Day 1&2: The Real Number System (5.1)

$\sqrt{x}$ means what number times itself is equal to x.
$\sqrt[3]{x}$ means what number times itself three times is equal to x.
$\sqrt[4]{x}$ means what number times itself four times is equal to x.
$\sqrt[n]{x}$ means what number times itself *n* times is equal to x.

<u>Radical –</u> The root of a quantity

$$r\sqrt[n]{x}$$

Where $\sqrt{\ }$ is the radical sign
r is the coefficient
x is the radicand
n is the index

Note –The index shows the type of root
ie- $\sqrt[4]{\ }$ is a fourth root

- If there is no number in the index, the root is a square root
- If n = 2 we do not write a number for the index

ie- $\sqrt{\ }$ is a square root, n = 2

Notes:
- *if there is no coefficient shown, it is equal to 1* *i.e.* $\sqrt{5}=1\sqrt{5}$ *or* $\sqrt[3]{7}=1\sqrt[3]{7}$
- *if there is no index shown, it is equal to 2* *i.e.* $\sqrt{5}=\sqrt[2]{5}$

Entire Radical (also known as a Pure Radical) - a radical with a coefficient of 1. $\sqrt{6}$

Mixed Radical - a radical with a coefficient other than 1. $2\sqrt{5}$

It is important to know your **perfect squares** and your **perfect cubes:**

Perfect Squares - 4, 9, 16, 25, 36, 49, 64, 91, 100, 121, 144, etc.

$2^2, 3^2, 4^2, 5^2, 6^2, 7^2, 8^2, 9^2, 10^2, 11^2, 12^2, etc.$

Perfect Cubes - 8, 27, 64, 125, 216 etc.

$2^3, 3^3, 4^3, 5^3, 6^3, etc.$

Properties of radicals:

1. $\sqrt{a}\times\sqrt{b}=\sqrt{ab}$
2. $\sqrt{\frac{a}{b}}=\frac{\sqrt{a}}{\sqrt{b}}$

$\therefore \frac{\sqrt{ab}}{\sqrt{ac}}=\sqrt{\frac{ab}{ac}}=\sqrt{\frac{b}{c}}=\frac{\sqrt{b}}{\sqrt{c}}$

Converting from Entire to Mixed Radicals:
Find a multiple of a perfect square within the radical and factor it out.
i.e. 4, 9, 16, 25, 36, 49 etc.

a) $\sqrt{8} = \sqrt{4\times 2} = \sqrt{4}\sqrt{2} = 2\sqrt{2}$

b) $\sqrt{75} = \sqrt{25\times 3} = \sqrt{25}\sqrt{3} = 5\sqrt{3}$

c) $\sqrt{80} = \sqrt{16\times 5} = \sqrt{16}\sqrt{5} = 4\sqrt{5}$

d) $\sqrt{20} = \sqrt{4\times 5} = \sqrt{4}\sqrt{5} = 2\sqrt{5}$

e) $2\sqrt{54} = 2\sqrt{9\times 6} = 2\sqrt{9}\sqrt{6} = 2\times 3\sqrt{6} = 6\sqrt{6}$

f) $-5\sqrt{45} = -5\sqrt{9\times 5} = -5\sqrt{9}\sqrt{5} = -5\times 3\sqrt{5} = -15\sqrt{5}$

g) $7\sqrt{\frac{20}{49}} = 7\frac{\sqrt{20}}{\sqrt{49}} = 7\frac{\sqrt{4}\sqrt{5}}{7} = \cancel{7}\frac{2\sqrt{5}}{\cancel{7}} = 2\sqrt{5}$

h) $\sqrt{\frac{6}{25}} = \frac{\sqrt{6}}{\sqrt{25}} = \frac{\sqrt{6}}{5}$ This is the best we can do. We have no issues with a radical in the numerator.

i) $\frac{2}{5}\sqrt{450} = \frac{2}{5}\sqrt{225\times 2} = \frac{2}{5}\sqrt{225}\sqrt{2} = \frac{2}{5}\times 15\sqrt{2} = 6\sqrt{2}$

j) $3\sqrt{8x} = 3\sqrt{4}\sqrt{2x} = 3\times 2\sqrt{2x} = 6\sqrt{2x}$

k) $5y\sqrt{7y^2} = 5y\sqrt{y^2}\sqrt{7} = 5yy\sqrt{7} = 5y^2\sqrt{7}$

Since the index is 2 (square root), the square root of y square cancels the square out, leaving only a y.

