MATH 10C

Unit 2

Chapter 4

ROOTS & POWERS

NAME:____

$\left(\sqrt[5]{32}\right)^3$

$\sqrt[3]{27}$

x^5

$25^{1/2}$

$3^{-4} = \frac{1}{3^4}$

$y^3 \times y^6 = y^9$

UNIT 2 Chapter 4: POWERS AND ROOTS

4.1 ESTIMATING ROOTS

Learning Outcome: Learn to explore decimal representations of different roots of numbers.

Since $3^2 = 9$, 3 is a square root of 9

We write: $3 = \sqrt{9}$

Since $3^3 = 27$, 3 is a cube root of 27.

We write: $3 = \sqrt[3]{27}$

Since $3^4 = 81$, 3 is a fourth root of 81.

We write: $3 = \sqrt[4]{81}$

The parts of a radical:

$$\sqrt[n]{a}$$

We know the following:

$\sqrt{4} = 2$ $\quad$ $\sqrt{81} = 9$ $\quad$ $\sqrt{225} = 15$

$\sqrt[3]{8} = 2$ $\quad$ $\sqrt[3]{216} = 6$ $\quad$ $\sqrt[3]{1331} = 11$

The above answers are **exact** because the **answers** (not the radical) **terminate.**

$\sqrt{\frac{16}{81}} = 0.4444 = 0.\bar{4} = \frac{4}{9}$ $\qquad$ $\sqrt[3]{\frac{343}{1728}} = 0.583333333 = 0.58\bar{3} = \frac{7}{12}$

This is also **exact** because a repeating decimal can be written as a **fraction.**

Type the above examples into your calculator. After entering the radical, covert into decimal form then hit MATH >1:>ENTER to convert to a fraction.

Rule: An answer is exact if it terminates and/or can be turned into a fraction.

Note: All terminating and repeating decimals can be turned into fractions.

Try these:

$\sqrt{5} = 2.23$ $\sqrt[3]{25} = 2.92$ These answers are approximate since the decimals continue forever and the answer can not be turned into a fraction.

Thus, an exact answer is NEVER rounded while an approximate answer must be rounded. If you are asked to state as an *exact answer,* it must either be a **terminating decimal, a fraction or a radical.**

Complete the table below (page 205) and #1-6, page 206

Radical	Value	Is the Value Exact or Approximate?
$\sqrt{16}$	4	Exact
$\sqrt{27}$	5.1962	Approximate
$\sqrt{\frac{16}{81}}$	$0.\overline{4}$	Exact
$\sqrt{0.64}$		
$\sqrt[3]{16}$		
$\sqrt[3]{27}$		
$\sqrt[3]{\frac{16}{81}}$		
$\sqrt[3]{0.64}$		
$\sqrt[3]{-0.64}$		
$\sqrt[4]{16}$		
$\sqrt[4]{27}$		
$\sqrt[4]{\frac{16}{81}}$		
$\sqrt[4]{0.64}$		

4.2 IRRATIONAL NUMBERS

Learning Outcome: Learn to identify and order irrational numbers.

These are rational numbers	These are not rational numbers
$\sqrt{100}$, $\sqrt{0.25}$, $\sqrt[3]{8}$, 0.5	$\sqrt{0.24}$, $\sqrt[3]{9}$, $\sqrt{2}$
$\frac{5}{6}$, $\sqrt{\frac{9}{64}}$, 0.8^2, $\sqrt[5]{-32}$	$\sqrt{\frac{1}{3}}$, $\sqrt[4]{12}$

1) How are radicals that are rational numbers different from radicals that are not rational numbers?

2) Which of these radicals are rational numbers? Which are not rational numbers? How do you know?

$$\sqrt{1.44},\ \sqrt{\frac{64}{81}},\ \sqrt[3]{-27},\ \sqrt{\frac{4}{5}},\ \sqrt{5}$$

Rational Numbers, Q a number that can be written as $\frac{a}{b}$, where $b \neq 0$.

All numbers that can be represented as a terminating or repeating decimal are in this group.

Irrational Numbers, $\overline{Q}$**:** is a number that <u>cannot</u> be expressed as a terminating or repeating decimal. **Irrational numbers are non-repeating decimals.** They cannot be expressed in the form $\frac{a}{b}$, where b≠0. Examples: $\sqrt{2} = 1.414213562...$

$$-\sqrt{7} = -2.645751311...$$

When an irrational number is written as a radical, **the radical is the exact value of the irrational number**; for example, $\sqrt{2}$ and $\sqrt[3]{-50}$. We can use the square root and cube root keys on a calculator to determine approximate values of these irrational numbers.

