.** 0, 

**5.** 0  **6.**   **7.** −3,  **8.** 0, ±9

**9.**  **10.** none

**II. Simplifying Rational Expressions.**

**1. Factor and/or simplify first.**

**2. List NPV’s**

**3. Reduce common *factors*…...easy, as long as you know what a factor is.**

**Examples:**

A. 

Solution



Use factored denominator  to determine N.P.V.’s





So our simplified expression is 

B. 

Solution

1



Non-Permissible Values

 Both factors are equal



So, 

**Practice:**

1.  2. 

3.  4. 

5.  6. 

7.  8. ,

9.  10. 

**Simplifying to get −1.**

Additive Inverse is an inverse or opposite number, that when added to the original number gives a sum 0. If N is the original number then additive inverse is -N such as N+(-N)=0

If the numerator and the denominator are additive inverses of one another, the expression can be simplified to -1.

Additive Inverse:  i.e.

Which of the following simplify to −1?

**1 2 3 4 5**



**Practice**: Simplify:

1. 

2. 

3. 

Plus: 1.  2. 

**6.2 Multiplying & Dividing Rational Expressions.** Symbols for Multiply****

1. Factor first.

2. List NPV’s

3. Reduce factors (watch for additive inverses)**.**

A. Recall the rule for multiplying fractions



ie. 

*Remember*: 

**Example:**



Solution

 Factor First

Factored form before simplifying, therefore, determine N.P.V.’s

N.P.V.’s





simplify

Additional Examples:

1. 

2. (Note: additive inverse)



**Practice:**

1. 

2. 

3. 

4. 

**Note :** 

5. 

6. 

7. 

8. Is there a difference? 

9.



10. 

11. Find an expression for the area.





**6.3 Addition & Subtraction of Rational Expressions.**

**Beware of Negatives! Pay special attention to them!**

1. Factor. List NPVs

2. Simplify if possible.

3. Determine L.C.D. (Lowest Common Denominator)

4. Convert to equivalent fractions.

5. Add or subtract numerators and place over common denominator.

6. Reduce if possible. Leave denominator in factored form.

**Examples**

Add/Subtract rational expression with common denominators

Determine the sum or difference. Express each answer in simplest form. Identify all non-permissible values.

1. 

Solution

Be sure to put the expression in brackets to distribute the -1 properly



Don’t forget any N.P.V.’s

Factor

1. 

Solution

 L.C.D. – Our L.C.D. is  \*\*Whatever term we multiply the denominator by to

create the L.C.D. we MUST also multiply the numerator by the same term. (Reduce when possible)



With some practice, you can eventually re-write the expression using the LCD this way:

N.P.V.’s



**Practice: Monomial Denominators**

1. 

2. 

3. 

4. 

5. 

6. 

7. 

**Binomial Denominators**

1. 

2. 

3. 

4.  (Factor first!)

Reduce Before

5. 

Reduce After

6. 

7. 

8. 

**Complex Expressions**

**Remember :** 

EXAMPLE



N.P.V.’s



**6.4 Rational Equations**

N.P.V.’s

\*\*Only new denominators



Rational Equation – An equation that contains one or more rational expressions

ie.  Are all rational equations.

\*\*\*NOTE\*\*\*

Working with rational equations is much the same as working with rational expressions, however:

* In an equation, whatever operation we perform on one side (of the equal sign) we must also perform on the other side.
* In an equation, we can clear fractions (rational expressions) by multiplying each term by the lowest common denominator (L.C.D). You will find this extremely helpful!!

To solve a rational equation we simply,

1. Factor all expressions. Factor first. First step is to factor.
2. Identify and state any non–permissible values (N.P.V.’s)
3. Multiply each expression by the L.C.D. to clear fractions (denominators).
4. Simplify remaining expression.
5. Solve expression by isolating variable to one side of our equation.
6. Check our answers (roots) for extraneous roots

Recall **An Extraneous Root** is a number (value) obtained when solving an equation that does not however satisfy the initial restrictions on the variable or satisfy the original equation

**Examples**

A. Solve a rational equation.

Solve and state any non-permissible values.

1. 

Solution

 Factor completely. That’s the first step…

N.P.V.’s



 State N.P.V.’s

 Multiply each expression by L.C.D.:  and simplify to clear fractions



Simplify

Factor

Set each factor equal to zero and solve for *x*.



N.P.V.

1. 
2. 

**Practice:**

1.  2. 

3.  4. 

5.  6. 

7. , x ≠ 0 8. , x ≠ 0

9. 

10. 

11. 

12. 

13. 

**Extra review for solving equations. Solve for x. Answers in { }**

1. 7x − 13 = 2(5x − 4) + 13 {−6} 2. ax + m = rt 

3. 3x + cx = m  4.  

5.  {14} 6.  

7.   8.  

**Using a rational equation to model and solve a problem**

1. Jeremy is training for a bike race by biking to his friend’s house. Jeremy biked 144km to Sam’s house and then Jeremy biked 144km back to his own house. He biked twice as fast on his way to Sam’s house. His total time was 22 hours. What rate, to the nearest tenth, did Jeremy bike both directions?

1. Susan takes 5h to paint a standard double garage. It takes her father George 3h to paint the same standard double garage. How long will it take then to paint on standard garage if they work together?
2. A plane flew from Chicago to Edmonton at 750km/h. If it had flown at 600km/h, the trip would have taken an hour longer. What is the flying distance from Chicago to Edmonton?

**More Practice for Problem Solving.**

1. Two friends share a paper route. Alex can deliver the papers in 40 min. Jordan takes 50 minutes for the same area. To the nearest minute, how long would it take them if they worked together?

2. Jim walked 12 km. He then biked 5 times as fast for another 100 km. The total time for the trip was 8 h. Determine the rates for each activity and the time Jim spent doing each.

3. Sheldon and Simon leave Sherwood Park in opposite directions. Simon is going 18

km/h faster. In 6 hours they are 972 km apart. Find the distance that each travelled.

4. John goes from A to B in 10 h and B to A in 8 h. Rate going back was 15 km/h more

than going there. How far from A to B?

5. Sandra left Calgary driving east at 90 km/h. Paul left Calgary 2 hours later

(10:00 a.m.) at 110 km/h. What time did Paul catch up to Sandra?

**Review**

I. Find NPV’s.

1.  2. 

II. Simplify:

1.  2.  3. 

4.  5.  6. 

7. 

III. Add and/or subtract as indicated. Simplify answers.

1.  2.  3. 

4.  5. 

6.  7. 

**ANS:** I. 1. m = 2. x = 0, 6 II. 1.  2.  3.  4. 

5.  6. 6x 7.  III. 1.  2. 

3.  4.  5.  6. 

7. 

**Additional Review**

**Simplify:**

1.  2. 

3.  4. 

5.  6. 

7. 

**ANS:**  1.  2.  3.  4. 

5.  6.  7. 

**Review**

1. Simplify and state NPV’s.

a.  **ANS=**, x ≠ 0 b.   **ANS=**, x ≠ ±1

2. Multiple and /or divide.

a.  **ANS=** b.  **ANS=**

c.  **ANS=**y + 2 d.  **ANS=**

e.  **ANS=**2

f.  **ANS=**

3. Add and/or subtract.

a.  **ANS=** b.  **ANS=**

c.  **ANS=**

d. **ANS=**

4. Solve the following equations.

1.  **ANS=**–2 b.  **ANS=**−3

c.  **ANS=** d.  **ANS=**

e.  **ANS=**5