l) $\sqrt{32x^5y} = \sqrt{16\times 2x^2x^2xy} = \sqrt{16}\sqrt{x^2}\sqrt{x^2}\sqrt{2xy} = 4xx\sqrt{2xy} = 4x^2\sqrt{2xy}$

Since the index is 2, break the x^5 into x^2x^2x.

m) $5\sqrt[3]{16x^6y^5} = 5\sqrt[3]{8\times 2x^3x^3y^3y^2} = 5\sqrt[3]{8}\sqrt[3]{x^3}\sqrt[3]{x^3}\sqrt[3]{y^3}\sqrt[3]{y^2} = 5\times 2xxy\sqrt[3]{2y^2} = 10x^2y\sqrt[3]{2y^2}$

n) $2x\sqrt[3]{108x^4} = 2x\sqrt[3]{27\times 4x^3x} = 2\sqrt[3]{27}\sqrt[3]{x^3}\sqrt[3]{4x} = 2\times 3x\sqrt[3]{4x} = 6x\sqrt[3]{4x}$

o) $7x^3y\sqrt[3]{3x^5y^4} = 7x^3y\sqrt[3]{3x^3x^2y^3y} = 7x^3y\sqrt[3]{3}\sqrt[3]{x^3}\sqrt[3]{y^3}\sqrt[3]{x^2}\sqrt[3]{y} = 7x^3yxy\sqrt[3]{3x^2y} = 7x^4y^2\sqrt[3]{3x^2y}$

Converting Mixed to Entire Radicals: put an equivalent form of the coefficient inside the radical.

a) $2\sqrt{6}$ $=\sqrt{2^2}\sqrt{6}=\sqrt{4}\sqrt{6}=\sqrt{24}$

b) $9\sqrt{10}$ $=\sqrt{9^2}\sqrt{10}=\sqrt{81}\sqrt{10}=\sqrt{810}$

c) $-4\sqrt{5}$ $=-\sqrt{16}\sqrt{5}=-\sqrt{80}$ **The negative stays OUT of the radical.**

d) $5\sqrt[3]{4}$ $=\sqrt[3]{5^3}\sqrt[3]{4}=\sqrt[3]{125}\sqrt[3]{4}=\sqrt[3]{500}$

f) $\frac{3}{2}\sqrt{8}$ $=\sqrt{\left(\frac{3}{2}\right)^2}\sqrt{8}=\sqrt{\frac{9}{4}}\sqrt{8}=\sqrt{\frac{9}{4}\times 8}=\sqrt{18}$

g) $x\sqrt[3]{80}$ $=\sqrt[3]{x^3}\sqrt[3]{80}=\sqrt[3]{80x^3}$

h) $x^2y\sqrt[3]{4xy}$ $=\sqrt[3]{\left(x^2\right)^3}\sqrt[3]{y^3}\sqrt[3]{4xy}=\sqrt[3]{x^6}\sqrt[3]{y^3}\sqrt[3]{4xy}=\sqrt[3]{4x^7y^4}$

i) $-3\sqrt[4]{3}$ $=-\sqrt[4]{3^4}\sqrt[4]{5}=-\sqrt[4]{81}\sqrt[4]{5}=-\sqrt[4]{405}$

j) $3x\sqrt{2}$ $=\sqrt{18x^2}$

k) $5y^3\sqrt{7y}$ $=\sqrt{175y^7}$

l) $8x^3y\sqrt{2xy}$ $=\sqrt{128x^7y^3}$

m) $5\sqrt[3]{4}$ $=\sqrt[3]{500}$

n) $2x\sqrt[3]{4x^2}$ $=\sqrt[3]{32x^5}$

o) $7x^3y\sqrt[3]{3xy^2}$ $=\sqrt[3]{1029x^{10}y^5}$

View a 5 minute video here
http://www.livescribe.com/cgi-bin/WebObjects/LDApp.woa/wa/MLSOverviewPage?sid=6HQlqtCZsv0V

Adding and Subtracting Radicals:

In the same way that 2x and 3x are "like terms" and can be added to 5x, radicals such as $2\sqrt{3}$ *and* $4\sqrt{3}$ are called "like terms" and can be added or subtracted.