Can you name the most famous irrational number?

Ex. Tell whether each number is rational or irrational. Explain your reasoning.

a. $\sqrt{\frac{25}{9}}$ b. $\sqrt[3]{-30}$

Together, the rational numbers and irrational numbers form the set of real numbers.

Numbers can be classified using a series of nested sets:

REAL

Ex. Use a number line to order these numbers from least to greatest.

$$\sqrt{2},\ \sqrt[3]{-2},\ \sqrt[3]{6},\ \sqrt{11},\ \sqrt[4]{30}$$

Number Systems

Please identify the following

	N	W	I	Q	$\overline{Q}$	R
1. 7.13						
2. $\sqrt{49}$						
3. $\frac{3}{5}$						
4. 0.12122…						
5. $3\sqrt{27}$						
6. 0.3333…						
7. $\frac{0}{5}$						
8. $0.\overline{78}$						
9. $-\sqrt{-27}$						
10 $\sqrt{-3}$						
11. $\sqrt{11}$						
12. π						
13. $3\sqrt{16}$						
14. 0.234						
15. 23						
16. $-\frac{1}{5}$						
17. 0						
18. $\frac{-4}{8}$						

4.3 MIXED AND ENTIRE RADICALS

Learning Outcome: Learn to express entire radicals as mixed radical, and vice versa.

We can name the fraction $\frac{3}{12}$ in many different ways:

$$\frac{1}{4} \quad \frac{5}{20} \quad \frac{30}{120} \quad \frac{100}{400}$$

How do you show that each fraction is equivalent to $\frac{3}{12}$?

Why is $\frac{1}{4}$ the simplest form of $\frac{3}{12}$?

Just as with fractions, equivalent expressions for any number have the same value.

1) $\sqrt{16 \bullet 9}$ is equivalent to $\sqrt{16} \bullet \sqrt{9}$ because:

$\sqrt{16 \bullet 9} = \sqrt{144}$ and $\sqrt{16} \bullet \sqrt{9} = \sqrt{144}$

$= 12$ $\qquad$ $= 12$

2) Similarly $\sqrt[3]{8 \bullet 27}$ is equivalent to $\sqrt[3]{8} \bullet \sqrt[3]{27}$ because:

$\sqrt[3]{8 \bullet 27} = \sqrt[3]{216}$ $\qquad$ $\sqrt[3]{8} \bullet \sqrt[3]{27} = 2 \bullet 3$

$= 6$ $\qquad$ $= 6$

Multiplication Property of Radicals:

$$\sqrt[n]{ab} = \sqrt[n]{a} \bullet \sqrt[n]{b}$$

Where n is a natural number, and a and b are real numbers.

We can use this property **to simplify square roots** and cube roots that are not perfect squares or perfect cubes, but have factors that are perfect squares or perfect cubes.

1. $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

2. $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

3. $\therefore \frac{\sqrt{ab}}{\sqrt{ac}} = \sqrt{\frac{ab}{ac}} = \sqrt{\frac{b}{c}} = \frac{\sqrt{b}}{\sqrt{c}}$

Changing Form of Radicals (Entire (Pure) $\longleftrightarrow$ Mixed).

Remember your perfect squares 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, etc.

and perfect cubes 8, 27, 64, 125 etc.

A. Entire to Mixed **Important**

1. $\sqrt{20}$ $= \sqrt{4 \times 5} = \sqrt{4}\sqrt{5} = 2\sqrt{5}$
2. $\sqrt{72}$ ____________________
3. $\sqrt{363}$ ____________________
4. $\sqrt{98}$ ____________________
5. $\sqrt{150}$ ____________________
6. $\sqrt[3]{135}$ $= \sqrt[3]{27 \times 5} = \sqrt[3]{27}\sqrt[3]{5} = 3\sqrt[3]{5}$
7. $3\sqrt[3]{32}$ ____________________

How can you confirm your answer is correct?

Type in the question into your calculator and hit enter. Type your answer into your calculator and hit enter.

If the answers are the same, your answer is correct!

