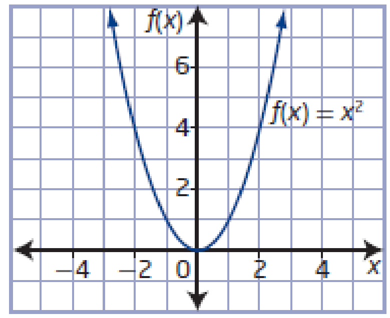
Chapter 4 Quadratic Equations

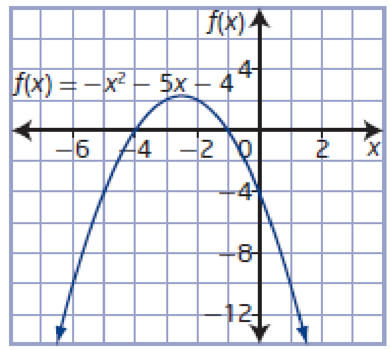
Section 4.1 Graphical Solutions of Quadratic Equations

Section 4.1 Page 215 Question 1

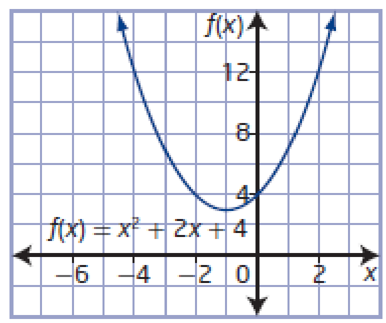
a) The quadratic function graph has one *x*-intercept.



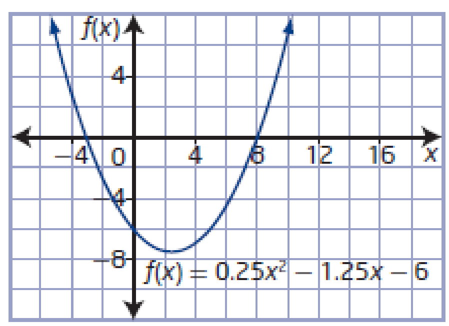
b) The quadratic function graph has two *x*-intercepts.



c) The quadratic function graph has no *x*-intercepts.



d) The quadratic function graph has two *x*-intercepts.



Section 4.1 Page 215 Question 2

a) The x-intercept of the graph in #1 a) is 0. So, the root of corresponding quadratic equation is 0.
Check by substituting $x = 0$ into $x^2 = 0$.

Left Side	Right Side
x^2	0
$= 0^2$	
$= 0$	

The solution is correct.

b) The x-intercepts of the graph in #1 b) are –4 and –1. So, the roots of corresponding quadratic equation are –4 and –1.
Check by substituting $x = -4$ and $x = -1$ into $-x^2 - 5x - 4 = 0$.

Left Side	Right Side	Left Side	Right Side
$-x^2 - 5x - 4$	0	$-x^2 - 5x - 4$	0
$= -(-4)^2 - 5(-4) - 4$		$= -(-1)^2 - 5(-1) - 4$	
$= -16 + 20 - 4$		$= -1 + 5 - 4$	
$= 0$		$= 0$	

Both solutions are correct.

c) The graph in #1 c) has no x-intercepts and so no roots.

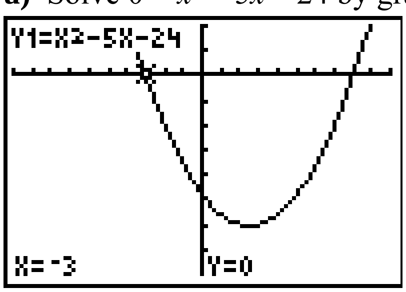
d) The x-intercepts of the graph in #1 d) are –3 and 8. So, the roots of corresponding quadratic equation are –3 and 8.
Check by substituting $x = -3$ and $x = 8$ into $0.25x^2 - 1.25x - 6 = 0$.

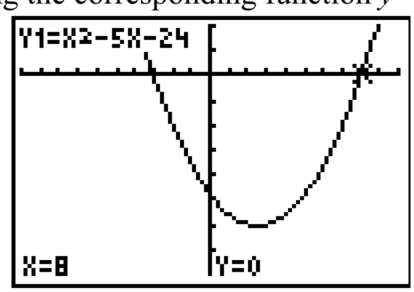
Left Side	Right Side	Left Side	Right Side
$0.25x^2 - 1.25x - 6$	0	$0.25x^2 - 1.25x - 6$	0
$= 0.25(-3)^2 - 1.25(-3) - 6$		$= 0.25(8)^2 - 1.25(8) - 6$	
$= 2.25 + 3.75 - 6$		$= 16 - 10 - 6$	
$= 0$		$= 0$	

Both solutions are correct.

Section 4.1 Page 215 Question 3

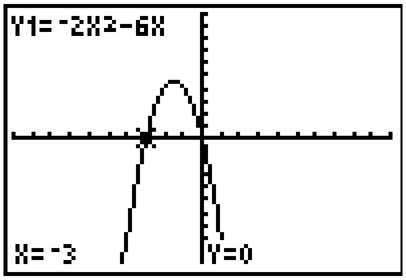
a) Solve $0 = x^2 - 5x - 24$ by graphing the corresponding function $y = x^2 - 5x - 24$.

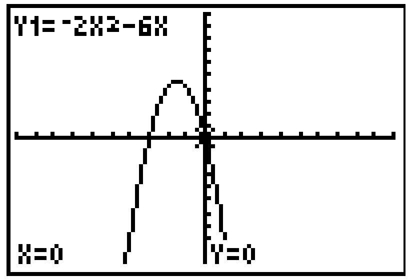




Since the x-intercepts of the graph are –3 and 8, the roots of the equation are –3 and 8.

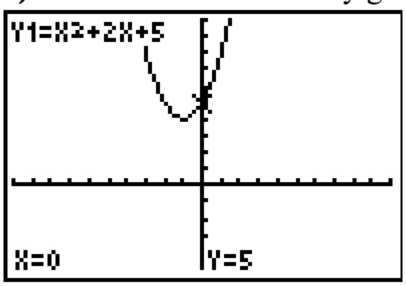
b) Solve $0 = -2r^2 - 6r$ by graphing the corresponding function $y = -2r^2 - 6r$.





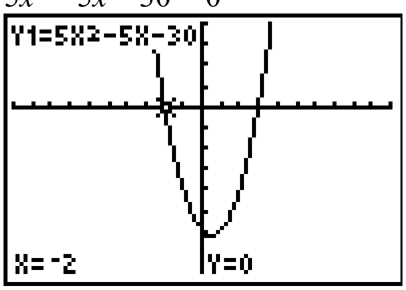
Since the x-intercepts of the graph are –3 and 0, the roots of the equation are –3 and 0.

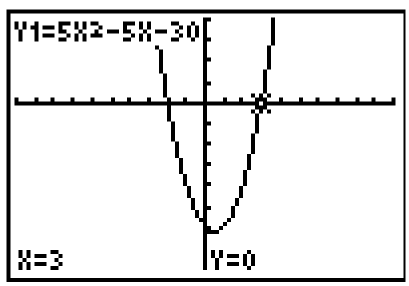
c) Solve $h^2 + 2h + 5 = 0$ by graphing the corresponding function $y = h^2 + 2h + 5 = 0$.



Since there are no x-intercepts, there are no real roots of the equation.

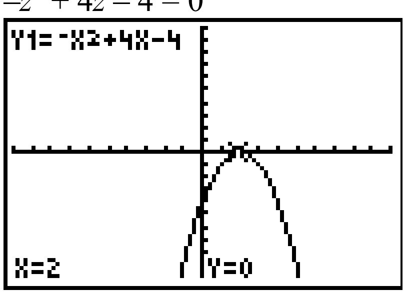
d) To solve $5x^2 - 5x = 30$, first rewrite the equation in the form $ax^2 + bx + c = 0$ then graph the corresponding function.
$5x^2 - 5x - 30 = 0$





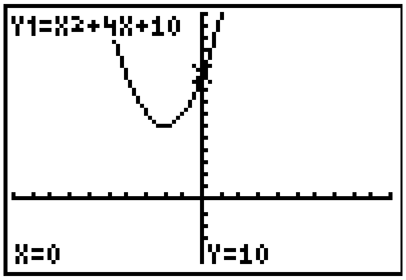
Since the x-intercepts of the graph are –2 and 3, the roots of the equation are –2 and 3.

e) To solve $-z^2 + 4z = 4$, first rewrite the equation in the form $ax^2 + bx + c = 0$ then graph the corresponding function.
$-z^2 + 4z - 4 = 0$



Since the x-intercept of the graph is 2, the root of the equation is 2.

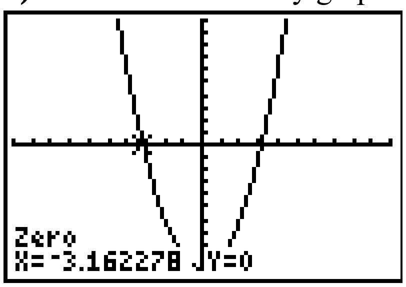
f) Solve $0 = t^2 + 4t + 10$ by graphing the corresponding function $y = t^2 + 4t + 10$.

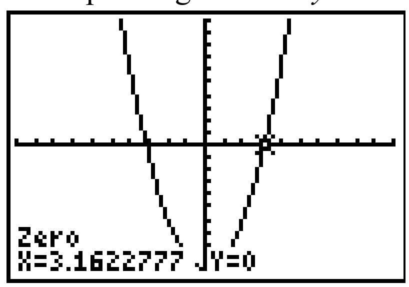


Since there are no x-intercepts, there are no real roots of the equation.

Section 4.1 Page 215 Question 4

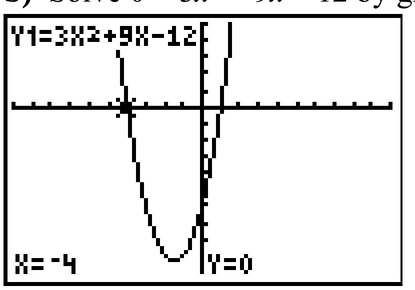
a) Solve $n^2 - 10 = 0$ by graphing the corresponding function $y = n^2 - 10$.

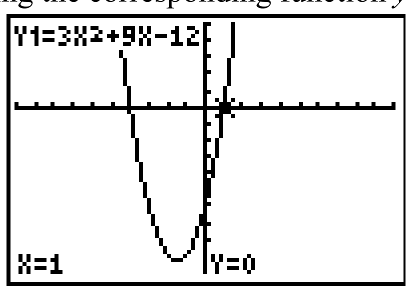




The roots of the equation are –3.2 and 3.2, to the nearest tenth.

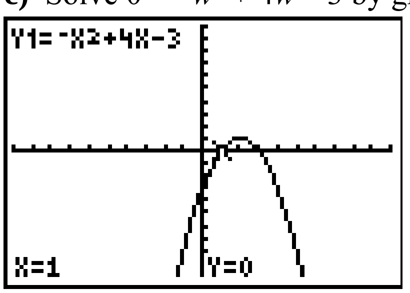
b) Solve $0 = 3x^2 + 9x - 12$ by graphing the corresponding function $y = 3x^2 + 9x - 12$.

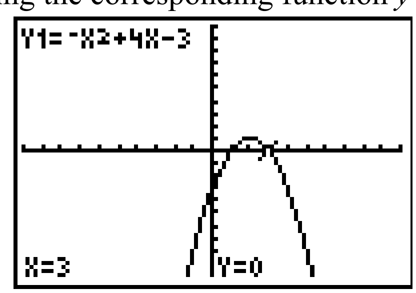




The roots of the equation are –4 and 1.

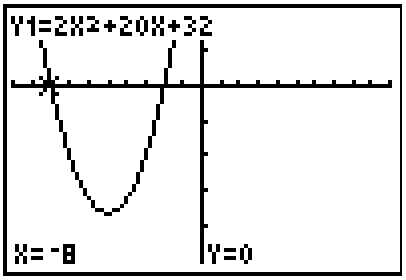
c) Solve $0 = -w^2 + 4w - 3$ by graphing the corresponding function $y = -w^2 + 4w - 3$.

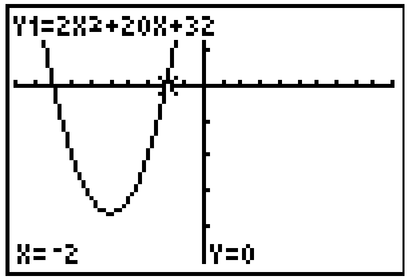




The roots of the equation are 1 and 3.

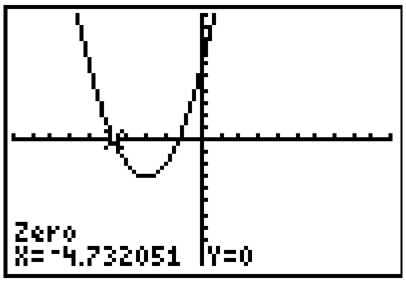
d) Solve $0 = 2d^2 + 20d + 32$ by graphing the corresponding function $y = 2d^2 + 20d + 32$.

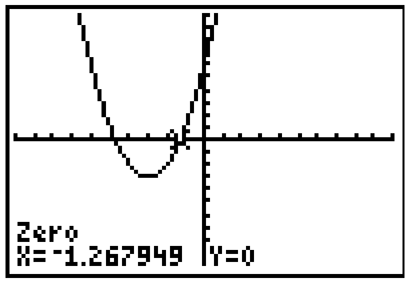




The roots of the equation are –8 and –2.

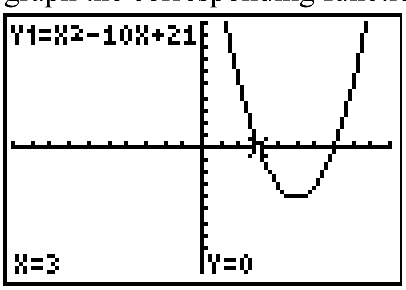
e) Solve $0 = v^2 + 6v + 6$ by graphing the corresponding function $y = v^2 + 6v + 6$.

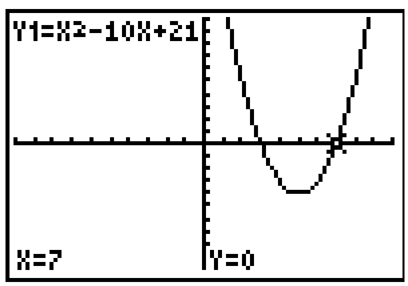




The roots of the equation are –4.7 and –1.3, to the nearest tenth.

f) To solve $m^2 - 10m = -21$, first rewrite the equation in the form $ax^2 + bx + c = 0$ then graph the corresponding function $m^2 - 10m + 21 = 0$.

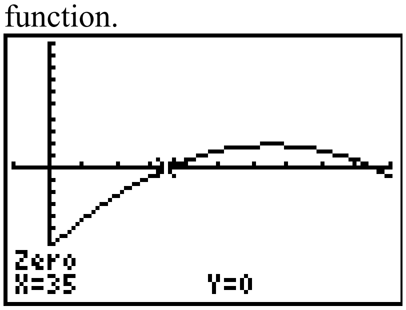


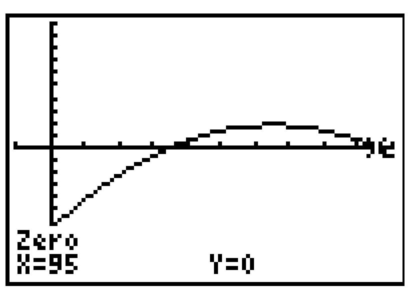


The roots of the equation are 3 and 7.

Section 4.1 Page 215 Question 5

The solutions to $-0.02d^2 + 2.6d - 66.5 = 0$ can be found by graphing the corresponding function.





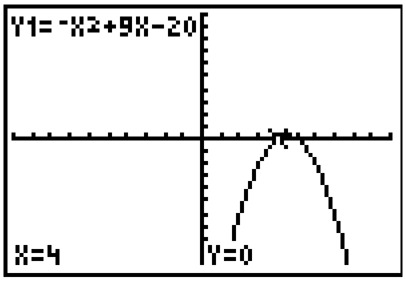
The ball travels 95 – 35, or 60 yd before hitting the ground.

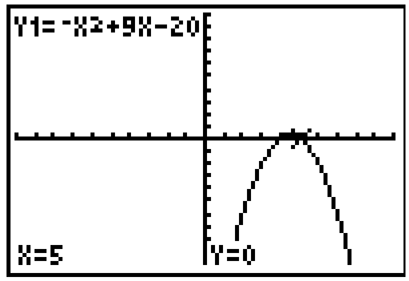
Section 4.1 Page 215 Question 6

a) Let x represent one number and then the other will be $9 - x$. For a product of 20,

$20 = x(9 - x)$
$20 = 9x - x^2$
$0 = -x^2 + 9x - 20$

b) The two numbers are 4 and 5.



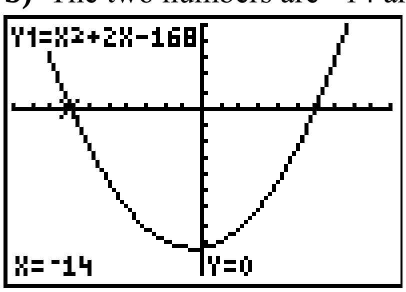


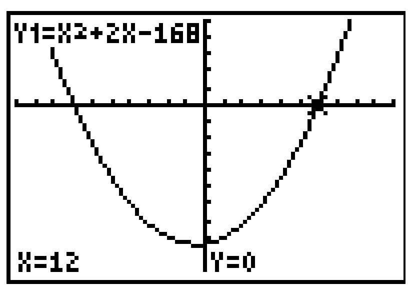
Section 4.1 Page 216 Question 7

a) Let x and $x + 2$ represent the two consecutive even integers. For a product of 168,

$168 = x(x + 2)$
$168 = x^2 + 2x$
$0 = x^2 + 2x - 168$

b) The two numbers are –14 and –12 or 12 and 14.