Radicals such as $2\sqrt{5}$ *and* $3\sqrt{7}$ are not like and cannot be combined.

(Radicands must be the same to add or subtract).

Note: $\sqrt{2}+\sqrt{3}\neq\sqrt{5}$

$$\sqrt{2}+\sqrt{2}+\sqrt{2}=3\sqrt{2}=3\times\sqrt{2}$$

Add and/or subtract. Express as exact values in simplest form.

1. $3\sqrt{5}-7\sqrt{5}$ $\quad =-4\sqrt{5}$

2. $6\sqrt{7}-3\sqrt{2}+5\sqrt{7}+\sqrt{2}$ $\quad =11\sqrt{7}-2\sqrt{2}$

3. $6\sqrt{11}-2\sqrt{5}-3\sqrt{5}-8\sqrt{11}$ $\quad =-2\sqrt{11}-5\sqrt{5}$

4. $\sqrt{8}+\sqrt{18}$ $\quad =\sqrt{4}\sqrt{2}+\sqrt{9}\sqrt{2}=2\sqrt{2}+3\sqrt{2}=5\sqrt{2}$

5. $\sqrt{20}-\sqrt{45}-\sqrt{125}$

$$=\sqrt{4}\sqrt{5}-\sqrt{9}\sqrt{5}-\sqrt{25}\sqrt{5}$$
$$=2\sqrt{5}-3\sqrt{5}-5\sqrt{5}$$
$$=-6\sqrt{5}$$

6. $2\sqrt{12}+\sqrt{50}-\sqrt{75}-6\sqrt{32}$

$$=2\sqrt{4}\sqrt{3}+\sqrt{25}\sqrt{2}-\sqrt{25}\sqrt{3}-6\sqrt{16}\sqrt{2}$$
$$=2\times2\sqrt{3}+5\sqrt{2}-5\sqrt{3}-6\times4\sqrt{2}$$
$$=4\sqrt{3}+5\sqrt{2}-5\sqrt{3}-24\sqrt{2}$$
$$=-\sqrt{3}-19\sqrt{2}$$

7. $\sqrt{50}+\sqrt[3]{250}$ $\quad =\sqrt{25}\sqrt{2}+\sqrt[3]{125}\sqrt[3]{2}=5\sqrt{2}+5\sqrt[3]{2}$

You can go no further.
$\sqrt{\ }$ *and* $\sqrt[3]{\ }$ can not combine.

Video 12 minutes - be sure to maximize http://bit.ly/rCkyKS

Day 1 Assignment	Read Page 278		
	Page 278	#1	Mixed to Entire, Entire to Mixed
		#2	Entire to Mixed
		#4	Mixed to Entire, Entire to Mixed
		#8-10	Adding and Subtracting
		#19	Find mistake
		#20	Equivalent radicals
Day 2 Assignment	M20-1 Ch 5 Radicals WA.doc		

Days 3

5.2 Multiplying and Dividing Radical Expressions

Rules:

a) solve the sign first

b) do the operation on the number outside of the radical. If there is no number in front of the radical sign put a coefficient of 1.

c) radicand

d) simplify

A. Simplify (Multiplication). Express as exact values in simplest form.

1. $\sqrt{3}\times\sqrt{5}$ $=\sqrt{15}$

2. $\sqrt{3}\times\sqrt{8}$ $=\sqrt{24}=\sqrt{4}\sqrt{6}=2\sqrt{6}$

3. $5\sqrt{2}\times7\sqrt{3}$ $=35\sqrt{6}$

4. $4\sqrt{6}\times5\sqrt{2}$ $=20\sqrt{12}=20\sqrt{4}\sqrt{3}=20\times2\sqrt{3}$

5. $3\sqrt{6}\times2\sqrt{3}$ $=6\sqrt{18}=6\sqrt{9}\sqrt{2}=6\times3\sqrt{2}=18\sqrt{2}$

Video - 10 minutes http://www.youtube.com/watch?v=6e5jwhznUoU

B. Simplify (Division). Express as exact values in simplest form.
Division of radicals is the inverse of multiplication.

A property of radicals:

$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}} \quad (a \ge 0, b > 0)$

1. $\frac{12\sqrt{6}}{6\sqrt{2}}$ $= 2\sqrt{\frac{6}{2}} = 2\sqrt{3}$

2. $\frac{24\sqrt{54}}{6\sqrt{6}}$ $= 4\sqrt{\frac{54}{6}} = 4\sqrt{9} = 4 \times 3 = 12$

3. $\frac{5\sqrt{20}}{10}$ $= \frac{\sqrt{4}\sqrt{5}}{2} = \frac{2\sqrt{5}}{2} = \sqrt{5}$

4. $\frac{3\sqrt{32}}{\sqrt{8}}$ $= 3\sqrt{\frac{32}{8}} = 3\sqrt{4} = 3 \times 2 = 6$

9 minute video - http://www.youtube.com/watch?v=r4GvnsrkClk

Multiplying Binomial Radicals

- Distributive Property: $a(b+c) = ab + ac$
  - multiply what is outside the bracket by each term in the bracket
  - $2\left(\sqrt{2}+1\right)$ $= 2\sqrt{2} + 2$
- FOIL: $(a+b)(c+d) = ac + ad + bc + bd$
  - $\left(\sqrt{2}+2\right)\left(\sqrt{2}+1\right)$ $= 2 + \sqrt{2} + 2\sqrt{2} + 2 = 4 + 3\sqrt{2}$

To **expand** an expression means to remove the brackets and **simplify.**

Examples. Simplify. Express as exact values in simplest form.

1. $2\sqrt{3}\left(5+4\sqrt{7}\right)$

$=10\sqrt{3}+8\sqrt{21}$

2. $5\sqrt{2}\left(3\sqrt{2}-\sqrt{6}\right)$

$=10\sqrt{4}-5\sqrt{12}=10\times2-5\sqrt{4}\sqrt{3}=20-10\sqrt{3}$

3. $\left(\sqrt{2}-\sqrt{6}\right)\left(3\sqrt{2}+\sqrt{6}\right)$

$=3\sqrt{4}+\sqrt{12}-3\sqrt{12}-\sqrt{36}$

$=6-2\sqrt{12}-6$

$=-2\sqrt{4}\sqrt{3}$

$=-4\sqrt{3}$

4. $\left(3+\sqrt{5}\right)\left(2+4\sqrt{5}\right)$

$=6+12\sqrt{5}+2\sqrt{5}+4\sqrt{25}$

$=6+12\sqrt{5}+20$

$=26+12\sqrt{5}$

5. $\left(\sqrt{7}+2\sqrt{3}\right)\left(\sqrt{7}-2\sqrt{3}\right)$

This one is neat, watch what happens to the outter and inner.

$=\sqrt{49}-2\sqrt{21}+2\sqrt{21}-4\sqrt{9}$

$=7-\cancel{2\sqrt{21}}+\cancel{2\sqrt{21}}-4\times3$

$=7-12$

$=-5$

6. $\left(3+5\sqrt{2}\right)^2$

$=\left(3+5\sqrt{2}\right)\left(3+5\sqrt{2}\right)$

$=9+15\sqrt{2}+15\sqrt{2}+25\sqrt{4}$

$=9+30\sqrt{2}+25\times2$

$=59+30\sqrt{2}$

Special Products.

> **Special Rules**
> ① $\sqrt{a}\times\sqrt{a}=a$
> ② $\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}-\sqrt{b}\right)=a-b$
>
> Conjugates:
> same but, but a different sign in the middle

1. $\sqrt{3}\times\sqrt{3}$ $=\sqrt{9}=3$

   Two square roots with same radicand cancel out the square root.

2. $5\sqrt{6}\times 2\sqrt{6}$ $=10\sqrt{36}=10\times 6=60$

3. $\left(\sqrt{6}+\sqrt{2}\right)\left(\sqrt{6}-\sqrt{2}\right)$

   $=\sqrt{36}-\sqrt{4}$
   $=6-2=4$

4. $\left(\sqrt{13}+2\right)\left(\sqrt{13}-2\right)$

   $=\sqrt{169}-4$
   $=13-4=9$

Assignment	Page 289	#1	mono x mono
		#2	mono x bi or tri
		#3	expressions
		#4	bi x bi
		#5	bi x bi
		#6	division

Day 4
Rationalizing the Denominator

$$\frac{3}{-4}=\frac{3\times -1}{-4\times -1}=\frac{-3}{4}$$

- normally fractions do **<u>NOT</u>** have a negative sign in the denominator
- normally fractions do **<u>NOT</u>** have a radical sign in the denominator

Rationalize the denominator (*Get rid of the radical in the bottom*).