B. Mixed to Entire

1. $5\sqrt{2}$ $= \sqrt{25} \times \sqrt{2} = \sqrt{25 \times 2} = \sqrt{50}$
2. $6\sqrt{5}$ ____________________
3. $5\sqrt{200}$ ____________________
4. $7\sqrt{18}$ ____________________
5. $2\sqrt[3]{2}$ $= \sqrt[3]{8} \times \sqrt[3]{2} = \sqrt[3]{8 \times 2} = \sqrt[3]{16}$
6. $4\sqrt[3]{56}$ ____________________

C. Arrange in ascending order (Change to entire).

1. $5\sqrt{2}$, $\sqrt{51}$, $2\sqrt{13}$, 7, $3\sqrt{5}$ ________________________________

2. $3\sqrt{10}$, $3\sqrt{8}$, $4\sqrt{6}$, 9 ________________________________

D. Simplifying radicals. (Divide)

1. $\frac{\sqrt{10}}{\sqrt{2}} = \frac{\sqrt{2}\sqrt{5}}{\sqrt{2}} = \sqrt{5}$

2. $\sqrt{\frac{11}{4}} = \frac{\sqrt{11}}{\sqrt{4}} = \frac{\sqrt{11}}{2}$

3. $\frac{\sqrt{12}}{\sqrt{3}}$

4. $\frac{\sqrt{12}}{2}$

5. $\frac{\sqrt{6}}{\sqrt{2}}$

6. $\sqrt{\frac{2}{9}}$

7. $\frac{\sqrt{20}}{\sqrt{5}}$

8. $\frac{\sqrt{54}}{\sqrt{2}}$

4.4 FRACTIONAL EXPONENTS AND RADICALS

Learning Outcome: Learn to relate rational exponents and radicals.

Complete each table. Use a calculator to start with.

x	$x^{\frac{1}{2}}$
1	$1^{\frac{1}{2}} =$
4	$4^{\frac{1}{2}} =$
9	$9^{\frac{1}{2}} =$
16	$16^{\frac{1}{2}} =$
25	$25^{\frac{1}{2}} =$

x	$x^{\frac{1}{3}}$
1	$1^{\frac{1}{3}} =$
8	$8^{\frac{1}{3}} =$
27	$27^{\frac{1}{3}} =$
64	$64^{\frac{1}{3}} =$
125	$125^{\frac{1}{3}} =$

1) What do you think the exponent $\frac{1}{2}$ means ?

2) What do you think the exponent $\frac{1}{3}$ means?

3) What do you think $a^{\frac{1}{4}}$ and $a^{\frac{1}{5}}$ mean?

Powers with fractional exponents can be written as radicals in the form $x^{\frac{1}{n}} = \sqrt[n]{x}$, where n≠ 0. **When n is even, x cannot be negative**, since the product of an even number of equal factors is always positive.

Rational Laws of Exponents:

$x^{\frac{1}{a}} = \sqrt[a]{x}$ $x^{\frac{a}{b}} = \sqrt[b]{x^a}$ or $x^{\frac{a}{b}} = \left(\sqrt[b]{x}\right)^a$

Rational exponents.

$$x^{\frac{m}{n}} = \sqrt[n]{x^m} \text{ or } \left(\sqrt[n]{x}\right)^m$$

$\sqrt{\ }$ = radical sign

x = radicand

n = order or index (n = 2 is assumed if no index is given)

– **the square root of a number is the inverse operation to squaring a number.**

– **a positive number has 2 square roots. The square roots of 9 are ±3.**

I. Changing Forms

As mentioned above, $x^{\frac{m}{n}} = \sqrt[n]{x^m}$ or $\left(\sqrt[n]{x}\right)^m$. One way to think of this is as follows: The numerator of the exponent (m) is like a wing man. A wing man never leaves his partner's side. So notice how *(m)* stays with *(x)* even when we change forms.