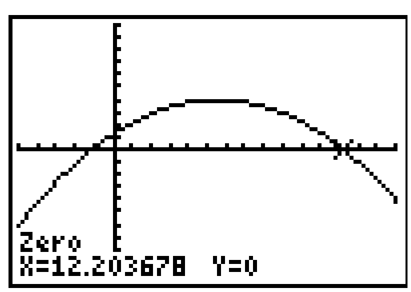




Section 4.1 Page 216 Question 8

a) Solving the equation $-0.09x^2 + x + 1.2 = 0$ gives the distance from the firefighter where the stream of water hits the ground.

b) The maximum distance that the firefighter can stand from the building is 12.2 m, to the nearest tenth of a metre.

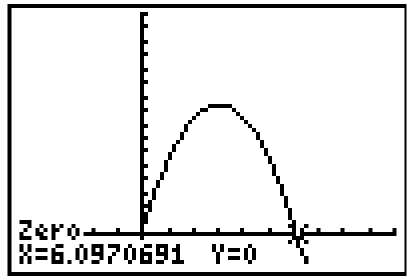


c) Example: Assume that the fire is located at ground level and environmental conditions are ideal (*i.e.*, no wind or obstructions).

Section 4.1 Page 216 Question 9

a) Solving the equation $0 = -4.9(t - 3)^2 + 47$ gives the time that the firework rocket hits the water.

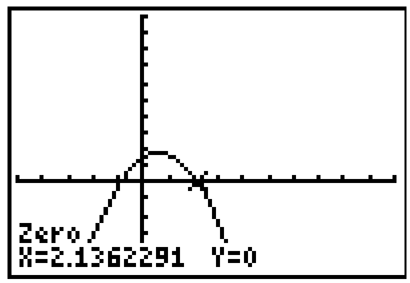
b) The fireworks rocket stays lit for 6.1 s, to the nearest tenth of a second.



Section 4.1 Page 216 Question 10

a) A quadratic equation that models the situation when the skateboarder lands is $0 = -0.75d^2 + 0.9d + 1.5$.

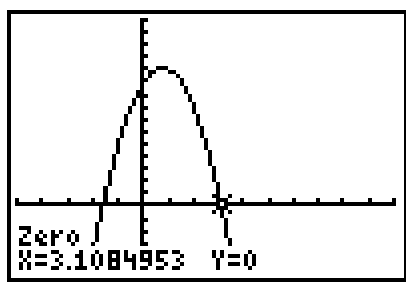
b) The skateboarder lands 2.1 m, to the nearest tenth of a metre, from the ledge.



Section 4.1 Page 216 Question 11

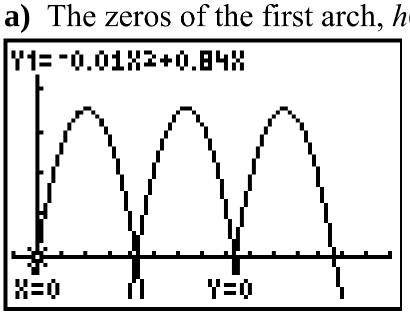
a) A quadratic equation to represent the situation when Émilie enters the water is $0 = -2d^2 + 3d + 10$.

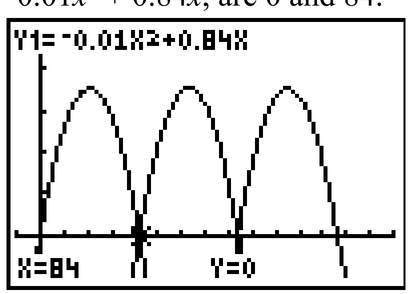
b) Émilie enters the water at a horizontal distance of 3.1 m, to the nearest tenth of a metre, from the end of the tower platform.



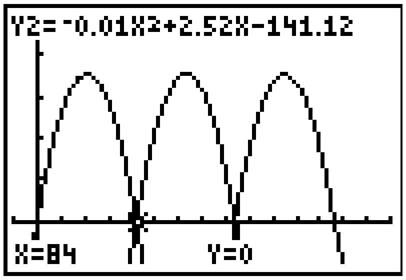
Section 4.1 Page 217 Question 12

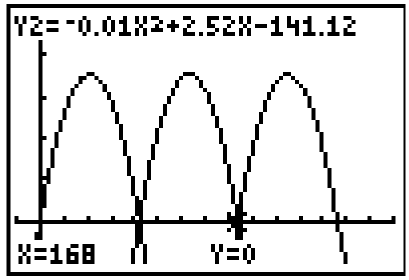
a) The zeros of the first arch, $h(x) = -0.01x^2 + 0.84x$, are 0 and 84.



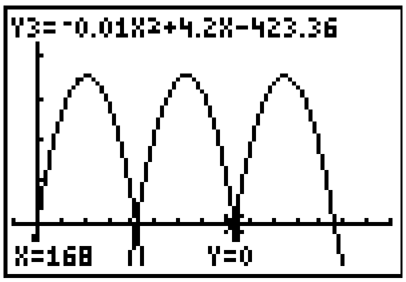


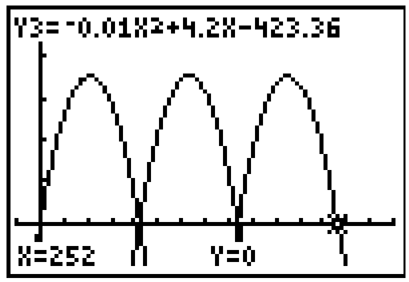
The zeros of the second arch, $h(x) = –0.01x^2 + 2.52x – 141.12$, are 84 and 168.





The zeros of the third arch, $h(x) = –0.01x^2 + 4.2x – 423.36$, are 168 and 252.





b) The zeros represent the locations where the arches meet the bridge deck.

c) The total span of the bridge is 252 m.

Section 4.1 Page 217 Question 13

a) The equation $x^2 + 6x + k = 0$ will have one real root when the graph of the corresponding function has its vertex on the x-axis, *i.e.* in the form $y = a(x - p)^2 + q$, $q = 0$.

Complete the square to get an expression for q.

$y = x^2 + 6x + k$

$y = (x^2 + 6x) + k$

$y = (x^2 + 6x + 9 – 9) + k$

$y = (x^2 + 6x + 9) – 9 + k$

$y = (x + 3)^2 – 9 + k$

So, $q = –9 + k$. Let $q = 0$ and solve for k.

$0 = –9 + k$

$k = 9$

b) For $a > 0$, the equation $x^2 + 6x + k = 0$ will have two real roots when the graph of the corresponding function has its vertex below the x-axis, $q < 0$. Using the expression for q from part a), $k < 9$.

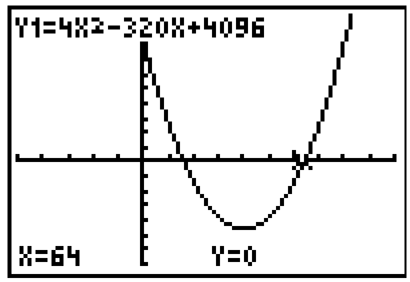
c) For $a > 0$, the equation $x^2 + 6x + k = 0$ will have no real roots when the graph of the corresponding function has its vertex above the x-axis, $q > 0$. Using the expression for q from part a), $k > 9$.

Section 4.1 Page 217 Question 14

a) To find the height of an arch with a span of 64 ft and a radius of 40 ft, substitute $s = 64$ and $r = 40$ into $4h^2 - 8hr + s^2 = 0$.

$4h^2 - 8h(40) + 64^2 = 0$

$4h^2 - 320h + 4096 = 0$



Graph the corresponding function to find h. The arch must be 64 ft high.

b) The equation would not change if all measurements were in metres. The input values would have to be given in metres.

Section 4.1 Page 217 Question 15

First, determine the distance of the Ultra Range after 5 s.

$d(t) = 1.5t^2$

$d(5) = 1.5(5)^2$

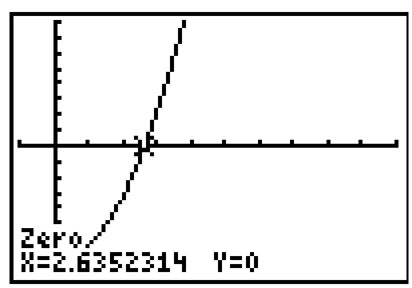
$d(5) = 37.5$

Next, determine the time it takes for the Edison to reach this distance.

$37.5 = 5.4t^2$

$0 = 5.4t^2 - 37.5$

Graph the corresponding function to solve for t.



It takes about 2.6 s for the Edison to reach the same distance as the Ultra Range after 5 s.

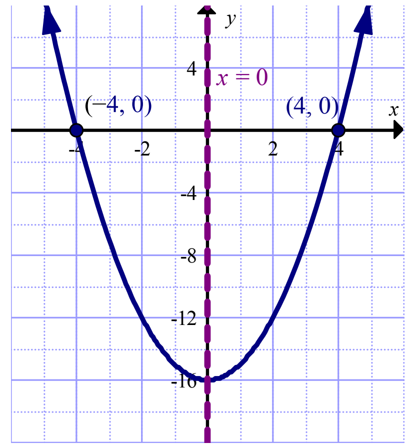
So, the Edison should start 2.4 s after the Ultra Range, to the nearest tenth of a second.

Section 4.1 Page 217 Question 16

For the value of the function to change from negative when $x = 1$ to positive when $x = 2$, it must cross the x-axis. So, there must be an x-intercept between the two values of x.

Section 4.1 Page 217 Question 17

For any point other than the vertex, there is a corresponding point that is equidistant from the axis of symmetry. For the axis of symmetry $x = 0$ and an x-intercept of –4, the corresponding point for (–4, 0) is (4, 0), the other x-intercept.



Section 4.1 Page 217 Question 18

The x-coordinate of the vertex is halfway between the two roots of 6 and –2. So, it is at 2. You can then substitute $x = 2$ into the equation to find that the y-coordinate of the vertex is –16.

Section 4.2 Factoring Quadratic Equations

Section 4.2 Page 229 Question 1

a) $x^2 + 7x + 10 = (x + 5)(x + 2)$

b) $5z^2 + 40z + 60 = 5(z^2 + 8z + 12)$
$= 5(z + 2)(z + 6)$

c) $0.2d^2 – 2.2d + 5.6 = 0.2(d^2 – 11d + 28)$
$= 0.2(d – 4)(d – 7)$

Section 4.2 Page 229 Question 2

a) $3y^2 + 4y – 7 = (3y + 7)(y – 1)$

b) $8k^2 – 6k – 5 = (2k + 1)(4k – 5)$

c) $0.4m^2 + 0.6m – 1.8 = 0.2(2m^2 + 3m – 9)$
$= 0.2(2m – 3)(m + 3)$

Section 4.2 Page 230 Question 3

a) $x^2 + x - 20 = (x + 5)(x - 4)$

b) $x^2 - 12x + 36 = (x - 6)(x - 6)$
$= (x - 6)^2$

c) $\frac{1}{4}x^2 + 2x + 3 = \frac{1}{4}(x^2 + 8x + 12)$
$= \frac{1}{4}(x + 2)(x + 6)$

d) $2x^2 + 12x + 18 = 2(x^2 + 6x + 9)$
$= 2(x + 3)(x + 3)$
$= 2(x + 3)^2$

Section 4.2 Page 230 Question 4

a) $4y^2 - 9x^2 = (2y - 3x)(2y + 3x)$

b) $0.36p^2 - 0.49q^2 = (0.6p - 0.7q)(0.6p + 0.7q)$

c) $\frac{1}{4}s^2 - \frac{9}{25}t^2 = \left(\frac{1}{2}s - \frac{3}{5}t\right)\left(\frac{1}{2}s + \frac{3}{5}t\right)$

d) $0.16t^2 - 16s^2 = (0.4t - 4s)(0.4t + 4s)$

Section 4.2 Page 230 Question 5

a) Let $r = x + 2$.
$(x + 2)^2 - (x + 2) - 42$
$= r^2 - r - 42$
$= (r - 7)(r + 6)$
$= (x + 2 - 7)(x + 2 + 6)$
$= (x - 5)(x + 8)$

b) Let $r = x^2 - 4x + 4$.
$6(x^2 - 4x + 4)^2 + (x^2 - 4x + 4) - 1$
$= 6r^2 + r - 1$
$= (3r - 1)(2r + 1)$
$= (3(x^2 - 4x + 4) - 1)(2(x^2 - 4x + 4) + 1)$
$= (3x^2 - 12x + 12 - 1)(2x^2 - 8x + 8 + 1)$
$= (3x^2 - 12x + 11)(2x^2 - 8x + 9)$

c) Use the pattern for factoring a difference of squares.
$(4j - 2)^2 - (2 + 4j)^2$
$= [(4j - 2) - (2 + 4j)][(4j - 2) + (2 + 4j)]$
$= (4j - 2 - 2 - 4j)(4j - 2 + 2 + 4j)$
$= (-4)(8j)$

Section 4.2 Page 230 Question 6

a) Let $r = 5b - 3$.

$4(5b - 3)^2 + 10(5b - 3) - 6$
$= 4r^2 + 10r - 6$
$= 2(2r^2 + 5r - 3)$
$= 2(2r - 1)(r + 3)$
$= 2(2(5b - 3) - 1)(5b - 3 + 3)$
$= 2(10b - 7)(5b)$

b) Use the pattern for factoring a difference of squares.

$16(x^2 + 1)^2 - 4(2x)^2$
$= [4(x^2 + 1) - 2(2x)][4(x^2 + 1) + 2(2x)]$
$= (4x^2 + 4 - 4x)(4x^2 + 4 + 4x)$
$= 16(x^2 - x + 1)(x^2 + x + 1)$

c) $-\frac{1}{4}(2x)^2 + 25(2y^3)^2$

$= -\frac{1}{4}\left[(2x)^2 - 100(2y^3)^2\right]$

$= -\frac{1}{4}[2x - 10(2y^3)][2x + 10(2y^3)]$

$= -\frac{1}{4}(2x - 20y^3)(2x + 20y^3)$

$= -(x - 10y^3)(x + 10y^3)$ or $(10y^3 - x)(10y^3 + x)$

Section 4.2 Page 230 Question 7

a) $(x + 3)(x + 4) = 0$

$x + 3 = 0$ or $x + 4 = 0$
$x = -3$ $x = -4$

The roots are –3 and –4.

b) $(x - 2)\left(x + \frac{1}{2}\right) = 0$

$x - 2 = 0$ or $x + \frac{1}{2} = 0$

$x = 2$ $x = -\frac{1}{2}$

The roots are 2 and $-\frac{1}{2}$.

c) $(x + 7)(x - 8) = 0$

$x + 7 = 0$ or $x - 8 = 0$

$x = -7$ $\quad$ $x = 8$

The roots are –7 and 8.

d) $x(x + 5) = 0$

$x = 0$ or $x + 5 = 0$

$x = -5$

The roots are 0 and –5.

e) $(3x + 1)(5x - 4) = 0$

$3x + 1 = 0$ or $5x - 4 = 0$

$3x = -1$ $\quad$ $5x = 4$

$x = -\frac{1}{3}$ $\quad$ $x = \frac{4}{5}$

The roots are $-\frac{1}{3}$ and $\frac{4}{5}$.

f) $2(x - 4)(7 - 2x) = 0$

$x - 4 = 0$ or $7 - 2x = 0$

$x = 4$ $\quad$ $-2x = -7$

$x = \frac{7}{2}$

The roots are 4 and $\frac{7}{2}$.

