**Unit 1 Chapter 2 TRIGONOMETRY 10C**

How to set up your triangles:

Angles are always upper case ( etc.) and sides are always lower case (a,b,c).

is always opposite side c. is always opposite side b. is always opposite side a.

A

a

C a B

c

b

B

C

**2.1 The Tangent Ratio**

When we get into trigonometry, we need a way to identify which side is being talked about. To do this, we always label the sides **according to the angle that is being discussed**. The hypotenuse is always the side opposite to the right angle. We call the side opposite the angle being talked about the **opposite**. Now that the hypotenuse and opposite are labeled, the last side is called the **adjacent**.

opposite (o)

adjacent (a)

hypotenuse (h)

θ

opposite (o)

adjacent (a)

hypotenuse (h)

θ

For solving triangles, there are set ratios set up for Math (Sine, Cosine, Tangent). These are used to solve triangles. These ratios apply to all right angle triangles. The first ratio we will be discussing is Tangent or Tan.

The Tangent Ratio is:  (The TOA in **SOH CAH TOA**…next lessons)

The tangent ratio for Angle A is written as tanA.

Please remember the order:

**Tan(angle) = value**

(value is also referred to as a ratio)

We usually write the value of tan as a fraction.

Although the tangent of an angle is defined as a ratio, you can think of it as a number that compares the two shorter sides (or legs) of a right triangle. For TAN, this ratio is

A

B

C

5 cm

12 cm

Ex. Given the triangle:

Determine Tan A: Determine Tan C:

Once we set up the ratio, we can find the angle we are talking about by doing **2nd Tan** and your ratio.

**NOTE: YOU MUST BE IN DEGREE MODE TO GET THE CORRECT ANGLE!**

Refer to the last example. If we want to figure out the angle created by Tan A and Tan C, we would need to use . Determine the measures of the angles to the nearest tenth:

Examples: Find the tangent ratio **and** then the angle indicated to the nearest tenth (±0.1)

1. Tan B =

10 cm

B

1. m

2. Tan A =

A

18.9 cm

Tan B =

C

B

22.6 cm

3.

3.45 mm

Y

X

Tan X =

2.78 mm

Z

Tan Z =

What do you notice about the tan ratio when the angle is greater than 45⁰? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

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(This allows us to make predictions about the angles created by the tangent ratios when we are evaluating our answers.)

Ex. 1 A small boat is 95m from the base of a lighthouse that has a height of 36m above sea level. Calculate the angle from the boat to the top of the lighthouse. Express your answer to the nearest degree.

Ex. 2 A ladder leans against the side of a building with its base 4ft. from the wall and its top 10ft from the base of the building. What angle does the ladder make with the ground?

10ft

4 ft

When we are using angles in real life situations, we will be talking about **an angle of inclination**. This is the acute angle made with the horizontal line.

Angle of inclination

Ex. 4. A ladder is 5 feet from the bottom of a wall. The ladder reaches 12 feet up the wall.

a) How long is the ladder?

b) What is the angle of inclination of the ladder?

Ex. 3 Mandy is standing outside in the sunshine. She is 5.5 ft tall and casts a 7ft shadow. How long of a shadow will a tree that is 20ft high make at this same time of day? After determining the angle of inclination of the sun, use the angle to determine the length of the shadow cast by the tree.

5.5 ft

7 ft

x

20 ft

20

5.5

**2.2 USING THE TANGENT RATIO TO CALCULATE LENGTHS**

Ex. In the triangle below, find the length of BC (which is also known as ‘a’)

A

B

C

5.0 cm

27˚

Ex. 1 Determine the length of VX to the nearest tenth of a centimetre.

7.2 cm

42˚

V

W

X

Ex. 2 In triangle ABC, ∠B = 90˚, and ∠C = 30˚, what is the measure of ∠A?



Ex. 3 A surveyor wants to determine the width of a river for a proposed bridge. The distance from the surveyor to the proposed bridge site is 400m. The surveyor uses a theodolite to measure angles. The surveyor measures a 31˚ angle to the bridge site across the river. What is the width of the river, to the nearest metre?

31˚

400m

Proposed bridge

When we are finding angles or sides using a formula it is called **indirect measurement**. When we use a measuring tool to get a measurement, it is called **direct measurement**. So, we used indirect measurement often in mathematics, as there are some things in this world that are very difficult to measure using a tool.

Ex. 4

2.3 m

F

G

89°

2.7 m

3.8 m

C

D

E

x

Solve for x.

Ex. 5 The angle of elevation (angle of inclinations) to the top of a building is 39.8°. If you are 28 m from the building how high is the building?

39.8°

28

x

Ex. 6 A plane is descending at an angle of depression (this is like the angle of inclination – it is the angle measured off the horizontal) of 7°. If the altitude of the plane is 1400 m, what is the horizontal distance between the plane and the runway?

7°

x

1400

7°

Ex. 7 If a kite is 25 m high, how far horizontally is the kite from the person flying it if the angle of inclination of the sun is 40⁰?

25 m

40⁰

**2.4, 2.5 THE SINE AND COSINE RATIOS**

***Learning Outcome:*** *Learn to develop and apply the sine and cosine ratios to determine angle measures.*

Point A

Point B

6.6 km

297 m

How would you determine the angle of inclination from point A to point B?