- To rationalize a single radical in the denominator, multiply numerator and denominator by that radical.

1. $\frac{3}{\sqrt{2}}$ $\quad =\frac{3}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}=\frac{3\sqrt{2}}{\sqrt{4}}=\frac{3\sqrt{2}}{2}$

2. $\frac{\sqrt{5}}{\sqrt{6}}$ $\quad =\frac{\sqrt{5}}{\sqrt{6}}\times\frac{\sqrt{6}}{\sqrt{6}}=\frac{\sqrt{30}}{\sqrt{36}}=\frac{\sqrt{30}}{6}$

3. $\sqrt{\frac{2}{3}}$ $\quad =\frac{\sqrt{2}}{\sqrt{3}}=\frac{\sqrt{2}}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{\sqrt{6}}{3}$

4. $\frac{6\sqrt{2}}{\sqrt{3}}$ $\quad =\frac{6\sqrt{2}}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{6\sqrt{6}}{3}=2\sqrt{6}$

5. $\frac{10\sqrt{2}}{\sqrt{8}}$ $\quad =\frac{10\sqrt{2}}{\sqrt{8}}\times\frac{\sqrt{8}}{\sqrt{8}}=\frac{10\sqrt{16}}{8}=\frac{10\times 4}{8}=5$

6. $\frac{2\sqrt{7}}{3\sqrt{2}}$ $\quad =\frac{2\sqrt{7}}{3\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}=\frac{2\sqrt{14}}{3\times 2}=\frac{2\sqrt{14}}{6}=\frac{\sqrt{14}}{3}$

7. $\frac{4\sqrt{8}}{2\sqrt{5}}$ $\quad =\frac{4\sqrt{8}}{2\sqrt{5}}=\frac{\cancel{4}^{2}\sqrt{8}}{\cancel{2}\sqrt{5}}=\frac{2\sqrt{8}}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}=\frac{2\sqrt{40}}{5}=\frac{2\sqrt{4}\sqrt{10}}{5}=\frac{4\sqrt{10}}{5}$

8. $\dfrac{8\sqrt{5}}{2\sqrt{10}}$ $\quad = 4\sqrt{\dfrac{5}{10}} = 4\sqrt{\dfrac{1}{2}} = \dfrac{4}{\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{4\sqrt{2}}{2} = 2\sqrt{2}$

9. $\dfrac{2\sqrt{5}+4}{\sqrt{10}}$ $\quad = \dfrac{\left(2\sqrt{5}+4\right)}{\sqrt{10}} \times \dfrac{\sqrt{10}}{\sqrt{10}} = \dfrac{2\sqrt{50}+4\sqrt{10}}{10} = \dfrac{2\sqrt{25}\sqrt{2}+4\sqrt{10}}{10} = \dfrac{10\sqrt{2}+4\sqrt{10}}{10}$

$= \dfrac{2\left(5\sqrt{2}+2\sqrt{10}\right)}{10} = \dfrac{5\sqrt{2}+2\sqrt{10}}{5}$

10. $\dfrac{2\sqrt{6}-4\sqrt{3}}{\sqrt{3}}$ $\quad = \dfrac{\left(2\sqrt{6}-4\sqrt{3}\right)}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{2\sqrt{18}-4\sqrt{9}}{3} = \dfrac{2\sqrt{9}\sqrt{2}-4\times 3}{3}$

$= \dfrac{6\sqrt{2}-12}{3} = 2\sqrt{2}-4$

8 minute video - http://www.youtube.com/watch?v=azaP5n6nmOY

- Two terms in denominator

  Multiply numerator and denominator by the *conjugate* of the denominator (change middle sign of denominator)

  If you have $\frac{7}{\sqrt{5}-2}$, multiply top and bottom by $\sqrt{5}+2$. Note change in sign.