Section 4.2 Page 230 Question 8

a)

$$10n^2 - 40 = 0$$
$$10(n^2 - 4) = 0$$
$$10(n - 2)(n + 2) = 0$$

$n - 2 = 0$ or $n + 2 = 0$

$n = 2$ $\quad$ $n = -2$

For $n = 2$:

Left Side	Right Side
$10n^2 - 40$	0
$= 10(2)^2 - 40$	
$= 40 - 40$	
$= 0$	

Left Side = Right Side

For $n = -2$:

Left Side	Right Side
$10n^2 - 40$	0
$= 10(-2)^2 - 40$	
$= 40 - 40$	
$= 0$	

Left Side = Right Side

The roots are 2 and –2.

b) $\frac{1}{4}x^2 + \frac{5}{4}x + 1 = 0$

$\frac{1}{4}(x^2 + 5x + 4) = 0$

$\frac{1}{4}(x + 4)(x + 1) = 0$

$x + 4 = 0$ or $x + 1 = 0$

$x = -4$ $\quad$ $x = -1$

For $x = -4$:

Left Side	Right Side
$\frac{1}{4}x^2 + \frac{5}{4}x + 1$	0
$= \frac{1}{4}(-4)^2 + \frac{5}{4}(-4) + 1$	
$= 4 - 5 + 1$	
$= 0$	

Left Side = Right Side

For $x = -1$:

Left Side	Right Side
$\frac{1}{4}x^2 + \frac{5}{4}x + 1$	0
$= \frac{1}{4}(-1)^2 + \frac{5}{4}(-1) + 1$	
$= \frac{1}{4} - \frac{5}{4} + 1$	
$= 0$	

Left Side = Right Side

The roots are –4 and –1.

c) $3w^2 + 28w + 9 = 0$

$(3w + 1)(w + 9) = 0$

$3w + 1 = 0$ or $w + 9 = 0$

$3w = -1$ $\quad$ $w = -9$

$w = -\frac{1}{3}$

For $w = -\frac{1}{3}$:

Left Side	Right Side
$3w^2 + 28w + 9$	0
$= 3\left(-\frac{1}{3}\right)^2 + 28\left(-\frac{1}{3}\right) + 9$	
$= \frac{1}{3} - \frac{28}{3} + \frac{27}{3}$	
$= 0$	

Left Side = Right Side

For $w = -9$:

Left Side	Right Side
$3w^2 + 28w + 9$	0
$= 3(-9)^2 + 28(-9) + 9$	
$= 243 - 252 + 9$	
$= 0$	

Left Side = Right Side

The roots are $-\frac{1}{3}$ and –9.

d) $8y^2 - 22y + 15 = 0$

$(4y - 5)(2y - 3) = 0$

$4y - 5 = 0$ or $2y - 3 = 0$

$4y = 5$ $\quad$ $2y = 3$

$y = \frac{5}{4}$ $\quad$ $y = \frac{3}{2}$

For $y = \frac{5}{4}$:

Left Side	Right Side
$8y^2 - 22y + 15$	0
$= 8\left(\frac{5}{4}\right)^2 - 22\left(\frac{5}{4}\right) + 15$	
$= \frac{25}{2} - \frac{55}{2} + \frac{30}{2}$	
$= 0$	

Left Side = Right Side

For $y = \frac{3}{2}$:

Left Side	Right Side
$8y^2 - 22y + 15$	0
$= 8\left(\frac{3}{2}\right)^2 - 22\left(\frac{3}{2}\right) + 15$	
$= 18 - 33 + 15$	
$= 0$	

Left Side = Right Side

The roots are $\frac{5}{4}$ and $\frac{3}{2}$.

e) $d^2 + \frac{5}{2}d + \frac{3}{2} = 0$

$\frac{1}{2}(2d^2 + 5d + 3) = 0$

$\frac{1}{2}(2d + 3)(d + 1) = 0$

$2d + 3 = 0$ or $d + 1 = 0$

$2d = -3$ $\quad$ $d = -1$

$d = -\frac{3}{2}$

For $d = -\frac{3}{2}$:

Left Side	Right Side
$d^2 + \frac{5}{2}d + \frac{3}{2}$	0
$= \left(-\frac{3}{2}\right)^2 + \frac{5}{2}\left(-\frac{3}{2}\right) + \frac{3}{2}$	
$= \frac{9}{4} - \frac{15}{4} + \frac{6}{4}$	
$= 0$	

Left Side = Right Side

For $y = -1$:

Left Side	Right Side
$d^2 + \frac{5}{2}d + \frac{3}{2}$	0
$= (-1)^2 + \frac{5}{2}(-1) + \frac{3}{2}$	
$= \frac{2}{2} - \frac{5}{2} + \frac{3}{2}$	
$= 0$	

Left Side = Right Side

The roots are $-\frac{3}{2}$ and -1.

f) $4x^2 - 12x + 9 = 0$
$(2x - 3)(2x - 3) = 0$
$2x - 3 = 0$
$2x = 3$
$x = \frac{3}{2}$

For $x = \frac{3}{2}$:

Left Side	Right Side
$4x^2 - 12x + 9$	0
$= 4\left(\frac{3}{2}\right)^2 - 12\left(\frac{3}{2}\right) + 9$	
$= 9 - 18 + 9$	
$= 0$	

Left Side = Right Side

The root is $\frac{3}{2}$.

Section 4.2 Page 230 Question 9

a) $k^2 - 5k = 0$
$k(k - 5) = 0$
$k = 0$ or $k - 5 = 0$
$k = 5$

For $k = 0$:

Left Side	Right Side
$k^2 - 5k$	0
$= 0^2 - 5(0)$	
$= 0 - 0$	
$= 0$	

Left Side = Right Side

For $k = 5$:

Left Side	Right Side
$k^2 - 5k$	0
$= 5^2 - 5(5)$	
$= 25 - 25$	
$= 0$	

Left Side = Right Side

The roots are 0 and 5.

b) $9x^2 = x + 8$
$9x^2 - x - 8 = 0$
$(9x + 8)(x - 1) = 0$
$9x + 8 = 0$ or $x - 1 = 0$
$9x = -8$ $x = 1$
$x = -\frac{8}{9}$

For $x = -\frac{8}{9}$:

Left Side	Right Side
$9x^2 - x - 8$	0
$= 9\left(-\frac{8}{9}\right)^2 - \left(-\frac{8}{9}\right) - 8$	
$= \frac{64}{9} + \frac{8}{9} - \frac{72}{9}$	
$= 0$	

Left Side = Right Side

For $x = 1$:

Left Side	Right Side
$9x^2 - x - 8$	0
$= 9(1)^2 - 1 - 8$	
$= 9 - 9$	
$= 0$	

Left Side = Right Side

The roots are $-\frac{8}{9}$ and 1.

c) $\frac{8}{3}t + 5 = -\frac{1}{3}t^2$

$\frac{1}{3}t^2 + \frac{8}{3}t + 5 = 0$

$\frac{1}{3}(t^2 + 8t + 15) = 0$

$\frac{1}{3}(t + 3)(t + 5) = 0$

$t + 3 = 0$ or $t + 5 = 0$

$t = –3$ $\quad$ $t = –5$

For $t = –3$:

Left Side	Right Side
$\frac{8}{3}t + 5$	$-\frac{1}{3}t^2$
$= \frac{8}{3}(-3) + 5$	$= -\frac{1}{3}(-3)^2$
$= -8 + 5$	$= -3$
$= -3$	

Left Side = Right Side

For $t = –5$:

Left Side	Right Side
$\frac{8}{3}t + 5$	$-\frac{1}{3}t^2$
$= \frac{8}{3}(-5) + 5$	$= -\frac{1}{3}(-5)^2$
$= -\frac{40}{3} + \frac{15}{3}$	$= -\frac{25}{3}$
$= -\frac{25}{3}$	

Left Side = Right Side

The roots are –3 and –5.

d) $\frac{25}{49}y^2 - 9 = 0$

$\left(\frac{5}{7}y - 3\right)\left(\frac{5}{7}y + 3\right) = 0$

$\frac{5}{7}y - 3 = 0$ or $\frac{5}{7}y + 3 = 0$

$\frac{5}{7}y = 3$ $\quad\quad$ $\frac{5}{7}y = -3$

$y = \frac{21}{5}$ $\quad\quad$ $y = -\frac{21}{5}$

For $y = \frac{21}{5}$:

Left Side	Right Side
$\frac{25}{49}y^2 - 9$	0
$= \frac{25}{49}\left(\frac{21}{5}\right)^2 - 9$	
$= \frac{441}{49} - \frac{441}{49}$	
$= 0$	

Left Side = Right Side

For $y = -\frac{21}{5}$:

Left Side	Right Side
$\frac{25}{49}y^2 - 9$	0
$= \frac{25}{49}\left(-\frac{21}{5}\right)^2 - 9$	
$= \frac{441}{49} - \frac{441}{49}$	
$= 0$	

Left Side = Right Side

The roots are $\frac{21}{5}$ and $-\frac{21}{5}$.

e) $2s^2 - 4s = 70$

$2s^2 - 4s - 70 = 0$

$2(s^2 - 2s - 35) = 0$

$2(s - 7)(s + 5) = 0$

$s - 7 = 0$ or $s + 5 = 0$

$s = 7$ $\quad\quad$ $s = -5$

For $s = 7$:

Left Side	Right Side
$2s^2 - 4s$	70
$= 2(7)^2 - 4(7)$	
$= 98 - 28$	
$= 70$	

Left Side = Right Side

For $s = -5$:

Left Side	Right Side
$2s^2 - 4s$	70
$= 2(-5)^2 - 4(-5)$	
$= 50 + 20$	
$= 70$	

Left Side = Right Side

The roots are 7 and –5.

f) $4q^2 - 28q = -49$

$4q^2 - 28q + 49 = 0$

$(2q - 7)(2q - 7) = 0$

$2q - 7 = 0$

$2q = 7$

$q = \frac{7}{2}$

For $q = \frac{7}{2}$:

Left Side | Right Side
$4q^2 - 28q$ | -49

$= 4\left(\frac{7}{2}\right)^2 - 28\left(\frac{7}{2}\right)$

$= 49 - 98$

$= -49$

Left Side = Right Side

The root is $\frac{7}{2}$.

Section 4.2 Page 230 Question 10

a) $42 = x^2 - x$

$0 = x^2 - x - 42$

$0 = (x - 7)(x + 6)$

$x - 7 = 0$ or $x + 6 = 0$

$x = 7$ $x = -6$

The roots are 7 and –6.

b) $g^2 = 30 - 7g$

$g^2 + 7g - 30 = 0$

$(g - 3)(g + 10) = 0$

$g - 3 = 0$ or $g + 10 = 0$

$g = 3$ $g = -10$

The roots are 3 and –10.

c) $y^2 + 4y = 21$

$y^2 + 4y - 21 = 0$

$(y - 3)(y + 7) = 0$

$y - 3 = 0$ or $y + 7 = 0$

$y = 3$ $y = -7$

The roots are 3 and –7.

d) $3 = 6p^2 - 7p$

$6p^2 - 7p - 3 = 0$

$(3p + 1)(2p - 3) = 0$

$3p + 1 = 0$ or $2p - 3 = 0$

$3p = -1$ $\qquad 2p = 3$

$p = -\frac{1}{3}$ $\qquad p = \frac{3}{2}$

The roots are $-\frac{1}{3}$ and $\frac{3}{2}$.

e) $3x^2 + 9x = 30$

$3x^2 + 9x - 30 = 0$

$3(x^2 + 3x - 10) = 0$

$3(x - 2)(x + 5) = 0$

$x - 2 = 0$ or $x + 5 = 0$

$x = 2$ $\qquad x = -5$

The roots are 2 and –5.

f) $2z^2 = 3 - 5z$

$2z^2 + 5z - 3 = 0$

$(2z - 1)(z + 3) = 0$

$2z - 1 = 0$ or $z + 3 = 0$

$2z = 1$ $\qquad z = -3$

$z = \frac{1}{2}$

The roots are $\frac{1}{2}$ and –3.

Section 4.2 Page 230 Question 11

a) Substitute the dimensions and given area into $A = \ell w$:

$54 = (x + 10)(2x - 3)$

$54 = 2x^2 + 17x - 30$

$0 = 2x^2 + 17x - 84$

b) Solve the equation from part a) to find the value of x.

$0 = 2x^2 + 17x - 84$

$0 = (2x - 7)(x + 12)$

$2x - 7 = 0$ or $x + 12 = 0$

$2x = 7$ $\qquad x = -12$

$x = \frac{7}{2}$

Since x represents a distance, it cannot be negative. So, reject the root –12.

The value of x is $\frac{7}{2}$, or 3.5 cm.

Section 4.2 Page 230 Question 12

a) To find the time it takes the osprey to reach a height of 20 m, solve the equation $20 = 5t^2 - 30t + 45$.

$20 = 5t^2 - 30t + 45$
$0 = 5t^2 - 30t + 25$
$0 = 5(t^2 - 6t + 5)$
$0 = 5(t - 1)(t - 5)$
$t - 1 = 0$ or $t - 5 = 0$
$t = 1$ $\quad$ $t = 5$

It takes the osprey 1 s to reach a height of 20 m above the water on its dive towards the salmon. It again is at this height at 5 s, flying away with its catch.

b) Example: Assume no winds and that the mass of the fish does not affect the speed at which the osprey flies after catching the fish.

Section 4.2 Page 231 Question 13

a) To find the time it takes the flare to return to the water, solve the equation $0 = 150t - 5t^2$.

b) $0 = 150t - 5t^2$
$0 = 5t(30 - t)$
$5t = 0$ or $30 - t = 0$
$t = 0$ $\quad$ $t = 30$

It takes 30 s for the flare to return to the water.

Section 4.2 Page 231 Question 14

Let the two consecutive even integers be x and $x + 2$. For a product of $8x + 16$,

$8x + 16 = x(x + 2)$
$8x + 16 = x^2 + 2x$
$0 = x^2 - 6x - 16$
$0 = (x - 8)(x + 2)$
$x - 8 = 0$ or $x + 2 = 0$
$x = 8$ $\quad$ $x = -2$

The two consecutive even integers are 8 and 10 or –2 and 0.

Section 4.2 Page 231 Question 15

Let x represent the side length of the square.
Then, the new dimensions are $(x + 10)$ and $(x + 12)$ and the new area is $3x^2$.
To determine the side length of the square, solve the equation $3x^2 = (x + 10)(x + 12)$.

$3x^2 = (x + 10)(x + 12)$
$3x^2 = x^2 + 22x + 120$
$0 = 2x^2 – 22x – 120$
$0 = 2(x^2 – 11x – 60)$
$0 = 2(x + 4)(x – 15)$
$x + 4 = 0$ or $x – 15 = 0$
$x = –4$ $x = 15$

Since x represents a side length, it cannot be negative. So, reject the root –4.
The side length is 15 cm.

Section 4.2 Page 231 Question 16

To find how long the ball was in the air before it is caught, solve the equation
$3 = 3 + 48t – 16t^2$.
$0 = 48t – 16t^2$
$0 = 16t(3 – t)$
$16t = 0$ or $3 – t = 0$
$t = 0$ $t = 3$

The ball was in the air for 3 s before it was caught.

Example: This time duration seems too long considering the ball went up to a maximum height of 39 ft. However, the initial velocity was 48 ft/s and the ball would be slowing down under the effects of gravity so 3 s may be realistic.

Section 4.2 Page 231 Question 17

a) To find the width of each strip, solve the equation $35 = (9 – 2x)(7 – 2x)$.
$35 = (9 – 2x)(7 – 2x)$
$35 = 63 – 32x + 4x^2$
$0 = 4x^2 – 32x + 28$
$0 = 4(x^2 – 8x + 7)$
$0 = 4(x – 7)(x – 1)$
$x – 7 = 0$ or $x – 1 = 0$
$x = 7$ $x = 1$

Since $x = 7$ would result in negative dimensions, reject this root.
The width of each strip is 1 cm.

b) The dimensions of the new rectangle are 7 cm by 5 cm.

Section 4.2 Page 232 Question 18

a) Check if $x = 5$ is a root of $x^2 - 5x - 36 = 0$.

Left Side	Right Side
$x^2 - 5x - 36$	0

$= 5^2 - 5(5) - 36$
$= 25 - 25 - 36$
$= -36$

Left Side ≠ Right Side

Since $x = 5$ is not a root, $x - 5$ is not a factor of $x^2 - 5x - 36$.

b) Check if $x = -3$ is a root of $x^2 - 2x - 15 = 0$.

Left Side	Right Side
$x^2 - 2x - 15$	0

$= (-3)^2 - 2(-3) - 15$
$= 9 + 6 - 15$
$= 0$

Left Side = Right Side

Since $x = -3$ is a root, $x + 3$ is a factor of $x^2 - 2x - 15$.

c) Check if $x = -\frac{1}{4}$ is a root of $6x^2 + 11x + 4 = 0$.

Left Side	Right Side
$6x^2 + 11x + 4$	0

$= 6\left(-\frac{1}{4}\right)^2 + 11\left(-\frac{1}{4}\right) + 4$
$= \frac{6}{16} - \frac{11}{4} + 4$
$= \frac{13}{8}$

Left Side ≠ Right Side

Since $x = -\frac{1}{4}$ is not a root, $4x + 1$ is not a factor of $6x^2 + 11x + 4$.

d) Check if $x = \frac{1}{2}$ is a root of $4x^2 + 4x - 3 = 0$.

Left Side	Right Side
$4x^2 + 4x - 3$	0

$= 4\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right) - 3$
$= 1 + 2 - 3$

$= 0$

Left Side = Right Side

Since $x = \frac{1}{2}$ is a root, $2x – 1$ is a factor of $4x^2 + 4x – 3$.

Section 4.2 Page 232 Question 19

a) $x(2x – 3) – 2(3 + 2x) = –4(x + 1)$

$2x^2 – 3x – 6 – 4x = –4x – 4$

$2x^2 – 3x – 2 = 0$

$(2x + 1)(x – 2) = 0$

$2x + 1 = 0$ or $x – 2 = 0$

$2x = –1$ $\quad x = 2$

$x = -\frac{1}{2}$

The roots are $-\frac{1}{2}$ and 2.

b) $3(x – 2)(x + 1) – 4 = 2(x – 1)^2$

$3(x^2 – x – 2) – 4 = 2(x^2 – 2x + 1)$

$3x^2 – 3x – 6 – 4 = 2x^2 – 4x + 2$

$x^2 + x – 12 = 0$

$(x + 4)(x – 3) = 0$

$x + 4 = 0$ or $x – 3 = 0$

$x = –4$ $\quad x = 3$

The roots are –4 and 3.

Section 4.2 Page 232 Question 20

Use the Pythagorean Theorem.

$x^2 + (x – 1)^2 = 29^2$

$x^2 + x^2 – 2x + 1 = 841$

$2x^2 – 2x – 840 = 0$

$2(x^2 – x – 420) = 0$

$2(x – 21)(x + 20) = 0$

$x – 21 = 0$ or $x + 20 = 0$

$x = 21$ $\quad x = –21$

Since x represents a leg of a right triangle, it cannot be negative. So, reject the root –21.
The lengths of the legs are 21 cm and 20 cm.