1. Radical ⟶ Exponential

a. $\sqrt[3]{x^5} = x^{\frac{5}{3}}$ b. $\sqrt{y^5} = y^{\frac{5}{2}}$

c. $\sqrt{x}$ = __________ d. $\sqrt[5]{y^3}$ = __________

e. $\left(\sqrt{x}\right)^3$ = ____________________

f. $\sqrt[3]{8y^7}$ = ____________________

2. Exponential ⟶ Radical

Remember, the numerator of the exponent STAYS WITH THE BASE.

a. $x^{\frac{2}{7}} = \sqrt[7]{x^2}$ b. $y^{\frac{1}{5}} =$ ____________________

c. $y^{\frac{3}{2}} =$ ______________ (The denominator of 2 is assumed in the square root.)

d. $(2x)^{0.2} =$ ____________________ (Change 0.2 to a fraction.)

e. $\left(3x^3\right)^{\frac{2}{7}} =$ ______________________________

f. $7^{\frac{-1}{2}} =$ ______________________________

3. Use the formula $b = 0.01m^{\frac{2}{3}}$ to estimate the brain mass of each animal:

a) a moose with a body mass of 512 kg

b) a cat with body mass of 5kg

II. Evaluating Radicals and Exponents

Calculator Buttons

$\sqrt{}$	Hit 2nd x^2
$\sqrt[3]{}$	Hit MATH 4
Any root such as $\sqrt[4]{}$	4 MATH 5
$\sqrt[5]{}$	5 MATH 5
$\sqrt[6]{}$	6 MATH 5
etc.	

A. Evaluate to the nearest hundredth. Use your calculator.

1. $\sqrt[3]{73}$

2. $\sqrt[4]{25}$

3. $\sqrt[100]{15\ 230}$

4. $\left(\sqrt[9]{55}\right)^{-2}$

5. $4\left(\sqrt[5]{216}\right)$

6. $-\sqrt[5]{231}$

7. $-2\sqrt[3]{-58}-3\sqrt[5]{21}$

B. Evaluate. (Exact) Can all be done on your calculator.

After your get a decimal answer, convert to a fraction if possible.

1. $8^{\frac{2}{3}} = 8 \wedge (2/3) = 4$

2. $64^{\frac{3}{2}} =$ ____________________

3. $(-27)^{\frac{1}{3}} =$ ____________________

4. $-4^{\frac{-3}{2}} =$ ____________________

5. $81^{-1.25} =$ ____________________

6. $(0.0009)^{\frac{1}{2}} =$ ____________________

7. $\left(\frac{27}{8}\right)^{\frac{2}{3}} =$ ____________________

8. $\left(\frac{4}{9}\right)^{-1.5} =$ ____________________

9. $5^{\frac{2}{3}} \times 5^{\frac{1}{3}} =$ ____________________

10. $\left(\sqrt[6]{25}\right)^{3} =$ ____________________

11. $(0.008)^{\frac{-2}{3}} =$ ____________________

12. $32^{0.4} =$ ____________________

4.5 NEGATIVE EXPONENTS AND RECIPROCALS

Learning Outcome: Learn to relate negative exponents to reciprocals.

Two numbers with a product of 1 are reciprocals.

Since $4 \bullet \frac{1}{4} = 1$, the numbers 4 and $\frac{1}{4}$ are reciprocals.

Also, $\frac{2}{3} \bullet \frac{3}{2} = 1$, so the numbers $\frac{2}{3}$ *and* $\frac{3}{2}$ are also reciprocals.

We define powers with negative exponents so that previously developed properties such as $a^m \bullet a^n = a^{m+n}$ and $a^0 = 1$ still apply.

Apply these properties:

$$5^{-2} \bullet 5^2 = 5^{-2+2} = 5^0 = 1$$

Since the product of 5^{-2} and 5^2 is 1, 5^{-2} and 5^2 are reciprocals.

So, $5^{-2} = \frac{1}{5^2}$ and $\frac{1}{5^{-2}} = 5^2$

That is, $5^{-2} = \frac{1}{25}$

x^{-n} is defined to be the reciprocal of x^n, that is, $x^{-n} = \frac{1}{x^n}$ $(x \neq 0)$

An easier way to think of a reciprocal, the numerator and denominator switch spots :-)

What does this have to do with exponents? Well...

$x^{-a} = \frac{1}{x^a}$ and $\frac{1}{x^{-a}} = x^a$

A negative exponent in numerator becomes a positive exponent in the denominator.

A negative exponent in denominator becomes a positive exponent in the numerator.