Remember: rationalize the denominator means get rid of the radical symbol from the bottom

Examples. Rationalize the denominator.

1. $\frac{2}{3+\sqrt{6}}$ $\quad = \frac{2}{3+\sqrt{6}} \times \frac{3-\sqrt{6}}{3-\sqrt{6}} = \frac{6-2\sqrt{6}}{9-\sqrt{36}} = \frac{6-2\sqrt{6}}{9-6} = \frac{6-2\sqrt{6}}{3}$

   Remember conjugates from, the previous lesson.

2. $\frac{3}{\sqrt{5}-\sqrt{2}}$ $\quad = \frac{3}{\sqrt{5}-\sqrt{2}} \times \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}} = \frac{3\sqrt{5}+3\sqrt{2}}{5-2} = \frac{3\sqrt{5}+3\sqrt{2}}{3} = \sqrt{5}+\sqrt{2}$

3. $\frac{\sqrt{3}}{\sqrt{5}-4}$ $\quad = \frac{\sqrt{3}}{\sqrt{5}-4} \times \frac{\sqrt{5}+4}{\sqrt{5}+4} = \frac{\sqrt{15}+4\sqrt{3}}{5-16} = \frac{\sqrt{15}+4\sqrt{3}}{-11} = -\frac{\sqrt{15}+4\sqrt{3}}{11} = \frac{-\sqrt{15}-4\sqrt{3}}{11}$

   Note: the answer is NOT $\frac{-\sqrt{15}+4\sqrt{3}}{11}$

   The negative is out front or in both parts of numerator.

4. $\frac{\sqrt{3}+\sqrt{5}}{\sqrt{6}-\sqrt{10}}$ $\quad = \frac{\sqrt{3}+\sqrt{5}}{\sqrt{6}-\sqrt{10}} \times \frac{\sqrt{6}+\sqrt{10}}{\sqrt{6}+\sqrt{10}} = \frac{\sqrt{18}+\sqrt{30}-\sqrt{30}+\sqrt{50}}{6-10}$

   $= \frac{\sqrt{9}\sqrt{2}+\sqrt{25}\sqrt{2}}{-4} = \frac{3\sqrt{2}+5\sqrt{2}}{-4} = \frac{8\sqrt{2}}{-4} = -2\sqrt{2}$

5. $\frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-3\sqrt{3}}$ $\quad = \frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-3\sqrt{3}} \times \frac{3\sqrt{2}+3\sqrt{3}}{3\sqrt{2}+3\sqrt{3}} =$

11 minute video - http://www.youtube.com/watch?v=ByNJ3T6Kb9I

Days 5&6 (5.3) Solving Radical Equations

Goal: Be able to solve radical equations and inequalities.

Read p 294 – 300
Do p 300- 303

Background

- Solve the following:
  - $x = \sqrt{25}$
  - $x^2 = 25$

- Why do you get different answers?

- Is it possible for $\sqrt{x-2} + 3 = 0$ to be true for any value of *x*?

- extraneous roots:

Examples.

- Solve and check (always check your answer(s) if you square both sides during the solving process).

1. $\sqrt{x+2} - 5 = 0$

Steps for solving radical equations.

① **Isolate the radical** $\sqrt{x+2} = 5$

② **Square both sides to remove radical** $\left(\sqrt{x+2}\right)^2 = 5^2$

$x + 2 = 25$

③ **Solve for variable. (Quadratic Formula if needed)** $x = 23$

④ **Check to ensure your answer is correct.**

It is possible that by squaring the equation, you may have created an extraneous root. (An answer that does not work) See questions 2, 4 & 7 below.