Section 4.2 Page 232 Question 21

Let x represent the length of one leg of the right triangle. Then, the other leg is $23 - x$.

$x^2 + (23 - x)^2 = 17^2$
$x^2 + 529 - 46x + x^2 = 289$
$2x^2 - 46x - 240 = 0$
$2(x^2 - 23x - 120) = 0$
$2(x - 8)(x - 15) = 0$
$x - 8 = 0$ or $x - 15 = 0$
$x = 8$ $x = 15$

The length of the legs of the right triangle are 8 cm and 15 cm.

Section 4.2 Page 232 Question 22

a) Let x represent the length of the notebook. Then, the width is $x - 7$.
An equation to represent the surface area of the notebook is $690 = x(x - 7)$.

b) $690 = x(x - 7)$
$690 = x^2 - 7x$
$0 = x^2 - 7x - 690$
$0 = (x - 30)(x + 23)$
$x - 30 = 0$ or $x + 23 = 0$
$x = 30$ $x = -23$

Since x represents a length, it cannot be negative. So, reject the root –23.
The dimensions of the top of the computer are 30 cm by 23 cm.

Section 4.2 Page 232 Question 23

To find the width of the walkway, solve the equation $(40 + 2x)(20 + 2x) - 20(40) = 700$.

$(40 + 2x)(20 + 2x) - 20(40) = 700$
$800 + 120x + 4x^2 - 800 = 700$
$4x^2 + 120x - 700 = 0$
$4(x^2 + 30x - 175) = 0$
$4(x + 35)(x - 5) = 0$
$x + 35 = 0$ or $x - 5 = 0$
$x = -35$ $x = 5$

Since x represents the width of the walkway, it cannot be negative. So, reject the root –35.
The width of the walkway is 5 m.

Section 4.2 Page 232 Question 24

Let x represent one leg of the right triangle. Then, the hypotenuse is $18 - x$.
Use the Pythagorean Theorem to find the height at which the tree broke.

$$
\begin{aligned}
x^2 + 12^2 &= (18 - x)^2 \\
x^2 + 144 &= 324 - 36x + x^2 \\
36x - 180 &= 0 \\
36(x - 5) &= 0 \\
x - 5 &= 0 \\
x &= 5
\end{aligned}
$$

The tree broke at a height of 5 m.

Section 4.2 Page 232 Question 25

Use the pattern for factoring a difference of squares.

$$P = \left(\frac{1}{2}d\right)(v_1)^2 - \left(\frac{1}{2}d\right)(v_2)^2$$

$$P = \frac{1}{2}d[(v_1)^2 - (v_2)^2]$$

$$P = \frac{1}{2}d(v_1 - v_2)(v_1 + v_2)$$

Section 4.2 Page 232 Question 26

Carlos's result $6x^2 - 16x + 8 = (x - 2)(6x - 4)$ is not completely factored.
The fully factored form is $6x^2 - 16x + 8 = 2(x - 2)(3x - 2)$.

Section 4.2 Page 232 Question 27

a) Let $r = 2z + 3$.

$$
\begin{aligned}
&3(2z + 3)^2 - 9(2z + 3) - 30 \\
&= 3r^2 - 9r - 30 \\
&= 3(r^2 - 3r - 10) \\
&= 3(r - 5)(r + 2) \\
&= 3(2z + 3 - 5)(2z + 3 + 2) \\
&= 3(2z - 2)(2z + 5) \\
&= 6(z - 1)(2z + 5)
\end{aligned}
$$

b) Use the pattern for factoring a difference of squares.

$16(m^2 - 4)^2 - 4(3n)^2$
$= 4[4(m^2 - 4)^2 - (3n)^2]$
$= 4[2(m^2 - 4) - 3n][2(m^2 - 4) + 3n]$
$= 4(2m^2 - 8 - 3n)(2m^2 - 8 + 3n)$

c) $\frac{1}{9}y^2 - \frac{1}{3}yx + \frac{1}{4}x^2$

$= \frac{1}{36}(4y^2 - 12yx + 9x^2)$

$= \frac{1}{36}(2y - 3x)(2y - 3x)$

$= \frac{1}{36}(2y - 3x)^2$

d) Use the pattern for factoring a difference of squares.

$-28\left(w + \frac{2}{3}\right)^2 + 7\left(3w - \frac{1}{3}\right)^2$

$= -7\left[4\left(w + \frac{2}{3}\right)^2 - \left(3w - \frac{1}{3}\right)^2\right]$

$= -7\left[2\left(w + \frac{2}{3}\right) - \left(3w - \frac{1}{3}\right)\right]\left[2\left(w + \frac{2}{3}\right) + \left(3w - \frac{1}{3}\right)\right]$

$= -7\left(2w + \frac{4}{3} - 3w + \frac{1}{3}\right)\left(2w + \frac{4}{3} + 3w - \frac{1}{3}\right)$

$= -7\left(-w + \frac{5}{3}\right)(5w + 1)$

$= 7\left(w - \frac{5}{3}\right)(5w + 1)$

Section 4.2 Page 232 Question 28

To find an expression for the side length of the square, factor $9x^2 + 30xy + 25y^2$.

$9x^2 + 30xy + 25y^2$
$= (3x + 5y)(3x + 5y)$

An expression for the perimeter of the square is $4(3x + 5y)$ centimetres.

Section 4.2 Page 233 Question 29

To find when the company will start to make a profit, solve the equation
$0 = 1125(t - 1)^2 - 10\,125$.

$0 = 1125(t - 1)^2 - 10\,125$
$0 = 1125[(t - 1)^2 - 9]$
$0 = 1125[(t - 1) - 3][(t - 1) + 3]$
$0 = 1125(t - 4)(t + 2)$
$t - 4 = 0$ or $t + 2 = 0$
$t = 4$ $t = -2$

Since t represents time, it cannot be negative. So, reject the root –2.
The company will start to make a profit after 4 years.

Section 4.2 Page 233 Question 30

a) For roots –3 and 3:
$(x + 3)(x - 3) = 0$
$x^2 - 9 = 0$

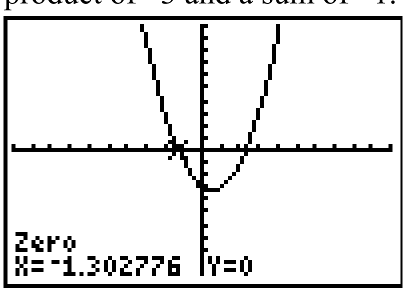
b) For root 2:
$(x - 2)^2 = 0$
$x^2 - 4x + 4 = 0$

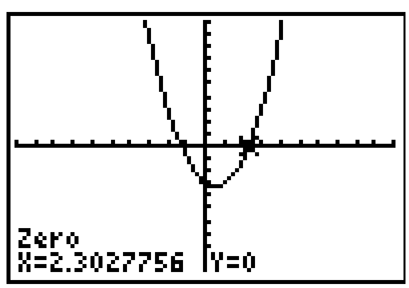
c) For roots $\frac{2}{3}$ and 4:
$(3x - 2)(x - 4) = 0$
$3x^2 - 14x + 8 = 0$

d) For roots $\frac{3}{5}$ and $-\frac{1}{2}$:
$(5x - 3)(2x + 1) = 0$
$10x^2 - x - 3 = 0$

Section 4.2 Page 233 Question 31

Example: $x^2 - x - 3 = 0$
This equation cannot be solved by factoring because there are no two integers with a product of –3 and a sum of –1.





Section 4.2 Page 233 Question 32

a) Instead of evaluating the difference of two numbers, 81 – 36, use the difference of squares pattern to rewrite the expression as (9 – 6)(9 + 6) and then simplify. You can only use this method when the two numbers are square numbers.

b) Examples:

$$\begin{aligned} 81 - 49 &= (9 - 7)(9 + 7) \\ &= 2(16) \\ &= 32 \end{aligned}$$

$$\begin{aligned} 121 - 36 &= (11 - 6)(11 + 6) \\ &= 5(17) \\ &= 85 \end{aligned}$$

Section 4.3 Solving Quadratic Equations by Completing the Square

Section 4.3 Page 240 Question 1

For each expression to be a perfect square, the value of c must be half the square of the coefficient of the x-term.

a) For $x^2 + x + c$, the coefficient of the x-term is 1. So, $c = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$.

b) For $x^2 - 5x + c$, the coefficient of the x-term is –5. So, $c = \left(\frac{-5}{2}\right)^2 = \frac{25}{4}$.

c) For $x^2 - 0.5x + c$, the coefficient of the x-term is –0.5. So, $c = \left(\frac{-0.5}{2}\right)^2 = 0.0625$.

d) For $x^2 + 0.2x + c$, the coefficient of the x-term is 0.2. So, $c = \left(\frac{0.2}{2}\right)^2 = 0.01$.

e) For $x^2 + 15x + c$, the coefficient of the x-term is 15. So, $c = \left(\frac{15}{2}\right)^2 = \frac{225}{4}$.

f) For $x^2 - 9x + c$, the coefficient of the x-term is –9. So, $c = \left(\frac{-9}{2}\right)^2 = \frac{81}{4}$.

Section 4.3 Page 240 Question 2

a)

$$\begin{aligned} 2x^2 + 8x + 4 &= 0 \\ x^2 + 4x + 2 &= 0 \\ x^2 + 4x &= -2 \\ x^2 + 4x + 4 &= -2 + 4 \\ (x + 2)^2 &= 2 \end{aligned}$$

b) $-3x^2 - 12x + 5 = 0$

$$x^2 + 4x - \frac{5}{3} = 0$$

$$x^2 + 4x = \frac{5}{3}$$

$$x^2 + 4x + 4 = \frac{5}{3} + 4$$

$$(x + 2)^2 = \frac{17}{3}$$

c) $\frac{1}{2}x^2 - 3x + 5 = 0$

$$x^2 - 6x + 10 = 0$$

$$x^2 - 6x = -10$$

$$x^2 - 6x + 9 = -10 + 9$$

$$(x - 3)^2 = -1$$

Section 4.3 Page 240 Question 3

a) $x^2 - 12x + 9 = 0$

$$(x^2 - 12x + 36) - 36 + 9 = 0$$

$$(x - 6)^2 - 27 = 0$$

b) $5x^2 - 20x - 1 = 0$

$$5(x^2 - 4x) - 1 = 0$$

$$5(x^2 - 4x + 4 - 4) - 1 = 0$$

$$5(x - 2)^2 - 20 - 1 = 0$$

$$5(x - 2)^2 - 21 = 0$$

c) $-2x^2 + x - 1 = 0$

$$-2\left(x^2 - \frac{1}{2}x\right) - 1 = 0$$

$$-2\left(x^2 - \frac{1}{2}x + \frac{1}{16} - \frac{1}{16}\right) - 1 = 0$$

$$-2\left(x - \frac{1}{4}\right)^2 + \frac{1}{8} - 1 = 0$$

$$-2\left(x - \frac{1}{4}\right)^2 - \frac{7}{8} = 0$$

d) $0.5x^2 + 2.1x + 3.6 = 0$

$$0.5(x^2 + 4.2x) + 3.6 = 0$$

$$0.5(x^2 + 4.2x + 4.41 - 4.41) + 3.6 = 0$$

$$0.5(x + 2.1)^2 - 2.205 + 3.6 = 0$$

$$0.5(x + 2.1)^2 + 1.395 = 0$$

e) $-1.2x^2 - 5.1x - 7.4 = 0$

$$-1.2(x^2 + 4.25x) - 7.4 = 0$$

$$-1.2(x^2 + 4.25x + 4.515\ 625 - 4.515\ 625) - 7.4 = 0$$

$$-1.2(x + 2.125)^2 + 5.418\ 75 - 7.4 = 0$$

$$-1.2(x + 2.125)^2 - 1.981\ 25 = 0$$

f)
$$\frac{1}{2}x^2 + 3x - 6 = 0$$
$$\frac{1}{2}(x^2 + 6x) - 6 = 0$$
$$\frac{1}{2}(x^2 + 6x + 9 - 9) - 6 = 0$$
$$\frac{1}{2}(x + 3)^2 - \frac{9}{2} - 6 = 0$$
$$\frac{1}{2}(x + 3)^2 - \frac{21}{2} = 0$$

Section 4.3 Page 240 Question 4

a) $x^2 = 64$

$x = \pm 8$

b) $2s^2 - 8 = 0$

$2s^2 = 8$

$s = \pm 2$

c) $\frac{1}{3}t^2 - 1 = 11$

$\frac{1}{3}t^2 = 12$

$t^2 = 36$

$t = \pm 6$

d) $-y^2 + 5 = -6$

$y^2 = 11$

$y = \pm\sqrt{11}$

Section 4.3 Page 241 Question 5

a) $(x - 3)^2 = 4$

$x - 3 = \pm 2$

$x = 3 \pm 2$

$x = 3 + 2$ or $x = 3 - 2$

$x = 5$ $x = 1$

b) $(x + 2)^2 = 9$

$x + 2 = \pm 3$

$x = -2 \pm 3$

$x = -2 + 3$ or $x = -2 - 3$

$x = 1$ $x = -5$

c) $\left(d + \frac{1}{2}\right)^2 = 1$

$d + \frac{1}{2} = \pm 1$

$d = -\frac{1}{2} \pm 1$

$d = -\frac{1}{2} + 1$ or $d = -\frac{1}{2} - 1$

$d = \frac{1}{2}$ $d = -\frac{3}{2}$

d) $\left(h - \frac{3}{4}\right)^2 = \frac{7}{16}$

$h - \frac{3}{4} = \pm\sqrt{\frac{7}{16}}$

$h = \frac{3}{4} \pm \frac{\sqrt{7}}{4}$

$h = \frac{3 \pm \sqrt{7}}{4}$

e) $(s + 6)^2 = \frac{3}{4}$

$$s + 6 = \pm\sqrt{\frac{3}{4}}$$

$$s = -6 \pm \frac{\sqrt{3}}{2}$$

$$s = \frac{-12 \pm \sqrt{3}}{2}$$

f) $(x + 4)^2 = 18$

$$x + 4 = \pm\sqrt{18}$$

$$x = -4 \pm 3\sqrt{2}$$

Section 4.3 Page 241 Question 6

a) $x^2 + 10x + 4 = 0$

$$x^2 + 10x = -4$$

$$x^2 + 10x + 25 = -4 + 25$$

$$(x + 5)^2 = 21$$

$$x + 5 = \pm\sqrt{21}$$

$$x = -5 \pm \sqrt{21}$$

b) $x^2 - 8x + 13 = 0$

$$x^2 - 8x = -13$$

$$x^2 - 8x + 16 = -13 + 16$$

$$(x - 4)^2 = 3$$

$$x - 4 = \pm\sqrt{3}$$

$$x = 4 \pm \sqrt{3}$$

c) $3x^2 + 6x + 1 = 0$

$$x^2 + 2x + \frac{1}{3} = 0$$

$$x^2 + 2x = -\frac{1}{3}$$

$$x^2 + 2x + 1 = -\frac{1}{3} + 1$$

$$(x + 1)^2 = \frac{2}{3}$$

$$x + 1 = \pm\sqrt{\frac{2}{3}}$$

$$x = -1 \pm \frac{\sqrt{6}}{3}$$

$$x = \frac{-3 \pm \sqrt{6}}{3}$$

d) $-2x^2 + 4x + 3 = 0$

$$x^2 - 2x - \frac{3}{2} = 0$$

$$x^2 - 2x = \frac{3}{2}$$

$$x^2 - 2x + 1 = \frac{3}{2} + 1$$

$$(x - 1)^2 = \frac{5}{2}$$

$$x - 1 = \pm\sqrt{\frac{5}{2}}$$

$$x = 1 \pm \frac{\sqrt{10}}{2}$$

$$x = \frac{2 \pm \sqrt{10}}{2}$$

e) $-0.1x^2 - 0.6x + 0.4 = 0$
$$x^2 + 6x - 4 = 0$$
$$x^2 + 6x = 4$$
$$x^2 + 6x + 9 = 4 + 9$$
$$(x + 3)^2 = 13$$
$$x + 3 = \pm\sqrt{13}$$
$$x = -3 \pm \sqrt{13}$$

f) $0.5x^2 - 4x - 6 = 0$
$$x^2 - 8x - 12 = 0$$
$$x^2 - 8x = 12$$
$$x^2 - 8x + 16 = 12 + 16$$
$$(x - 4)^2 = 28$$
$$x - 4 = \pm\sqrt{28}$$
$$x = 4 \pm 2\sqrt{7}$$

Section 4.3 Page 241 Question 7

a) $x^2 - 8x - 4 = 0$
$$x^2 - 8x = 4$$
$$x^2 - 8x + 16 = 4 + 16$$
$$(x - 4)^2 = 20$$
$$x - 4 = \pm\sqrt{20}$$
$x - 4 = \sqrt{20}$ or $x - 4 = -\sqrt{20}$
$x = 4 + \sqrt{20}$ $\quad$ $x = 4 - \sqrt{20}$
$x \approx 8.5$ $\quad$ $x \approx -0.5$

b) $-3x^2 + 4x + 5 = 0$
$$x^2 - \frac{4}{3}x = \frac{5}{3}$$
$$x^2 - \frac{4}{3}x + \frac{4}{9} = \frac{5}{3} + \frac{4}{9}$$
$$\left(x - \frac{2}{3}\right)^2 = \frac{19}{9}$$
$$x - \frac{2}{3} = \pm\sqrt{\frac{19}{9}}$$
$$x = \frac{2 \pm \sqrt{19}}{3}$$
$x = \frac{2 + \sqrt{19}}{3}$ or $x = \frac{2 - \sqrt{19}}{3}$
$x \approx 2.1$ $\quad$ $x \approx -0.8$

c) $\frac{1}{2}x^2 - 6x - 5 = 0$

$$x^2 - 12x = 10$$
$$x^2 - 12x + 36 = 10 + 36$$
$$(x - 6)^2 = 46$$
$$x - 6 = \pm\sqrt{46}$$