1. Rewrite the following with **positive** exponents only.

a. x^{-3} = ____________________ b. $\frac{1}{m^{-4}}$ = ____________________

c. $4x^{-5}$ = ____________________ (The constant 4 is not affected by the exponent!)

d. $\frac{1}{7m^{-2}}$ = ____________________ (The constant 7 is not affected by the exponent!)

e. 9^{-8} = ____________________

f. $(4n)^{-3}$ = ____________________ (The four is part of the base.)

g. $(3x)^{\frac{-4}{5}}$ = ____________________ (The *3x* is part of the base.)

h. $\frac{5}{x^{\frac{-6}{7}}}$ = ____________________

i. $\frac{1}{-3^{-2}}$ = ____________________

2. **Evaluate:**

a) $2^{-1} + 3^{-1}$ =

b) $(3^2 + 5^{-1})^0$=

c) $\frac{2^{-1}+6^{-1}}{3^{-1}}$ =

d) $\frac{6+6^{-1}}{6-6^{-1}}$ =

e) $\frac{3^{-2}}{3^{-1}-3^{-2}}$ =

3. **Applications** of exponents.

a) The formula $D=\sqrt[3]{830t^3}$ is used to describe violent storms where D is the diameter of the storm in kilometers and *t* is the number of hours the storm will last. If a storm lasts 4 hours, what is its diameter? (± 0.1)

b) The skin area A, in square metres, of a person's body can be estimated from the formula $A=0.025h^{0.42}w^{0.5}$, where h is the person's height in centimetres and w the mass in kilograms. Estimate the skin area of Mr. Lazaruk if he is 180 cm tall and has a mass of 70 kg to the nearest 100th.

4.6 APPLYING THE EXPONENT LAWS

Learning Outcome: Learn to apply the exponent laws to simplify expressions.

Exponent Law Description	Algebraic representation
Product of Powers To multiply powers with the same base, add the exponents	$n^a \times n^b = n^{a+b}$
Quotient of Powers To divide powers with the same base, subtract the exponents	$n^a \div n^b = n^{a-b}, n \neq 0$
Power of a Power To determine the power of a power, multiply the exponents	$\left(n^a\right)^b = n^{ab}$
Power of a Product The power of a product is equal to the product of the powers	$\left(m \times n\right)^a = m^a \times n^a$
Power of a Quotient The power of a quotient is equal to the quotient of the powers	$\left(\frac{m}{n}\right)^a = \frac{m^a}{n^a}, n \neq 0$

NOTE: $(-2)^2 = 4$ $-2^2 = -4$

Work on your own:

What is the value of $\left(\frac{a^6b^9}{a^5b^8}\right)^{-2}$ when a = -3 and b = 2?

Compare your answer and strategies with a partner.

If you used the same strategy, find a different strategy.

Which strategy is more efficient, and why?

Ex. Simplify by writing as a single power.

a) $\left(\frac{2x^3y^2}{4x^2y^6}\right)^{-3}$

$=\left(\frac{x}{2y^4}\right)^{-3}$

$=\left(\frac{2y^4}{x}\right)^{3}$

$=\frac{8y^{12}}{x^3}$

First simplify the exponents in the original equation. Then use the negative exponent law to create a positive exponent. Then use the power law to simplify the expression.

b) $(3a^3b^{-2})(15a^2b^5)$

c) $(25a^4b^2)^{\frac{3}{2}}$

d) $\left(\frac{50x^2y^4}{2x^4y^7}\right)^{\frac{1}{2}}$

Ex. A cone with height and radius that are equal, has volume $18cm^3$. What are the radius and height of the cone to the nearest tenth of a centimetre?

$$V = \frac{1}{3}\pi r^2 h$$

More Practice Simplify. Positive exponents only in answers.

1. -2^4

2. $(-3)^2$

3. 7^0

4. -5^0

5. $(-3x^2)^0$

6. $7x^0 - 2^2$

7. $(3^{-7})(3^4)$

8. $\frac{x^5}{x^{-4}}$

9. $(3x^5y)(-2x^6y^{-4})$

10. $(-x^2)(5x^{-4})$

11. $\left(\frac{3}{4}\right)^{-2}$

12. $(-4p^3q^{-2})^3$

13. $\dfrac{-15x^6y^4}{3x^3y^7}$

14. $\left(\dfrac{10x^6y}{-2x^{-3}y^5}\right)$

15. $\dfrac{(-6)^0}{\left(-2a^4\right)^{-3}}$

16. $\left(\dfrac{45x^2y^3}{9xy^8}\right)^4$ **(Clean up bracket first.)**

17. $(-3x^{-1}y^2z)^{-3}$

18. $\left(\dfrac{3}{5x}\right)^{-2}$

ROOTS AND POWERS REVIEW

Section 4.1

1. Name the **precise** (smallest) set of numbers to which each number belongs : Natural, Whole, Integer, Rational, Irrational, Real or none of these.