$\sqrt{x+2} - 5 = 0$

$\sqrt{23+2} - 5 = 0$

$\sqrt{25} - 5 = 0$

$5 - 5 = 0$

$0 = 0$

2. $\sqrt{x+2}+5=0$

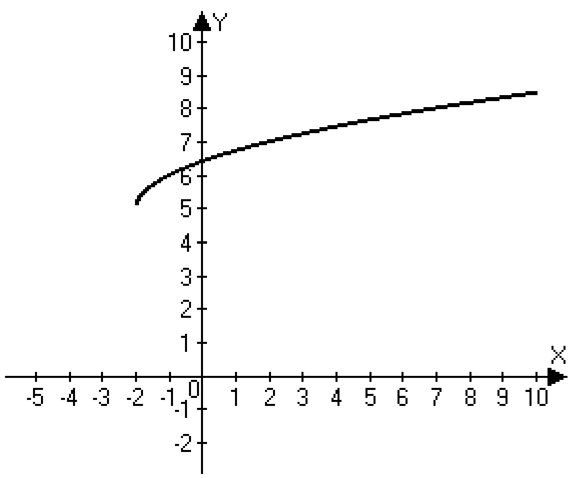
$$\sqrt{x+2}=-5$$
$$\left(\sqrt{x+2}\right)^{2}=\left(-5\right)^{2}$$
$$x+2=25$$
$$x=23$$

But, does 23 work in the original equation?

$$\sqrt{x+2}=-5$$
$$\sqrt{23+2}=-5$$
$$\sqrt{25}=-5$$
$$5\neq-5$$

This equation has no solution.

As proof, graph $\sqrt{x+2}+5$ in y_1 on your calculator

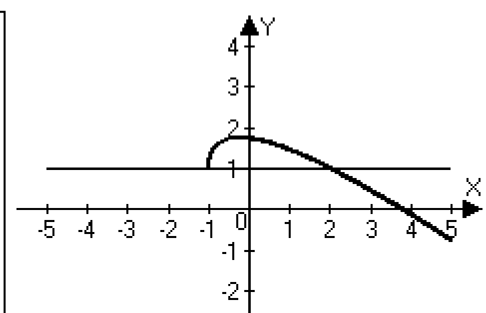


Notice how the graph NEVER equals 0.
No solution.

3. Find the roots. $\sqrt{3x+3}-x=1$

$$\sqrt{3x+3}=x+1$$
$$\left(\sqrt{3x+3}\right)^{2}=\left(x+1\right)^{2}$$
$$3x+3=x^{2}+2x+1$$
$$0=x^{2}-x-2$$
$$0=\left(x-2\right)\left(x+1\right)$$
$$x=2,-1$$

Insert 2 or –1 into the original equation and both work. The graph also shows both values of x (2 & –1) work.



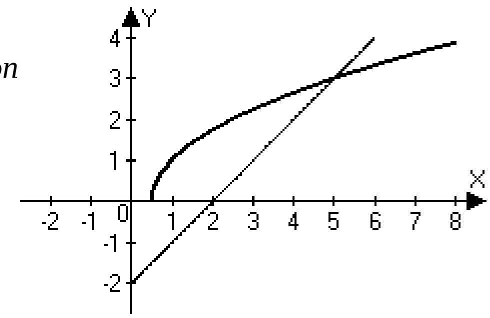
4. Solve for *x*. $\sqrt{2x-1} = x-2$

$$\left(\sqrt{2x-1}\right)^2 = (x-2)^2$$
$$2x-1 = x^2-4x+4$$
$$0 = x^2-6x+5$$
$$0 = (x-5)(x-1)$$
$$x = 5,1$$

Put 1 *back into the equation*

$$\sqrt{2x-1} = x-2$$
$$\sqrt{2(1)-1} = 1-2$$
$$\sqrt{2-1} = -1$$
$$1 \neq -1$$

1 *does not work*



5. Solve: $\sqrt{x} + \sqrt{x+16} = 8$

More Questions

When squaring to get rid of radical sign, always check answer

6. $\sqrt{2x-3}=3$ **ANS**: 6

$$\sqrt{2x-3}=3$$
$$\left[\sqrt{2x-3}\right]^2=3^2 \quad \textit{square both sides}$$
$$2x-3=9$$
$$2x=12$$
$$x=6$$

7. $\sqrt{2x-3}+3=x$ **ANS**: 6, ~~2~~

$$\sqrt{2x-3}+3=x$$
$$\sqrt{2x-3}=x-3$$
$$\left[\sqrt{2x-3}\right]^2=\left[x-3\right]^2$$
$$2x-3=x^2-6x+9$$
$$0=x^2-8x+12$$
$$x=6, \cancel{2}$$

8. $\sqrt{3x-2}=2\sqrt{x-3}$ **ANS**: 10

$$\sqrt{3x-2}=2\sqrt{x-3}$$
$$\left[\sqrt{3x-2}\right]^2=\left[2\sqrt{x-3}\right]^2$$
$$3x-2=4(x-3)$$
$$3x-2=4x-12$$
$$x=10$$