$x - 6 = \sqrt{46}$ or $x - 6 = -\sqrt{46}$

$x = 6 + \sqrt{46}$ $\quad$ $x = 6 - \sqrt{46}$

$x \approx 12.8$ $\quad$ $x \approx -0.8$

d) $0.2x^2 + 0.12x - 11 = 0$

$$x^2 + 0.6x = 55$$
$$x^2 + 0.6x + 0.09 = 55 + 0.09$$
$$(x + 0.3)^2 = 55.09$$
$$x + 0.3 = \pm\sqrt{55.09}$$

$x + 0.3 = \sqrt{55.09}$ or $x + 0.3 = -\sqrt{55.09}$

$x = -0.3 + \sqrt{55.09}$ $\quad$ $x = -0.3 - \sqrt{55.09}$

$x \approx 7.1$ $\quad$ $x \approx -7.7$

e) $-\frac{2}{3}x^2 - x + 2 = 0$

$$x^2 + \frac{3}{2}x = 3$$
$$x^2 + \frac{3}{2}x + \frac{9}{16} = 3 + \frac{9}{16}$$
$$\left(x + \frac{3}{4}\right)^2 = \frac{57}{16}$$
$$x + \frac{3}{4} = \pm\sqrt{\frac{57}{16}}$$
$$x = \frac{-3 \pm \sqrt{57}}{4}$$

$x = \frac{-3 + \sqrt{57}}{4}$ or $x = \frac{-3 - \sqrt{57}}{4}$

$x \approx 1.1$ $\quad$ $x \approx -2.6$

f) $\frac{3}{4}x^2 + 6x + 1 = 0$

$$x^2 + 8x = -\frac{4}{3}$$

$$x^2 + 8x + 16 = -\frac{4}{3} + 16$$

$$(x + 4)^2 = \frac{44}{3}$$

$$x + 4 = \pm\sqrt{\frac{44}{3}}$$

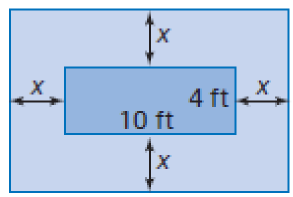
$x + 4 = \sqrt{\frac{44}{3}}$ or $x + 4 = -\sqrt{\frac{44}{3}}$

$x = -4 + \sqrt{\frac{44}{3}}$ $x = -4 - \sqrt{\frac{44}{3}}$

$x \approx -0.2$ $x \approx -7.8$

Section 4.3 Page 241 Question 8

a) Let x represent the distance added to each side of the kennel.



b) An equation that models the new area is $80 = (4 + 2x)(10 + 2x)$ or $0 = 4x^2 + 28x - 40$.

c) $4x^2 + 28x - 40 = 0$

$$x^2 + 7x = 10$$

$$x^2 + 7x + \frac{49}{4} = 10 + \frac{49}{4}$$

$$\left(x + \frac{7}{2}\right)^2 = \frac{89}{4}$$

$$x + \frac{7}{2} = \pm\sqrt{\frac{89}{4}}$$

$$x = \frac{-7 \pm \sqrt{89}}{2}$$

$x = \frac{-7 + \sqrt{89}}{2}$ or $x = \frac{-7 - \sqrt{89}}{2}$

$x \approx 1.2$ $x \approx -8.2$

Since the distance added to the kennel cannot be negative, $x = -8.2$ is an extraneous root. The dimensions of the new kennel are 4 + 2(1.2), or 6.4 ft, by 10 + 2(1.2), or 12.4 ft.

Section 4.3 Page 241 Question 9

a) Use the quadratic equation $0 = -0.02d^2 + 0.4d + 1$ to determine how far the disc will travel if no one catches it.

b)

$$-0.02d^2 + 0.4d + 1 = 0$$
$$d^2 - 20d = 50$$
$$d^2 - 20d + 100 = 50 + 100$$
$$(d - 10)^2 = 150$$
$$d - 10 = \pm\sqrt{150}$$

$d = 10 + \sqrt{150}$ or $d = 10 - \sqrt{150}$

$d \approx 22.2$ $\quad$ $d \approx -2.2$

Since distance cannot be negative, $d = -2.2$ is an extraneous root.

The disc will travel 22.2 m, to the nearest tenth of a metre, if no one catches it.

Section 4.3 Page 241 Question 10

Solve $0 = -0.01d^2 + 2d + 1$ to determine how far the rocket lands from its launch position.

$$-0.01d^2 + 2d + 1 = 0$$
$$d^2 - 200d = 100$$
$$d^2 - 200d + 10\,000 = 100 + 10\,000$$
$$(d - 100)^2 = 10\,100$$
$$d - 100 = \pm\sqrt{10\,100}$$

$d = 100 + \sqrt{10\,100}$ or $d = 100 - \sqrt{10\,100}$

$d \approx 200.5$ $\quad$ $d \approx -0.5$

Since distance cannot be negative, $d = -0.5$ is an extraneous root.

The rocket lands 200.5 m, to the nearest tenth of a metre, from its launch position.

Section 4.3 Page 242 Question 11

Solve a quadratic equation to determine the dimensions of the photograph.

$$(12 - 2x)(12 - 4x) = 54$$
$$144 - 72x + 8x^2 - 54 = 0$$
$$8x^2 - 72x + 90 = 0$$
$$x^2 - 9x = -11.25$$
$$x^2 - 9x + 20.25 = -11.25 + 20.25$$
$$(x - 4.5)^2 = 9$$
$$x - 4.5 = \pm 3$$

$x - 4.5 = 3$ or $x - 4.5 = -3$

$x = 4.5 + 3$ $\quad$ $x = 4.5 - 3$

$x = 7.5$ $\quad$ $x = 1.5$

Since $x = 7.5$ would result in negative dimensions, it is an extraneous root.
The dimensions of the photograph are 12 – 2(1.5), or 9 in., by 12 – 4(1.5), or 6 in.

Section 4.3 Page 242 Question 12

Solve $0 = -0.04x^2 + 2x + 8$ to determine how far away the debris lands.

$$-0.04x^2 + 2x + 8 = 0$$
$$x^2 - 50x = 200$$
$$x^2 - 50x + 625 = 200 + 625$$
$$(x - 25)^2 = 825$$
$$x - 25 = \pm\sqrt{825}$$

$x - 25 = \sqrt{825}$ or $x - 25 = -\sqrt{825}$

$x = 25 + \sqrt{825}$ $\quad$ $x = 25 - \sqrt{825}$

$x \approx 53.7$ $\quad$ $x \approx -3.7$

Since distance cannot be negative, $x = -3.7$ is an extraneous root.
The debris lands 53.7 m, to the nearest tenth of a metre, from the launch site.

Section 4.3 Page 242 Question 13

a) For roots $\sqrt{7}$ and $-\sqrt{7}$,

$$x = \pm\sqrt{7}$$
$$x^2 = 7$$
$$x^2 - 7 = 0$$

b) For roots $1 + \sqrt{3}$ and $1 - \sqrt{3}$,

$$x = 1 \pm\sqrt{3}$$
$$x - 1 = \pm\sqrt{3}$$
$$(x - 1)^2 = 3$$
$$x^2 - 2x + 1 = 3$$
$$x^2 - 2x - 2 = 0$$

c) For roots $\frac{5+\sqrt{11}}{2}$ and $\frac{5-\sqrt{11}}{2}$,

$$x = \frac{5\pm\sqrt{11}}{2}$$
$$x - \frac{5}{2} = \pm\frac{\sqrt{11}}{2}$$
$$\left(x - \frac{5}{2}\right)^2 = \frac{11}{4}$$
$$x^2 - 5x + \frac{25}{4} = \frac{11}{4}$$

$$x^2 - 5x + \frac{14}{4} = 0$$
$$4x^2 - 20x + 14 = 0$$

Section 4.3 Page 242 Question 14

a) $x^2 + 2x = k$

$$x^2 + 2x + 1 = k + 1$$
$$(x + 1)^2 = k + 1$$
$$x + 1 = \pm\sqrt{k+1}$$
$$x = -1 \pm \sqrt{k+1}$$

b) $kx^2 - 2x = k$

$$x^2 - \frac{2}{k}x = 1$$
$$x^2 - \frac{2}{k}x + \frac{1}{k^2} = 1 + \frac{1}{k^2}$$
$$\left(x - \frac{1}{k}\right)^2 = \frac{k^2+1}{k^2}$$
$$x - \frac{1}{k} = \pm\sqrt{\frac{k^2+1}{k^2}}$$
$$x = \frac{1}{k} \pm \frac{\sqrt{k^2+1}}{k}$$
$$x = \frac{1 \pm \sqrt{k^2+1}}{k}$$

c) $x^2 = kx + 1$

$$x^2 - kx = 1$$
$$x^2 - kx + \frac{k^2}{4} = 1 + \frac{k^2}{4}$$
$$\left(x - \frac{k}{2}\right)^2 = \frac{4+k^2}{4}$$
$$x - \frac{k}{2} = \pm\sqrt{\frac{4+k^2}{4}}$$
$$x = \frac{k}{2} \pm \frac{\sqrt{4+k^2}}{2}$$
$$x = \frac{k \pm \sqrt{4+k^2}}{2}$$

Section 4.3 Page 242 Question 15

$$ax^2 + bx + c = 0$$
$$ax^2 + bx = -c$$
$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$
$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2}$$
$$\left(x + \frac{b}{2a}\right)^2 = \frac{-4ac + b^2}{4a^2}$$
$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$
$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This is the quadratic formula. You can use this result to solve any quadratic equation with real roots.

Section 4.3 Page 242 Question 16

a) Substitute $S_n = 3870$, $t_1 = 6$, and $d = 4$ into $S_n = \frac{n}{2}[2t_1 + (n - 1)d]$.

$$S_n = \frac{n}{2}[2t_1 + (n - 1)d]$$
$$3870 = \frac{n}{2}[2(6) + (n - 1)4]$$
$$7740 = n(4n + 8)$$
$$7740 = 4n^2 + 8n$$
$$1935 = n^2 + 2n$$
$$1935 + 1 = n^2 + 2n + 1$$
$$1936 = (n + 1)^2$$
$$\pm 44 = n + 1$$
$$n = -1 \pm 44$$

$n = -1 + 44$ or $n = -1 - 44$

$n = 43$ $\quad$ $n = -45$

Since the number of terms cannot be negative, $n = -45$ is an extraneous root.
There are 43 terms in the sum.

b) Substitute $S_n = 780$, $t_1 = 1$, and $d = 1$ into $S_n = \frac{n}{2}[2t_1 + (n - 1)d]$.

$$S_n = \frac{n}{2}[2t_1 + (n - 1)d]$$
$$780 = \frac{n}{2}[2(1) + (n - 1)1]$$
$$1560 = n(n + 1)$$
$$1560 = n^2 + n$$
$$1560 + 0.25 = n^2 + n + 0.25$$
$$1560.25 = (n + 0.5)^2$$
$$\pm 44 = n + 0.5$$
$$n = -0.5 \pm 39.5$$

$n = -0.5 + 39.5$ or $n = -0.5 - 39.5$

$n = 39$ $\quad$ $n = -40$

Since the number of terms cannot be negative, $n = -40$ is an extraneous root.
There are 39 terms in the sum.

Section 4.3 Page 242 Question 17

a) Use the cosine law $c^2 = a^2 + b^2 - 2ab \cos C$.

$$12^2 = 4^2 + x^2 - 2(4)x \cos 60°$$
$$144 = 16 + x^2 - 8x(0.5)$$
$$0 = x^2 - 4x - 128$$

b)
$$0 = x^2 - 4x - 128$$
$$128 = x^2 - 4x$$
$$128 + 4 = x^2 - 4x + 4$$
$$132 = (x - 2)^2$$
$$\pm\sqrt{132} = x - 2$$
$$x = 2 \pm \sqrt{132}$$

$x = 2 + \sqrt{132}$ or $x = 2 - \sqrt{132}$

$x \approx 13.5$ $\quad$ $x \approx -9.5$

Since x represents a length, it cannot be negative.
The length of the rod is 4 + 13.5, or 17.5 m, to the nearest tenth of a metre.

Section 4.3 Page 242 Question 18

Example: The solutions to $x^2 = 9$ and $x = \sqrt{9}$ are different because the first equations requires taking the square root of both sides, while the second is simply asking for the principle square root.

Section 4.3 Page 243 Question 19

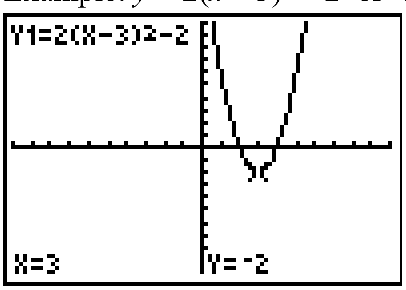
Example: Allison completed the square on the right side of the equation of the function. Riley completed the square of the corresponding quadratic equation, which involves changes to both sides of the equation. Allison has converted the function from standard form to vertex form for easier graphing. Riley has found the roots of the equation or x-intercepts of the graph of the corresponding function.

Section 4.3 Page 243 Question 20

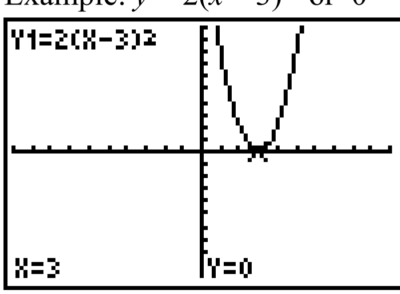
Example:
• Completing the square requires operations with rational numbers, which could lead to arithmetic errors. This method gives exact roots.
• Graphing the corresponding function using technology may lead to approximate roots.
• Factoring can only be used when the equation can be factored.
All of the methods can lead to the same answers.

Section 4.3 Page 243 Question 21

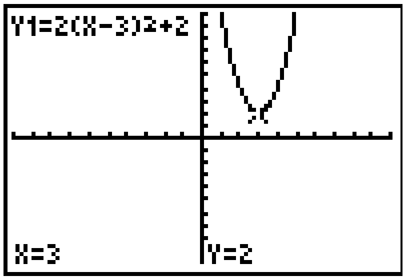
a) A quadratic function of the form $y = a(x - p)^2 + q$ will have two real roots if $a > 0$ and the vertex is below the x-axis.
Example: $y = 2(x - 3)^2 - 2$ or $0 = 2x^2 - 12x + 16$



b) A quadratic function of the form $y = a(x - p)^2 + q$ will have one real root if $a > 0$ and the vertex is on the x-axis.
Example: $y = 2(x - 3)^2$ or $0 = 2x^2 - 12x + 18$



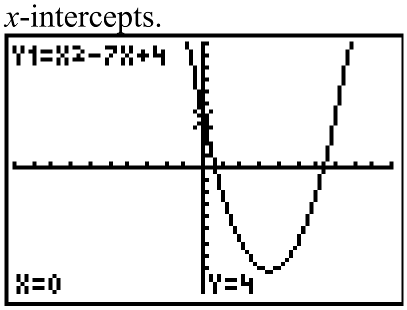
c) A quadratic function of the form $y = a(x - p)^2 + q$ will have no real roots if $a > 0$ and the vertex is above the x-axis.
Example: $y = 2(x - 3)^2 + 2$ or $0 = 2x^2 - 12x + 20$



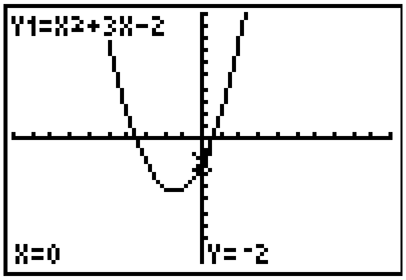
Section 4.4 The Quadratic Formula

Section 4.4 Page 254 Question 1

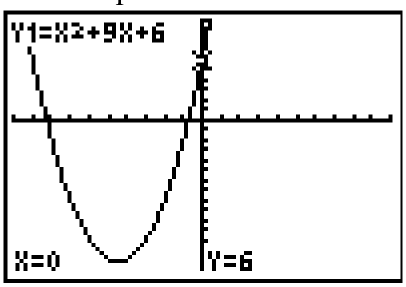
a) For $x^2 - 7x + 4 = 0$, $a = 1$, $b = -7$, and $c = 4$.
$b^2 - 4ac = (-7)^2 - 4(1)(4)$
$b^2 - 4ac = 49 - 16$
$b^2 - 4ac = 33$
Since the value of the discriminant is positive, there are two distinct real roots.
The graph of the corresponding quadratic function confirms that there are two distinct x-intercepts.