a) $-\sqrt{144}$ __________ b) -5 __________ c) $\sqrt[3]{27}$ __________

d) 4.2 __________ e) 7.123123..... __________ f) $\sqrt{-10}$ __________

Section 4.2

2. Arrange the following radicals in order on the number line:

$$5\sqrt{5},\ 2\sqrt{6},\ 3\sqrt{7},\ 4\sqrt{10},\ 2\sqrt{12}$$

3. Evaluate. Round to the nearest hundredth as needed: (± 0.01)

a) $\sqrt[3]{64}-\sqrt[3]{-27}$ __________

b) $\sqrt{0.02359}$ __________

c) $\left(\sqrt{7^4}\right)^3$ __________

4. The formula T = $\sqrt[3]{\frac{d^3}{830}}$ is used to describe violent storms, where D is the diameter of the storm in kilometres, and t is the number of hours the storm will last. If a storm has a diameter of 37 km, how long will it last? Round to the nearest tenth of an hour.

____

5. Patricia is investing some money to help fund the rebellion. She knows that her Accumulated Amount (A) can be determined using the formula $A = P(1+i)^n$ where (i) is the interest rate as a decimal and (n) is the number of years she invests.

How much money will Patricia have (A) is she invests \$4444 (P), for 11 years (n) at an interest rate of 6.5%? . $(i = 0.065)$ Round to the nearest dollar.

____

Section 4.3

6. Express as a mixed radical.

a) $\sqrt{63}$ __________ b) $\sqrt{108}$ __________ c) $\sqrt[3]{135}$ __________

d) $\sqrt{175}$ __________ e) $\sqrt{96}$ __________ f) $2\sqrt[3]{24}$ __________

7. Express as an entire radical.

a) $4\sqrt{7}$ __________ b) $3\sqrt[3]{2}$ __________ c) $3\sqrt{3125}$ __________

d) $5\sqrt{6}$ __________ e) $5\sqrt[3]{2}$ __________ f) $3\sqrt[4]{2}$ __________

Section 4.4

8. Evaluate:

a) $\left(4^{-3}\right)^{\frac{2}{3}}$ ________ b) $(-7x)^0$ ________ c) $\left(\frac{81}{16}\right)^{\frac{5}{4}}$ ________

d) $13x^0$ ________ e) $\left(\frac{1}{9}\right)^{2.5}$ ________ f) $(-2)^4$ ________

g) $\left(\sqrt{125}\right)^4$ ________ h) -3^{-2} ________ i) $3^{\frac{6}{7}} \times 3^{\frac{1}{7}}$ ________

j) -4^2 ________ k) $3^{-2} - 3^{-3}$ ________

9. Express each power as a radical.

a) $4^{\frac{2}{3}}$ ________ b) $7^{\frac{1}{2}}$ ________ c) $81^{\frac{5}{4}}$ ________

d) $x^{\frac{m}{n}}$ ________ e) $\left(\frac{1}{9}\right)^{\frac{1}{9}}$ ________ f) $(-2)^{\frac{4}{3}}$ ________

10. Express each radical as a power.

a) $\sqrt{m^3}$ ________ b) $\sqrt{\left(m^5\right)^3}$ ________ c) $\sqrt[7]{5^3}$ ________

d) $\sqrt[3]{x^8}$ ________ e) $\sqrt[n]{8}$ ________ f) $\sqrt{y^{\frac{5}{3}}}$ ________

Section 4.6

11. Simplify. Answers must contain positive exponents only.

a) $(-2x^3y^3)(-5x^4y^2)$ ________ b) $\left(\frac{2x^3}{3y^{-2}}\right)^2$ ________

c) $(x^{-2}y^3)^{-2}$ ________ d) $3m^{-2} \times 4m^6$ ________

e) $(y^3)^0$ ________ f) $\frac{3ab^4}{2a^3b^2} \times \frac{12a^5b}{15ab}$ ________