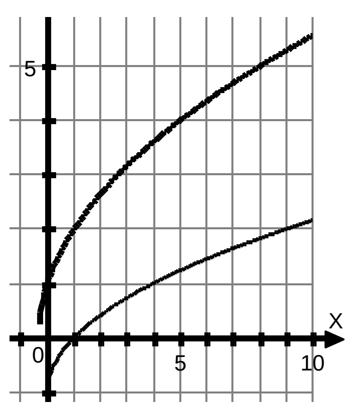
9. $\sqrt{x}+1=\sqrt{2x-2}$ **(IB)** **ANS**: ~~1~~, 9

10. $\sqrt{x+7}-\sqrt{2x}=\sqrt{x-1}$ **(IB)** **ANS**: ~~−8~~, 2

11. $\sqrt{3x+1}=\sqrt{x}-1$

$$\left(\sqrt{3x+1}\right)^2=\left(\sqrt{x}-1\right)^2$$
$$3x+1=x-2\sqrt{x}+1$$
$$2x=-2\sqrt{2x}$$
$$(x)^2=\left(-\sqrt{x}\right)^2$$
$$x^2=x$$
$$x^2-x=x(x-1)=0$$
$$x=0 \text{ or } 1$$

neither solution works



Examples (Graphically):

Get everything to one side of equation and then type into y=.
Solve for x-intercepts (zeros)

12. $\sqrt{x+5}=\sqrt{2x+7}-1$ **ANS**: 5.47

Solve algebraically.

a. $\sqrt{2x-3}=3$ **ANS:** 6

b. $\sqrt{2x-3}+3=x$ **ANS:** 6, ~~2~~

c. $\sqrt{3x-2}=2\sqrt{x-3}$ **ANS:** 10

d. $\sqrt{x}+1=\sqrt{2x-2}$ **(IB)** **ANS:** ~~1~~, 9

e. $\sqrt{x+7}-\sqrt{2x}=\sqrt{x-1}$ **(IB)** **ANS:** ~~−8~~, 2

f. Solve graphically: $\sqrt{x+5}=\sqrt{2x+7}-1$ **ANS:** 5.47

Word Problem: Baseballs and Tennis Balls

The radius, r, of a sphere is related to the surface area, A, by the equation

$$r=\frac{1}{2}\sqrt{\frac{A}{\pi}}$$

A. The surface area of a baseball is about 172 cm^2
Find the radius of a baseball, to the nearest tenth of a cm.

B. The radius of a tennis ball is about 3.3 cm. Find the surface area to the nearest square cm.

<u>Solution</u>

A. $r=\frac{1}{2}\sqrt{\frac{A}{\pi}}$ r = ? when A = 172 cm^2

$$r=\frac{1}{2}\sqrt{\frac{172}{\pi}}$$

$$r=3.69963...$$

$$\boxed{r=3.7cm}$$

B. $r=\frac{1}{2}\sqrt{\frac{A}{\pi}}$ A = ? when r = 3.3cm

$$3.3=\frac{1}{2}\sqrt{\frac{A}{\pi}}$$

$$6.6=\sqrt{\frac{A}{\pi}}$$

$$(6.6)^2=\left(\sqrt{\frac{A}{\pi}}\right)^2$$

$$43.56=\frac{A}{\pi}$$

$$43.56\pi=A$$

$$A=136.84777...$$

$$\boxed{A=137cm^2}$$

Assignment

pg. 300 #3–9 Solving equations

#12-18 Word problems

plus: **(IB)**

1. $\sqrt{2x}+3=\sqrt{5x+9}$ **ANS**: 0, 8

2. $\sqrt{x}+\sqrt{2x-7}=\sqrt{x+5}$ **ANS**: ~~-9~~, 4

3. $\sqrt{3x+2}=3\sqrt{x}-\sqrt{2}$ **ANS**: ~~0~~, 2

4. $\sqrt{\sqrt{x^2+6x}}=2$ **ANS**: –8, 2

5. $\sqrt[3]{x^2-12x}-4=0$ **ANS**: 16, –4

Day 7

Day 8

Day 9 Quiz