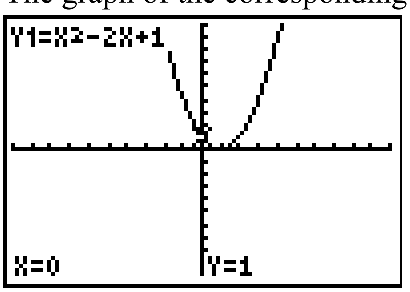
b) For $s^2 + 3s - 2 = 0$, $a = 1$, $b = 3$, and $c = -2$.
$b^2 - 4ac = 3^2 - 4(1)(-2)$
$b^2 - 4ac = 9 + 8$
$b^2 - 4ac = 17$
Since the value of the discriminant is positive, there are two distinct real roots.
The graph of the corresponding quadratic function confirms that there are two distinct x-intercepts.



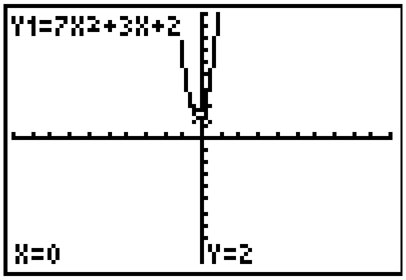
c) For $r^2 + 9r + 6 = 0$, $a = 1$, $b = 9$, and $c = 6$.
$b^2 - 4ac = 9^2 - 4(1)(6)$
$b^2 - 4ac = 81 - 24$
$b^2 - 4ac = 57$
Since the value of the discriminant is positive, there are two distinct real roots.
The graph of the corresponding quadratic function confirms that there are two distinct x-intercepts.



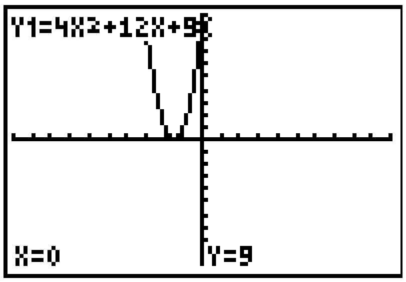
d) For $n^2 - 2n + 1 = 0$, $a = 1$, $b = -2$, and $c = 1$.
$b^2 - 4ac = (-2)^2 - 4(1)(1)$
$b^2 - 4ac = 4 - 4$
$b^2 - 4ac = 0$
Since the value of the discriminant is zero, there is one distinct real root.
The graph of the corresponding quadratic function confirms that there is one x-intercept.



e) For $7y^2 + 3y + 2 = 0$, $a = 7$, $b = 3$, and $c = 2$.
$b^2 - 4ac = 3^2 - 4(7)(2)$
$b^2 - 4ac = 9 - 56$
$b^2 - 4ac = -47$
Since the value of the discriminant is negative, there are no real roots.
The graph of the corresponding quadratic function confirms that there are no x-intercepts.



f) For $4t^2 + 12t + 9 = 0$, $a = 4$, $b = 12$, and $c = 9$.
$b^2 - 4ac = 12^2 - 4(4)(9)$
$b^2 - 4ac = 144 - 144$
$b^2 - 4ac = 0$
Since the value of the discriminant is zero, there is one distinct real root.
The graph of the corresponding quadratic function confirms that there is one x-intercept.



Section 4.4 Page 254 Question 2

a) For $f(x) = x^2 - 2x - 14$, $a = 1$, $b = -2$, and $c = -14$.
$b^2 - 4ac = (-2)^2 - 4(1)(-14)$
$b^2 - 4ac = 4 + 56$
$b^2 - 4ac = 60$
Since the value of the discriminant is positive, the function has two zeros.

b) For $g(x) = -3x^2 + 0.06x + 4$, $a = -3$, $b = 0.06$, and $c = 4$.
$b^2 - 4ac = 0.06^2 - 4(-3)(4)$
$b^2 - 4ac = 0.0036 + 48$
$b^2 - 4ac = 48.0036$
Since the value of the discriminant is positive, the function has two zeros.

c) For $f(x) = \frac{1}{4}x^2 - 3x + 9$, $a = \frac{1}{4}$, $b = -3$, and $c = 9$.
$b^2 - 4ac = (-3)^2 - 4\left(\frac{1}{4}\right)(9)$
$b^2 - 4ac = 9 - 9$
$b^2 - 4ac = 0$
Since the value of the discriminant is zero, the function has one zero.

d) For $f(v) = -v^2 + 2v - 1$, $a = -1$, $b = 2$, and $c = -1$.
$b^2 - 4ac = 2^2 - 4(-1)(-1)$
$b^2 - 4ac = 4 - 4$
$b^2 - 4ac = 0$
Since the value of the discriminant is zero, the function has one zero.

e) For $f(x) = \frac{1}{2}x^2 - x + \frac{5}{2}$, $a = \frac{1}{2}$, $b = -1$, and $c = \frac{5}{2}$.
$b^2 - 4ac = (-1)^2 - 4\left(\frac{1}{2}\right)\left(\frac{5}{2}\right)$
$b^2 - 4ac = 1 - 20$
$b^2 - 4ac = -19$
Since the value of the discriminant is negative, the function has no zeros.

f) For $g(y) = -6y^2 + 5y - 1$, $a = -6$, $b = 5$, and $c = -1$.
$b^2 - 4ac = 5^2 - 4(-6)(-1)$
$b^2 - 4ac = 25 - 24$
$b^2 - 4ac = 1$
Since the value of the discriminant is positive, the function has two zeros.

Section 4.4 Page 254 Question 3

a) For $7x^2 + 24x + 9 = 0$, $a = 7$, $b = 24$, and $c = 9$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-24 \pm \sqrt{24^2 - 4(7)(9)}}{2(7)}$$
$$x = \frac{-24 \pm \sqrt{324}}{14}$$
$$x = \frac{-24 \pm 18}{14}$$

$$x = \frac{-24 + 18}{14} \quad \text{or} \quad x = \frac{-24 - 18}{14}$$
$$x = \frac{-6}{14} \qquad\qquad x = \frac{-42}{14}$$
$$x = -\frac{3}{7} \qquad\qquad x = -3$$

The roots are $-\frac{3}{7}$ and -3.

b) For $4p^2 - 12p - 9 = 0$, $a = 4$, $b = -12$, and $c = -9$.

$$p = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$p = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(4)(-9)}}{2(4)}$$

$$p = \frac{12 \pm \sqrt{144 + 144}}{8}$$

$$p = \frac{12 \pm \sqrt{288}}{8}$$

$$p = \frac{12 \pm 12\sqrt{2}}{8}$$

$$p = \frac{3 \pm 3\sqrt{2}}{2}$$

The roots are $\frac{3 + 3\sqrt{2}}{2}$ and $\frac{3 - 3\sqrt{2}}{2}$.

c) For $3q^2 + 5q - 1 = 0$, $a = 3$, $b = 5$, and $c = -1$.

$$q = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$q = \frac{-5 \pm \sqrt{5^2 - 4(3)(-1)}}{2(3)}$$

$$q = \frac{-5 \pm \sqrt{25 + 12}}{6}$$

$$q = \frac{-5 \pm \sqrt{37}}{8}$$

The roots are $\frac{-5 + \sqrt{37}}{6}$ and $\frac{-5 - \sqrt{37}}{6}$.

d) For $2m^2 + 4m - 7 = 0$, $a = 2$, $b = 4$, and $c = -7$.

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$m = \frac{-4 \pm \sqrt{4^2 - 4(2)(-7)}}{2(2)}$$

$$m = \frac{-4 \pm \sqrt{72}}{4}$$

$$m = \frac{-4 \pm 6\sqrt{2}}{4}$$

$$m = \frac{-2 \pm 3\sqrt{2}}{2}$$

The roots are $\frac{-2+3\sqrt{2}}{2}$ and $\frac{-2-3\sqrt{2}}{2}$.

e) For $2j^2 - 7j + 4 = 0$, $a = 2$, $b = -7$, and $c = 4$.

$$j = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$j = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(4)}}{2(2)}$$

$$j = \frac{7 \pm \sqrt{49 - 32}}{4}$$

$$j = \frac{7 \pm \sqrt{17}}{4}$$

The roots are $\frac{7+\sqrt{17}}{4}$ and $\frac{7-\sqrt{17}}{4}$.

f) For $16g^2 + 24g + 9 = 0$, $a = 16$, $b = 24$, and $c = 9$.

$$g = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$g = \frac{-24 \pm \sqrt{24^2 - 4(16)(9)}}{2(16)}$$

$$g = \frac{-24 \pm \sqrt{0}}{32}$$

$$g = -\frac{3}{4}$$

The root is $-\frac{3}{4}$.

Section 4.4 Page 254 Question 4

a) For $3z^2 + 14z + 5 = 0$, $a = 3$, $b = 14$, and $c = 5$.

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$z = \frac{-14 \pm \sqrt{14^2 - 4(3)(5)}}{2(3)}$$

$$z = \frac{-14 \pm \sqrt{196 - 60}}{6}$$

$$z = \frac{-14 \pm \sqrt{136}}{6}$$

$$z = \frac{-14 + \sqrt{136}}{6} \quad \text{or} \quad z = \frac{-14 - \sqrt{136}}{6}$$

$$z \approx -0.39 \qquad z \approx -4.28$$

The roots are –0.39 and –4.28, to the nearest hundredth.

b) For $4c^2 - 7c - 1 = 0$, $a = 4$, $b = -7$, and $c = -1$.

$$c = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$c = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(4)(-1)}}{2(4)}$$

$$c = \frac{7 \pm \sqrt{49 + 16}}{8}$$

$$c = \frac{7 \pm \sqrt{65}}{8}$$

$$c = \frac{7 + \sqrt{65}}{8} \quad \text{or} \quad c = \frac{7 - \sqrt{65}}{8}$$

$$c \approx 1.88 \qquad c \approx -0.13$$

The roots are 1.88 and –0.13, to the nearest hundredth.

c) For $-5u^2 + 16u - 2 = 0$, $a = -5$, $b = 16$, and $c = -2$.

$$u = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$u = \frac{-16 \pm \sqrt{16^2 - 4(-5)(-2)}}{2(-5)}$$

$$u = \frac{-16 \pm \sqrt{216}}{-10}$$

$$u = \frac{-16 + \sqrt{216}}{-10} \quad \text{or} \quad u = \frac{-16 - \sqrt{216}}{-10}$$

$$u \approx 0.13 \qquad\qquad u \approx 3.07$$

The roots are 0.13 and 3.07, to the nearest hundredth.

d) For $8b^2 + 12b + 1 = 0$, $a = 8$, $b = 12$, and $c = 1$.

$$b = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$b = \frac{-12 \pm \sqrt{12^2 - 4(8)(1)}}{2(8)}$$

$$b = \frac{-12 \pm \sqrt{112}}{16}$$

$$b = \frac{-12 + \sqrt{112}}{16} \quad \text{or} \quad b = \frac{-12 - \sqrt{112}}{16}$$

$$b \approx -0.09 \qquad\qquad b \approx -1.41$$

The roots are –0.09 and –1.41, to the nearest hundredth.

e) For $10w^2 - 45w - 7 = 0$, $a = 10$, $b = -45$, and $c = -7$.

$$w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$w = \frac{-(-45) \pm \sqrt{(-45)^2 - 4(10)(-7)}}{2(10)}$$

$$w = \frac{45 \pm \sqrt{2025 + 280}}{20}$$

$$w = \frac{45 \pm \sqrt{2305}}{20}$$

$$w = \frac{45 + \sqrt{2305}}{20} \quad \text{or} \quad w = \frac{45 - \sqrt{2305}}{20}$$

$$w \approx 4.65 \qquad\qquad w \approx -0.15$$

The roots are 4.65 and –0.65, to the nearest hundredth.

f) For $-6k^2 + 17k + 5 = 0$, $a = -6$, $b = 17$, and $c = 5$.

$$k = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$k = \frac{-17 \pm \sqrt{17^2 - 4(-6)(5)}}{2(-6)}$$

$$k = \frac{-17 \pm \sqrt{289 + 120}}{-12}$$

$$k = \frac{-17 \pm \sqrt{409}}{-12}$$

$$k = \frac{-17 + \sqrt{409}}{-12} \quad \text{or} \quad k = \frac{-17 - \sqrt{409}}{-12}$$

$$k \approx -0.27 \qquad\qquad k \approx 3.10$$

The roots are –0.27 and 3.10, to the nearest hundredth.

Section 4.4 Page 254 Question 5

a) For $3x^2 + 6x + 1 = 0$, $a = 3$, $b = 6$, and $c = 1$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(3)(1)}}{2(3)}$$

$$x = \frac{-6 \pm \sqrt{36 - 12}}{6}$$

$$x = \frac{-6 \pm \sqrt{24}}{6}$$

$$x = \frac{-6 \pm 2\sqrt{6}}{6}$$

$$x = \frac{-3 \pm \sqrt{6}}{3}$$

$$x = \frac{-3 + \sqrt{6}}{3} \quad \text{or} \quad x = \frac{-3 - \sqrt{6}}{3}$$

$$x \approx -0.18 \qquad\qquad x \approx -1.82$$

The roots are $\frac{-3 + \sqrt{6}}{3}$ and $\frac{-3 - \sqrt{6}}{3}$ or –0.18 and –1.82, to the nearest hundredth.

b) For $h^2 + \frac{h}{6} - \frac{1}{2} = 0$, $a = 1$, $b = \frac{1}{6}$, and $c = -\frac{1}{2}$.

$$h = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$h = \frac{-\frac{1}{6} \pm \sqrt{\left(\frac{1}{6}\right)^2 - 4(1)\left(-\frac{1}{2}\right)}}{2(1)}$$

$$h = \frac{-\frac{1}{6} \pm \sqrt{\frac{1}{36} + 2}}{2}$$

$$h = \frac{-\frac{1}{6} \pm \sqrt{\frac{73}{36}}}{2}$$

$$h = \frac{-1 \pm \sqrt{73}}{12}$$

$$h = \frac{-1 + \sqrt{73}}{12} \quad \text{or} \quad h = \frac{-1 - \sqrt{73}}{12}$$

$$h \approx 0.63 \qquad\qquad h \approx -0.80$$

The roots are $\frac{-1 + \sqrt{73}}{12}$ and $\frac{-1 - \sqrt{73}}{12}$ or 0.63 and –0.80, to the nearest hundredth.

c) For $0.2m^2 + 0.3m - 0.1 = 0$, $a = 0.2$, $b = 0.3$, and $c = -0.1$.

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$m = \frac{-0.3 \pm \sqrt{0.3^2 - 4(0.2)(-0.1)}}{2(0.2)}$$

$$m = \frac{-0.3 \pm \sqrt{0.09 + 0.08}}{0.4}$$

$$m = \frac{-0.3 \pm \sqrt{0.17}}{0.4}$$

$$m = \frac{-0.3 + \sqrt{0.17}}{0.4} \quad \text{or} \quad m = \frac{-0.3 - \sqrt{0.17}}{0.4}$$

$$m \approx 0.28 \qquad\qquad m \approx -1.78$$

The roots are $\frac{-0.3 + \sqrt{0.17}}{0.4}$ and $\frac{-0.3 - \sqrt{0.17}}{0.4}$ or 0.28 and –1.78, to the nearest hundredth.

d) For $4y^2 - 12y + 7 = 0$, $a = 4$, $b = -12$, and $c = 7$.

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(4)(7)}}{2(4)}$$

$$y = \frac{12 \pm \sqrt{144 - 112}}{8}$$

$$y = \frac{12 \pm \sqrt{32}}{8}$$

$$y = \frac{12 \pm 4\sqrt{2}}{8}$$

$$y = \frac{3 \pm \sqrt{2}}{2}$$

$y = \frac{3 + \sqrt{2}}{2}$ or $y = \frac{3 - \sqrt{2}}{2}$

$y \approx 2.21$ $\quad$ $y \approx 0.79$

The roots are $\frac{3 + \sqrt{2}}{2}$ and $\frac{3 - \sqrt{2}}{2}$ or 2.21 and 0.79, to the nearest hundredth.

e) First multiply the equation $\frac{7x^2}{2} - \frac{x}{2} - 1 = 0$ by 2 to eliminate fractions.

For $7x^2 - x - 2$, $a = 7$, $b = -1$, and $c = -2$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(7)(-2)}}{2(7)}$$

$$x = \frac{1 \pm \sqrt{1 + 56}}{14}$$

$$x = \frac{1 \pm \sqrt{57}}{14}$$

$x = \frac{1 + \sqrt{57}}{14}$ or $x = \frac{1 - \sqrt{57}}{14}$

$x \approx 0.61$ $\quad$ $x \approx -0.47$

The roots are $\frac{1 + \sqrt{57}}{14}$ and $\frac{1 - \sqrt{57}}{14}$ or 0.61 and –0.47, to the nearest hundredth.

f) For $2z^2 - 6z + 1 = 0$, $a = 2$, $b = -6$, and $c = 1$.

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$z = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(1)}}{2(2)}$$

$$z = \frac{6 \pm \sqrt{28}}{4}$$

$$z = \frac{6 \pm 2\sqrt{7}}{4}$$

$$x = \frac{3 \pm \sqrt{7}}{2}$$

$$z = \frac{3 + \sqrt{7}}{2} \quad \text{or} \quad z = \frac{3 - \sqrt{7}}{2}$$

$$z \approx 2.82 \qquad\qquad z \approx 0.18$$

The roots are $\frac{3+\sqrt{7}}{2}$ and $\frac{3-\sqrt{7}}{2}$ or 2.82 and 0.18, to the nearest hundredth.

Section 4.4 Page 254 Question 6

Example: I disagree with Marge. Using the quadratic formula to solve quadratic equations that are easily factored is not efficient . It is much quicker to solve equations such as $x^2 + x - 2 = 0$ and $x^2 - 4 = 0$ by factoring.