g) $\left(\dfrac{m^3}{n^2}\right)^5$ ____________

h) $\dfrac{10x^{-2}}{-2x^{-3}}$ ____________

i) $(2x^4y^{-2})^{-3}$ ____________

j) $\left(\dfrac{6x^7}{2x}\right)^{-2}$ ____________

12. Evaluate. Exact answers where possible, otherwise, round to the nearest hundredth. (± 0.01)

a) $\left(3^{\frac{1}{6}}\right)\left(3^{\frac{5}{6}}\right)$ ________

b) $\left(\dfrac{36}{25}\right)^{1.5}$ ________

c) $\dfrac{6^{-2}}{36^{\frac{-1}{2}}}$ ________

d) $\left(\dfrac{2^3}{8^2}\right)^{\frac{2}{3}}$ ________

e) $\left(\dfrac{-64}{6^{\frac{1}{2}}}\right)^{\frac{4}{3}}$ ________

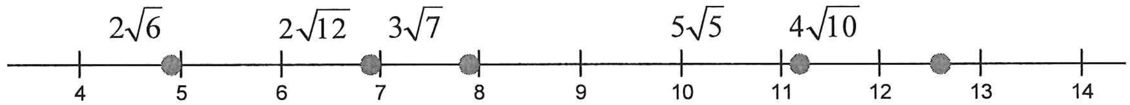
f) $\left(81^{\frac{3}{4}}\right)^{\frac{4}{3}}$ ________

13. Barb incorrectly simplified $\left(27x^4\right)^{\frac{2}{3}}$ as $18x^{\frac{8}{3}}$. What error did she make? What is the correct answer?

ANSWERS

1. a) −12 is an Integer **b)** Integer **c)** 3 is a Natural **d)** Rational **e)** Irrational **f)** None

2.



3. a) 7 **b)** 0.15 **c)** 117649 **4.** 3.9 hours **5.** $8884

6. a) $\sqrt{9}\sqrt{7} = 3\sqrt{7}$ **b)** $\sqrt{36}\sqrt{3} = 6\sqrt{3}$ **c)** $\sqrt[3]{27}\sqrt[3]{5} = 3\sqrt[3]{5}$ **d)** $\sqrt{25}\sqrt{7} = 5\sqrt{7}$

e) $\sqrt{16}\sqrt{6} = 4\sqrt{6}$ **f)** $2\sqrt[3]{8}\sqrt[3]{3} = 2\times 2\sqrt[3]{3} = 4\sqrt[3]{3}$

7. a) $\sqrt{16}\sqrt{7} = \sqrt{112}$ **b)** $\sqrt[3]{27}\sqrt[3]{2} = \sqrt[3]{54}$ **c)** $\sqrt{9}\sqrt{3125} = \sqrt{28125}$

d) $\sqrt{25}\sqrt{6} = \sqrt{150}$ **e)** $\sqrt[3]{125}\sqrt[3]{2} = \sqrt[3]{250}$ **f)** $\sqrt[4]{81}\sqrt[4]{2} = \sqrt[4]{162}$

8. a) $\frac{1}{16}$ **b)** 1 **c)** $\frac{243}{32}$ **d)** 13 **e)** $\frac{1}{243}$ **f)** 16

g) 15625 **h)** $\frac{-1}{9}$ **i)** 3 **j)** −16 **k)** $\frac{2}{27}$

9. a) $\sqrt[3]{4^2}$ **b)** $\sqrt{7}$ **c)** $\sqrt[4]{81^5}$ **d)** $\sqrt[n]{x^m}$ **e)** $\sqrt[9]{\frac{1}{9}}$ **f)** $\sqrt[3]{(-2)^4}$ (Brackets required!)

10. a) $m^{\frac{3}{2}}$ **b)** $\sqrt{m^{15}} = m^{\frac{15}{2}}$ **c)** $5^{\frac{3}{7}}$ **d)** $x^{\frac{8}{3}}$ **e)** $8^{\frac{1}{n}}$ **f)** $y^{\frac{\frac{5}{3}}{2}} = y^{\frac{5}{6}}$

11. a) $10x^7y^5$ **b)** $\frac{4x^6y^4}{9}$ **c)** $\frac{x^4}{y^6}$ **d)** $12m^4$ **e)** 1

f) $\frac{6a^2b^2}{5}$ **g)** $\frac{m^{15}}{n^{10}}$ **h)** $-5x$ **i)** $\frac{y^6}{8x^{12}}$ **j)** $\frac{1}{9x^{12}}$

12. a) 3 **b)** 1.728 **c)** $\frac{1}{6}$ **d)** $\frac{1}{4}$ **e)** 77.53 **f)** 81

13. $27^{\frac{2}{3}} = 9$, not 18. The correct answer is $9x^{\frac{8}{3}}$.