In a right triangle, the ratios that relate each leg to the hypotenuse depend only on the measure of the acute angle, and not on the size of the triangle. These ratios are called the sine ratio and the cosine ratio.

A

B

C

The tangent, sine and cosine are called primary trigonometric ratios. The trigonometry means “three angle measure.”

A

C

B

4

3

5

Ex. 1 Write each trigonometric ratio

a) sin A b) cos A

c) sin B d) cos B

Ex. 2 Determine the measures of ∠K and ∠M to the nearest tenth of a degree. Angle N is a right angle.

M

K

N

3

8

Ex. 3 In the World Cup Downhill held at Panorama Mountain Village in BC, the skiers raced 3514m down the mountain. If the vertical height of the course was 984m, determine the average angle of the ski course with the ground.

984m

3514m

COMPLETE 2.4 ASSIGNMENT

Ex. 4 Determine the length of PQ to the nearest tenth of a centimetre.

P

Q

R

10.4 cm

67˚

Ex. 5 In right triangle PQR,  and PQ = 7.5cm. Calculate the lengths of RQ and PR to the nearest tenth of a centimetre.

Ex. 6 A pilot starts his takeoff and climbs steadily at an angle of 12.2˚. Determine the horizontal distance the plane has travelled when it has climbed 5.4km along its flight path. Express your answer to the nearest tenth of a kilometre.

**2.6 APPLYING AND CHOOSING THE TRIGONOMETRIC RATIOS**

**Solving a triangle** means that we calculate the measures of all the angles and all the lengths in a right triangle. We can use any of the three primary trigonometric ratios to do this.

The basic strategy to solve triangle is:

1. Choose one of the acute angles and label the sides opposite, adjacent and hypotenuse in relation to that angle. Choose wisely!

2. Decide on a trigonometric ratio that can be used to solve the remaining sides and angles.

Remember SOH CAH TOA and Pythagorean:

Ex. 1 Solve the following triangle:

12 cm

5 cm

A

B

C

Ex. 2 Solve the following triangle:

22cm

42˚

B

C

A

**2.7 PROBLEM SOLVING WITH TRIGONOMETRY**

Ex. Calculate the length of CD to the nearest tenth of a centimetre.

B

C

D

26˚

A

47˚

4.2cm

**Angle of elevation (inclination) and angle of depression:**

Both angles of elevation and angles of depression are always measured from the horizontal. The angle of elevation looks from the horizontal upwards:

Angle of elevation

And the angle of depression looks from the horizontal downwards:

Angle of depression

Ex. 1 From a height of 50m in his fire tower, a ranger observes the beginnings of two fires. One fire is due west at an angle of depression of 9˚. The other fire is due east at an angle of depression of 7˚. What is the distance between the two fires?

Ex. 2 A surveyor stands at a window on the 9th floor of an office tower. He uses a clinometer to measure the angles of elevation and depression of the top and the base of a taller building. The surveyor sketches this plan of his measurements. Determine the height of the taller building to the nearest tenth of a metre.

39m

31˚

42˚

Ex. 3 From the top of a short building, the angle of elevation to the top of a tall building is . The angle of depression to the base of the tall building is . The buildings are 35m apart. Find the height of the two buildings.

40°

56

y

35 m

x

z



Review \*\* Round all answers to the **nearest hundredth** unless stated otherwise.

I. Find x and/or y in each of the following.

1296

6.7°

x

1. 2.

37°

83

x

y

23

x

40

3. 4.

67.8°

16

x

x

41°

72°

53

5. 6.

33°

x

y

20

15

II. Solve the following problems.

1. From a lookout tower a fire is observed at an angle of depression of 6.7°. The fire is 1276 m from the base of the tower. How high is the tower?

2. A road in the mountains is inclined at 6°. You travel 253 m along the road. What is the increase in elevation?

3. Instead of doing his math homework, Marcus is flying a kite. The kite is 56 m high when the string makes an angle of 70° with the ground. How high is the kite when the string makes an angle of 50° with the ground. The length of the string is the same in both cases.

4. A 16 m wire is attached to the tops of two vertical poles. The longer pole is 19 m high. If the wire makes an angle of 67.8° with the longer pole, find the height of the shorter pole and

the distance between the two poles.

5. Jason is 28 m from the base of a building. He finds the angle of elevation to the top of the

building to be 35°. He also finds the angle of elevation to the top of a flagpole on top of the building to be 42°. How tall is the building? How tall is the flagpole?

1. A ladder makes an angle of 28° with a wall and reaches 7.8 m up the wall. If the ladder slides

1 m down the wall, what angle does it now make with the floor?

**(Challenge!)** 7. Curt finds the angle of elevation to the top of a building to be 29°. He walks 17 m directly towards the building and now finds the angle of elevation to be 37°. How high is the building?

**ANSWERS**

I. 1. 152.25 2. 137.92 3. 6.05 4. x = 35.10°, y = 32.73 5. x = 41.41°, y = 15.59° 6. 43.75

II. 1. 149.90 2. 26.45 m 3. 45.65 m 4. h = 12.95, d = 14.81 5. B = 19.61, F = 5.61 6. 50.33° 7. 35.64 m