Section 4.4 Page 254 Question 7

a) Solve $n^2 + 2n - 2 = 0$ by completing the square; it cannot be factored and coefficients are easy to work with.

$$n^2 + 2n - 2 = 0$$
$$n^2 + 2n = 2$$
$$n^2 + 2n + 1 = 2 + 1$$
$$(n + 1)^2 = 3$$
$$n + 1 = \pm\sqrt{3}$$
$$n = -1 \pm \sqrt{3}$$

b) Solve $-y^2 + 6y - 9 = 0$ by factoring.

$$-y^2 + 6y - 9 = 0$$
$$y^2 - 6y + 9 = 0$$
$$(y - 3)^2 = 0$$
$$y - 3 = 0$$
$$y = 3$$

c) Solve $-2u^2 + 16 = 0$ by taking the square root.

$$-2u^2 + 16 = 0$$
$$u^2 - 8 = 0$$
$$u^2 = 8$$
$$u = \pm\sqrt{8}$$
$$u = \pm 2\sqrt{2}$$

d) Solve $\frac{x^2}{2} - \frac{x}{3} = 1$ using the quadratic formula since some of the coefficients are rational numbers.

For $\frac{x^2}{2} - \frac{x}{3} - 1 = 0$, $a = \frac{1}{2}$, $b = -\frac{1}{3}$, and $c = -1$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-\left(-\frac{1}{3}\right) \pm \sqrt{\left(-\frac{1}{3}\right)^2 - 4\left(\frac{1}{2}\right)(-1)}}{2\left(\frac{1}{2}\right)}$$
$$x = \frac{\frac{1}{3} \pm \sqrt{\frac{1}{9} + 2}}{1}$$
$$x = \frac{1}{3} \pm \frac{\sqrt{19}}{3}$$
$$x = \frac{1 \pm \sqrt{19}}{3}$$

e) Solve $x^2 - 4x + 8 = 0$ by completing the square; it cannot be factored and coefficients are easy to work with.

$$x^2 - 4x + 8 = 0$$
$$x^2 - 4x = -8$$
$$x^2 - 4x + 4 = -8 + 4$$
$$(x - 2)^2 = -4$$
$$x - 2 = \pm\sqrt{-4}$$

There are no real roots, since you cannot take the square root of a negative number.

Section 4.4 Page 254 Question 8

Let x represent the width of the corral. Then, $30 - 2x$ represents the length. Solve $x(30 - 2x) = 100$ to find the dimensions.

$$x(30 - 2x) = 100$$
$$-2x^2 + 30x - 100 = 0$$
$$x^2 - 15x + 50 = 0$$
$$(x - 10)(x + 5) = 0$$

$x - 10 = 0$ or $x + 5 = 0$

$x = 10$ $\quad$ $x = -5$

Since the width cannot be negative, $x = -5$ is an extraneous root.

The dimensions of the corral are 10 m by 10 m.

Section 4.4 Page 255 Question 9

Let x represent the width of the border. Then, the dimensions of the mural are $15 - 2x$ by $12 - 2x$ with an area of 135 m^2. Solve $(15 - 2x)(12 - 2x) = 135$ to find the width of the border.

$$(15 - 2x)(12 - 2x) = 135$$
$$4x^2 - 54x + 180 = 135$$
$$4x^2 - 54x + 45 = 0$$

Substitute into the quadratic formula, $a = 4$, $b = -54$, $c = 45$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-54) \pm \sqrt{(-54)^2 - 4(4)(45)}}{2(4)}$$
$$x = \frac{54 \pm \sqrt{2916 - 720}}{8}$$
$$x = \frac{54 \pm \sqrt{2196}}{8}$$

$x = \frac{54 + \sqrt{2196}}{8}$ or $x = \frac{54 - \sqrt{2196}}{8}$

$x \approx 12.61$ $\quad$ $x \approx 0.89$

Since the border cannot be wider than one of the dimensions, $x = 12.61$ is an extraneous root.

The width of the border is 0.89 m, to the nearest hundredth of a metre.

Section 4.4 Page 255 Question 10

Let x represent the number. Solve $\frac{1}{2}x^2 - x = 11$ to find the number.

$\frac{1}{2}x^2 - x - 11 = 0$

Substitute into the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4\left(\frac{1}{2}\right)(-11)}}{2\left(\frac{1}{2}\right)}$$

$$x = \frac{1 \pm \sqrt{1 + 22}}{1}$$

$x = 1 \pm \sqrt{23}$

$x = 1 + \sqrt{23}$ or $x = 1 - \sqrt{23}$

$x \approx 5.80$ $\quad$ $x \approx -3.80$

The exact number is $1 + \sqrt{23}$ or $1 - \sqrt{23}$. The number, to the nearest hundredth, is 5.80 or –3.80.

Section 4.4 Page 255 Question 11

Solve $0 = -0.4(d - 2.5)^2 + 2.5$ to find the width of the arch.

$0 = -0.4(d - 2.5)^2 + 2.5$

$0 = -0.4d^2 + 2d$

$0 = d^2 - 5d$

$0 = d(d - 5)$

$d = 0$ or $d - 5 = 0$

$d = 5$

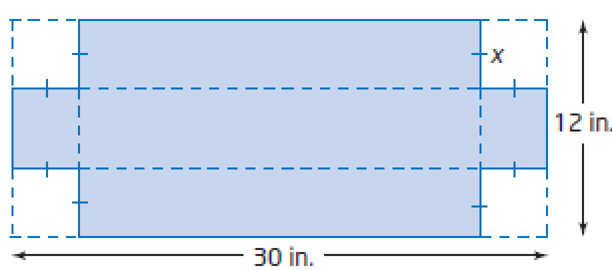
The arch is 5 – 0, or 5 m wide.

Section 4.4 Page 255 Question 12

a) $SA_{base} = (12 - 2x)(30 - 2x)$

$208 = 4x^2 - 84x + 360$

$0 = 4x^2 - 84x + 152$



b) $0 = 4x^2 - 84x + 152$
$0 = x^2 - 21x + 38$
$0 = (x - 19)(x - 2)$
$x - 19 = 0$ or $x - 2 = 0$
$x = 19$ $x = 2$

Since the side length of the corner square cannot be greater than a dimension of the cardboard, $x = 19$ is an extraneous root. The side length of the square cut from each corner is 2 in.

c) The dimensions of the box are 26 in. by 8 in. by 2 in.

Section 4.4 Page 255 Question 13

a) $42 = 0.0067v^2 + 0.15v$
$0 = 0.0067v^2 + 0.15v - 42$

Substitute into the quadratic formula.

$$v = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$v = \frac{-0.15 \pm \sqrt{0.15^2 - 4(0.0067)(-42)}}{2(0.0067)}$$

$$v = \frac{-0.15 \pm \sqrt{0.0225 + 1.1256}}{0.0134}$$

$$v = \frac{-0.15 \pm \sqrt{1.1481}}{0.0134}$$

$$v = \frac{-0.15 + \sqrt{1.1481}}{0.0134} \quad \text{or} \quad v = \frac{-0.15 - \sqrt{1.1481}}{0.0134}$$

$v \approx 68.8$ $v \approx -91.2$

Since speed cannot be negative, $x = -91.2$ is an extraneous root.
The car can be travelling at approximately 68.8 km/h to be able to stop in 42 m.

b) $75 = 0.0067v^2 + 0.15v$
$0 = 0.0067v^2 + 0.15v - 75$

Substitute into the quadratic formula.

$$v = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$v = \frac{-0.15 \pm \sqrt{0.15^2 - 4(0.0067)(-75)}}{2(0.0067)}$$
$$v = \frac{-0.15 \pm \sqrt{0.0225 + 2.01}}{0.0134}$$
$$v = \frac{-0.15 \pm \sqrt{2.0325}}{0.0134}$$
$$v = \frac{-0.15 + \sqrt{2.0325}}{0.0134} \quad \text{or} \quad v = \frac{-0.15 - \sqrt{2.0325}}{0.0134}$$
$$v \approx 95.2 \qquad\qquad v \approx -117.6$$

Since speed cannot be negative, $x = -117.6$ is an extraneous root.
The car can be travelling at approximately 95.2 km/h to be able to stop in 75 m.

c) $135 = 0.0067v^2 + 0.15v$
$0 = 0.0067v^2 + 0.15v - 135$
Substitute into the quadratic formula.

$$v = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$v = \frac{-0.15 \pm \sqrt{0.15^2 - 4(0.0067)(-135)}}{2(0.0067)}$$
$$v = \frac{-0.15 \pm \sqrt{0.0225 + 3.618}}{0.0134}$$
$$v = \frac{-0.15 \pm \sqrt{3.6405}}{0.0134}$$
$$v = \frac{-0.15 + \sqrt{3.6405}}{0.0134} \quad \text{or} \quad v = \frac{-0.15 - \sqrt{3.6405}}{0.0134}$$
$$v \approx 131.2 \qquad\qquad v \approx -153.6$$

Since speed cannot be negative, $x = -153.6$ is an extraneous root.
The car can be travelling at approximately 131.2 km/h to be able to stop in 135 m.

Section 4.4 Page 255 Question 14

a) $A(t) = 0.3t^2 + 0.1t + 4.2$
$A(0) = 0.3(0)^2 + 0.1(0) + 4.2$
$A(0) = 4.2$
At $t = 0$, the level of carbon dioxide in the air is 4.2 ppm.

b) Solve $8 = 0.3t^2 + 0.1t + 4.2$ using the quadratic formula.

$8 = 0.3t^2 + 0.1t + 4.2$

$0 = 0.3t^2 + 0.1t - 3.8$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-0.1 \pm \sqrt{0.1^2 - 4(0.3)(-3.8)}}{2(0.3)}$$

$$t = \frac{-0.1 \pm \sqrt{4.57}}{0.6}$$

$$t = \frac{-0.1 + \sqrt{4.57}}{0.6} \quad \text{or} \quad t = \frac{-0.1 - \sqrt{4.57}}{0.6}$$

$t \approx 3.4$ $\qquad$ $t \approx -3.7$

Since time cannot be negative, $x = -3.7$ is an extraneous root.

In 3.4 years, to the nearest tenth of a year, the carbon monoxide level will be 8 ppm.

Section 4.4 Page 256 Question 15

Let n represent the number of price decreases. The new price is $275 - 15n$.

The new number of ski jackets sold is $90 + 5n$.

The revenue is $19 600.

Revenue = (price)(number of sessions)

$19\,600 = (275 - 15n)(90 + 5n)$

$19\,600 = -75n^2 + 25n + 24\,750$

$0 = -75n^2 + 25n + 5150$

$0 = 3n^2 - n - 206$

Substitute into the quadratic formula.

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$n = \frac{-1 \pm \sqrt{1^2 - 4(3)(-206)}}{2(3)}$$

$$n = \frac{-1 \pm \sqrt{2473}}{6}$$

$$n = \frac{-1 + \sqrt{2473}}{6} \quad \text{or} \quad n = \frac{-1 - \sqrt{2473}}{6}$$

$n \approx 8$ $\qquad$ $n \approx -8$

Since the number of price decreases must be positive, $x = -8$ is an extraneous root.

The lowest price that would produce revenues of at least $19 600 is 275 – 15(8), or $155.

At this price, 90 + 5(8), or 130 jackets would be sold.

Section 4.4 Page 256 Question 16

Let x represent the height of the tower. Then, the length of the guy wire is $x + 20$ and the horizontal distance from the base of the tower to where the guy wire is anchored to the ground is $0.5x$. Use the Pythagorean Theorem.

$$(x + 20)^2 = x^2 + (0.5x)^2$$
$$x^2 + 40x + 400 = x^2 + 0.25x^2$$
$$-0.25x^2 + 40x + 400 = 0$$

Substitute into the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-40 \pm \sqrt{40^2 - 4(-0.25)(400)}}{2(-0.25)}$$
$$x = \frac{-40 \pm \sqrt{2000}}{-0.5}$$
$$x = \frac{-40 + \sqrt{2000}}{-0.5} \quad \text{or} \quad x = \frac{-40 - \sqrt{2000}}{-0.5}$$
$$x \approx -9.4 \qquad\qquad x \approx 169.4$$

Since the height cannot be negative, $x = -9.4$ is an extraneous root.

The tower is 169.4 m tall, to the nearest tenth of a metre.

Section 4.4 Page 256 Question 17

Since one root is –8, substitute $x = 8$ into $2x^2 + bx - 24 = 0$ and solve for b.

$$2x^2 + bx - 24 = 0$$
$$2(-8)^2 + b(-8) - 24 = 0$$
$$104 - 8b = 0$$
$$-8b = -104$$
$$b = \frac{-104}{-8}$$
$$b = 13$$

Find the other root.

$$2x^2 + 13x - 24 = 0$$
$$(x + 8)(2x - 3) = 0$$
$$x + 8 = 0 \quad \text{or} \quad 2x - 3 = 0$$
$$x = -8 \qquad\qquad x = \frac{3}{2}$$

The value of b is 13 and the other root is $\frac{3}{2}$.

Section 4.4 Page 256 Question 18

Use the formula for the surface area of a cylinder.

$SA = 2\pi r^2 + 2\pi rh$

$100 = 2\pi r^2 + 2\pi r(5)$

$0 = 2\pi r^2 + 10\pi r - 100$

Substitute into the quadratic formula.

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$r = \frac{-10\pi \pm \sqrt{(10\pi)^2 - 4(2\pi)(-100)}}{2(2\pi)}$$

$$r = \frac{-10\pi \pm \sqrt{100\pi^2 + 800\pi}}{4\pi}$$

$$r = \frac{-10\pi + \sqrt{100\pi^2 + 800\pi}}{4\pi} \quad \text{or} \quad r = \frac{-10\pi - \sqrt{100\pi^2 + 800\pi}}{4\pi}$$

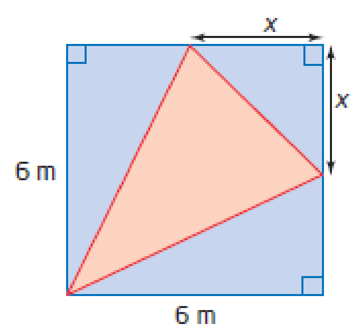
$r \approx 2.2$ $\qquad$ $r \approx -7.2$

Since the radius cannot be negative, $x = -7.2$ is an extraneous root.

The radius of the cylinder is 2.2 cm, to the nearest tenth of a centimetre.

Section 4.4 Page 256 Question 19

a) Use the formula for the area of a triangle.



Solve $\frac{1}{2}x^2 = \frac{1}{2}(6)(6 - x)$.

$x^2 + 6x - 36 = 0$

Substitute into the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(-36)}}{2(1)}$$

$$x = \frac{-6 \pm \sqrt{180}}{2}$$

$$x = \frac{-6 \pm 6\sqrt{5}}{2}$$

$x = -3 \pm 3\sqrt{5}$

$x = -3 + 3\sqrt{5}$ or $x = -3 - 3\sqrt{5}$

$x \approx 3.7$ $\qquad$ $x \approx -9.7$

Since x represents a length it must be positive. The exact value of x is $(-3 + 3\sqrt{5})$ m.

b) Subtract the areas of the three right triangle from the area of the square.

$A = 6(6) - 3\left[\frac{1}{2}\left(-3+3\sqrt{5}\right)^2\right]$

$A = 36 - \frac{3}{2}(9 - 18\sqrt{5} + 45)$

$A = 36 - 81 + 27\sqrt{5}$

$A = -45 + 27\sqrt{5}$

The exact area of the acute isosceles triangle is $(-45 + 27\sqrt{5})$ m^2.

Section 4.4 Page 256 Question 20

Let x represent the flying time of the second plane. Then, the distance flown by the first plane is represented by $150(x + 2)$ and the distance flown by the second plane is represented by $200x$.

Use the Pythagorean Theorem with $c = 600$.

$600^2 = [150(x + 2)]^2 + (200x)^2$

$360\ 000 = 22\ 500x^2 + 90\ 000x + 90\ 000 + 40\ 000x^2$

$0 = 62\ 500x^2 + 90\ 000x - 270\ 000$

$0 = 6.25x^2 + 9x - 27$

Substitute into the quadratic formula.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-9 \pm \sqrt{9^2 - 4(6.25)(-27)}}{2(6.25)}$

$x = \frac{-9 \pm \sqrt{81 + 675}}{12.5}$

$x = \frac{-9 \pm \sqrt{756}}{12.5}$

$x = \frac{-9 + \sqrt{756}}{12.5}$ or $x = \frac{-9 - \sqrt{756}}{12.5}$

$x \approx 1.5$ $x \approx -2.9$

Since time cannot be negative, $x = -2.9$ is an extraneous root.

The planes will be 600 km apart in 1.5 + 2, or 3.5 h, to the nearest tenth of an hour.

Section 4.4 Page 256 Question 21

In Line 1, the wrong value for b was substituted outside the radical.
In Line 2, the expression $-4(-3)(2)$ was incorrectly evaluated.
The corrected solution is as follows.
For $-3x^2 - 7x + 2 = 0$, $a = -3$, $b = -7$, and $c = 2$.

Line 1: $x = \dfrac{-(-7) \pm \sqrt{(-7)^2 - 4(-3)(2)}}{2(-3)}$

Line 2: $x = \dfrac{7 \pm \sqrt{49 + 24}}{-6}$

Line 3: $x = \dfrac{7 \pm \sqrt{73}}{-6}$

Line 4: $x = \dfrac{-7 \pm \sqrt{73}}{6}$

Line 5: So, $x = \dfrac{-7 + \sqrt{73}}{6}$ or $x = \dfrac{-7 - \sqrt{73}}{6}$.

Section 4.4 Page 256 Question 22

a) The roots of a quadratic equation are the same as the x-intercepts of the graph of the corresponding quadratic function. So, the x-intercepts are $x = \dfrac{3 \pm \sqrt{25}}{2}$, or $x = 4$ and $x = -1$.

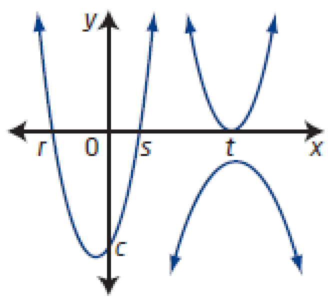
b) The axis of symmetry is halfway between the two roots of -1 and 4. So, the equation of the axis of symmetry is $x = 2$.

Section 4.4 Page 257 Question 23

Example: If the quadratic is easily factored, then factoring is faster. If it cannot be factored, then completing the square or applying the quadratic formula are other ways to determine exact answers. Graphing with technology is a quick way of finding out if there are real solutions. However, the roots found may be approximate.

Section 4.4 Page 257 Question 24

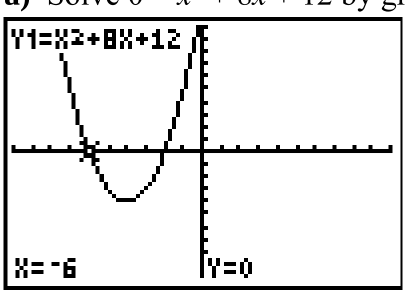
Example:

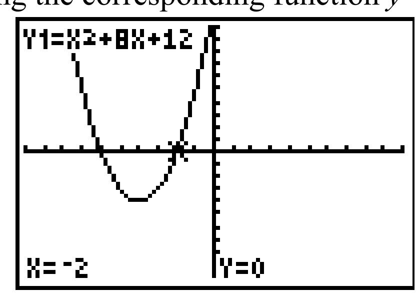
Quadratic Functions		Quadratic Equations
For $y = ax^2 + bx + c$, • $a > 0$ means the parabola opens upward • $a < 0$ means the parabola opens downward • c is the y-intercept For $y = a(x - p)^2 + q$, • has vertex (p, q) • has axis of symmetry with equation $x = p$ • parameter a determines the direction of the opening and the width of the parabola • parameter p determines the horizontal translation • parameter q determines the vertical translation You can convert a quadratic function from standard form to vertex form by completing the square.	 x-intercepts ↔ zeros ↔ roots	A quadratic equation $ax^2 + bx + c = 0$ can have no, one, or two real roots. You can solve a quadratic equation by • graphing the corresponding function • determining the square roots • factoring • completing the square • applying the quadratic formula

Chapter 4 Review

Chapter 4 Review Page 258 Question 1

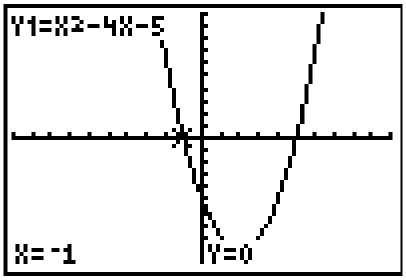
a) Solve $0 = x^2 + 8x + 12$ by graphing the corresponding function $y = x^2 + 8x + 12$.

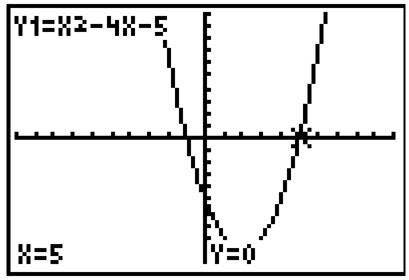




Since the x-intercepts of the graph are –6 and –2, the roots of the equation are –6 and –2.

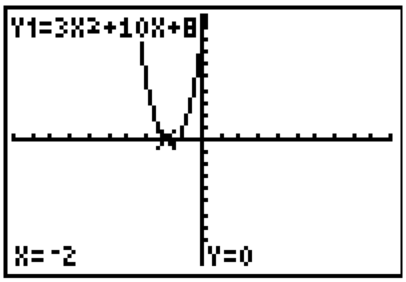
b) Solve $0 = x^2 - 4x - 5$ by graphing the corresponding function $y = x^2 - 4x - 5$.

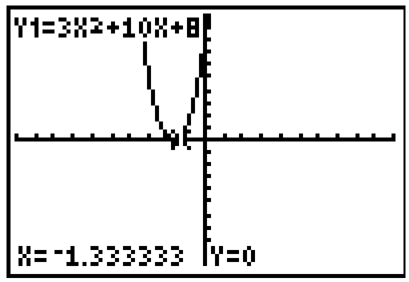




Since the x-intercepts of the graph are –1 and 5, the roots of the equation are –1 and 5.

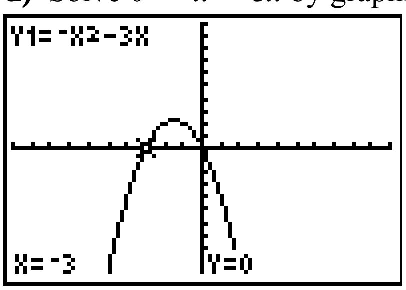
c) Solve $0 = 3x^2 + 10x + 8$ by graphing the corresponding function $y = 3x^2 + 10x + 8$.

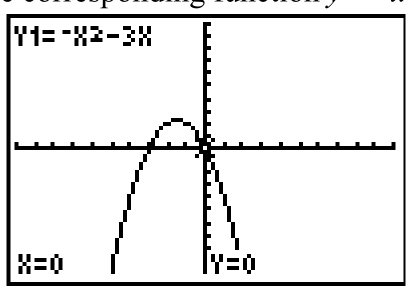




Since the x-intercepts of the graph are –2 and $-\frac{4}{3}$, the roots of the equation are –2 and $-\frac{4}{3}$.

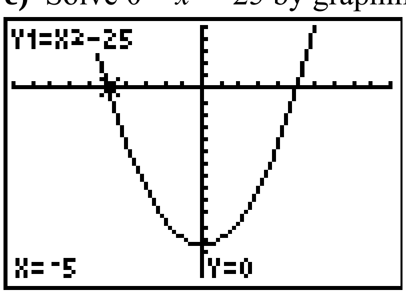
d) Solve $0 = -x^2 - 3x$ by graphing the corresponding function $y = -x^2 - 3x$.

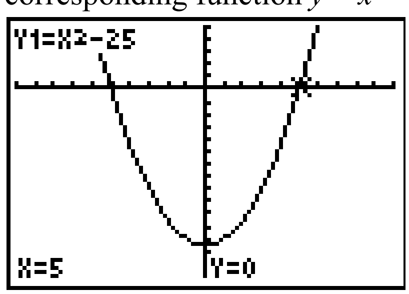




Since the x-intercepts of the graph are –3 and 0, the roots of the equation are –3 and 0.

e) Solve $0 = x^2 - 25$ by graphing the corresponding function $y = x^2 - 25$.

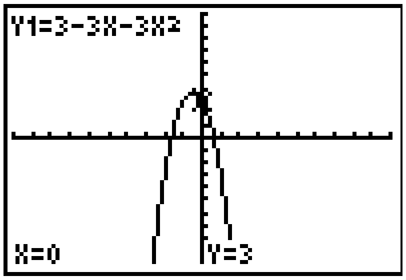




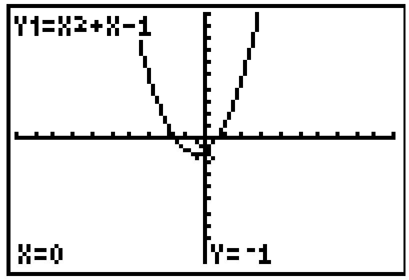
Since the x-intercepts of the graph are –5 and 5, the roots of the equation are –5 and 5.

Chapter 4 Review Page 258 Question 2

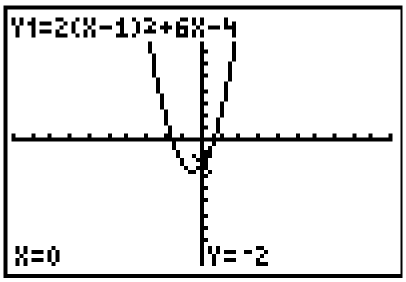
A: $0 = 3 - 3x - 3x^2$



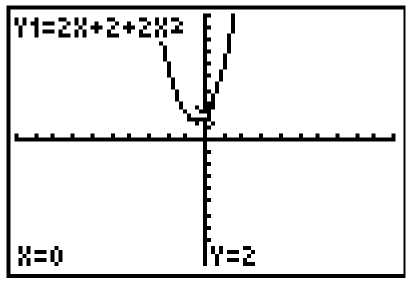
B: $0 = x^2 + x - 1$



C: $0 = 2(x - 1)^2 + 6x - 4$



D: $2x + 2 + 2x^2$



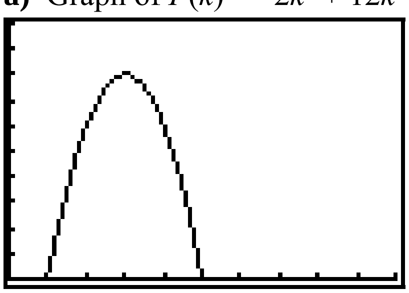
From the graphs of the corresponding functions, choice D has different roots from the other three. It has no real roots.

Chapter 4 Review Page 258 Question 3

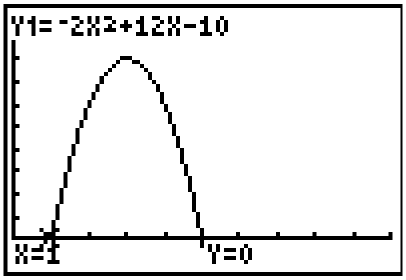
Example: For a quadratic equation to have no real roots, its corresponding graph must open upward and have a vertex above the x-axis or open downward and have a vertex below the x-axis.

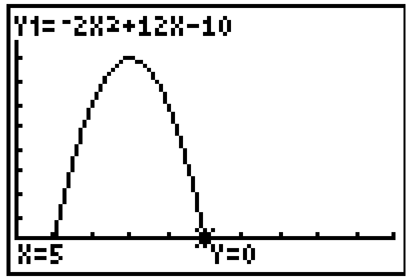
Chapter 4 Review Page 258 Question 4

a) Graph of $P(k) = -2k^2 + 12k - 10$.



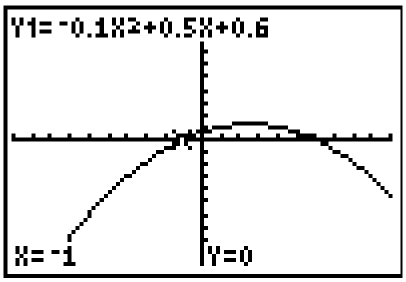
b) Since the x-intercepts are 1 and 5, then 1000 or 5000 keys will result in no profit or loss.

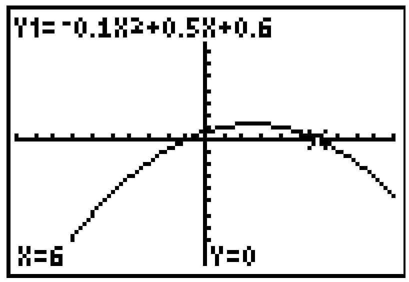




Chapter 4 Review Page 258 Question 5

a) Since the x-intercepts are –1 and 6, the zeros of the function are –1 and 6.





b) Since distance is positive, the ball travelled 6 m downfield before it hit the ground.

Chapter 4 Review Page 258 Question 6

a) $4x^2 - 13x + 9 = (4x - 9)(x - 1)$

b) $\frac{1}{2}x^2 - \frac{3}{2}x - 2 = \frac{1}{2}(x^2 - 3x - 4)$

$= \frac{1}{2}(x - 4)(x + 1)$

c) Let $r = v + 1$.

$3(v + 1)^2 + 10(v + 1) + 7$

$= 3r^2 + 10r + 7$

$= (3r + 7)(r + 1)$

$= (3(v + 1) + 7)(v + 1 + 1)$

$= (3v + 10)(v + 2)$

d) Use the pattern for factoring a difference of squares.

$9(a^2 - 4)^2 - 25(7b)^2$

$= [3(a^2 - 4) - 5(7b)][3(a^2 - 4) + 5(7b)]$

$= (3a^2 - 12 - 35b)(3a^2 - 12 + 35b)$

a) $x^2 + 10x + 21 = 0$

$(x + 3)(x + 7) = 0$

$x + 3 = 0$ or $x + 7 = 0$

$x = -3$ $\quad$ $x = -7$

For $x = -3$:

Left Side	Right Side
$x^2 + 10x + 21$	0
$= (-3)^2 + 10(-3) + 21$	
$= 9 - 30 + 21$	
$= 0$	

Left Side = Right Side

For $x = -7$:

Left Side	Right Side
$x^2 + 10x + 21$	0
$= (-7)^2 + 10(-7) + 21$	
$= 49 - 70 + 21$	
$= 0$	

Left Side = Right Side

The roots are –3 and –7.

b) $\frac{1}{4}m^2 + 2m - 5 = 0$

$\frac{1}{4}(m^2 + 8m - 20) = 0$

$\frac{1}{4}(m + 10)(m - 2) = 0$

$m + 10 = 0$ or $m - 2 = 0$

$m = -10$ $\quad$ $m = 2$

For $m = -10$:

Left Side	Right Side
$\frac{1}{4}m^2 + 2m - 5$	0
$= \frac{1}{4}(-10)^2 + 2(-10) - 5$	
$= 25 - 20 - 5$	
$= 0$	

Left Side = Right Side

For $m = 2$:

Left Side	Right Side
$\frac{1}{4}m^2 + 2m - 5$	0
$= \frac{1}{4}(2)^2 + 2(2) - 5$	
$= 1 + 4 - 5$	
$= 0$	

Left Side = Right Side

The roots are –10 and 2.

c) $5p^2 + 13p - 6 = 0$

$(5p - 2)(p + 3) = 0$

$5p - 2 = 0$ or $p + 3 = 0$

$p = \frac{2}{5}$ $\quad$ $p = -3$

For $p = \frac{2}{5}$:

Left Side | Right Side
$5p^2 + 13p - 6$ | 0

$= 5\left(\frac{2}{5}\right)^2 + 13\left(\frac{2}{5}\right) - 6$

$= \frac{4}{5} + \frac{26}{5} - \frac{30}{5}$

$= 0$

Left Side = Right Side

For $p = -3$:

Left Side | Right Side
$5p^2 + 13p - 6$ | 0

$= 5(-3)^2 + 13(-3) - 6$

$= 45 - 39 - 6$

$= 0$

Left Side = Right Side

The roots are $\frac{2}{5}$ and -3.

d) $6z^2 - 21z + 9 = 0$

$3(2z^2 - 7z + 3) = 0$

$3(2z - 1)(z - 3) = 0$

$2z - 1 = 0$ or $z - 3 = 0$

$z = \frac{1}{2}$ $\qquad z = 3$

For $z = \frac{1}{2}$:

Left Side | Right Side
$6z^2 - 21z + 9$ | 0

$= 6\left(\frac{1}{2}\right)^2 - 21\left(\frac{1}{2}\right) + 9$

$= \frac{3}{2} - \frac{21}{2} + \frac{18}{2}$

$= 0$

Left Side = Right Side

For $z = 3$:

Left Side | Right Side
$6z^2 - 21z + 9$ | 0

$= 6(3)^2 - 21(3) + 9$

$= 54 - 63 + 9$

$= 0$

Left Side = Right Side

The roots are $\frac{1}{2}$ and 3.

Chapter 4 Review Page 259 Question 8

a) $-4g^2 + 6 = -10g$

$-4g^2 + 10g + 6 = 0$

$-2(2g^2 - 5g - 3) = 0$

$-2(2g + 1)(g - 3) = 0$

$2g + 1 = 0$ or $g - 3 = 0$

$g = -\frac{1}{2}$ $\qquad g = 3$

b) $8y^2 = -5 + 14y$

$8y^2 - 14y + 5 = 0$

$(2y - 1)(4y - 5) = 0$

$2y - 1 = 0$ or $4y - 5 = 0$

$y = \frac{1}{2}$ $\qquad y = \frac{5}{4}$

c) $30k - 25k^2 = 9$
$-25k^2 + 30k - 9 = 0$
$-(25k^2 - 30k + 9) = 0$
$-(5k - 3)(5k - 3) = 0$
$5k - 3 = 0$
$k = \frac{3}{5}$

d) $0 = 2x^2 - 9x - 18$
$0 = (2x + 3)(x - 6)$
$2x + 3 = 0$ or $x - 6 = 0$
$x = -\frac{3}{2}$ $x = 6$

Chapter 4 Review Page 259 Question 9

a) For roots 2 and 3:
$(x - 2)(x - 3) = 0$
$x^2 - 5x + 6 = 0$

b) For roots –1 and –5:
$(x + 1)(x + 5) = 0$
$x^2 + 6x + 5 = 0$

c) For roots $\frac{3}{2}$ and –4:
$(2x - 3)(x + 4) = 0$
$2x^2 + 5x - 12 = 0$

Chapter 4 Review Page 259 Question 10

$-\frac{1}{4}t^2 + t + 3 = 0$
$-\frac{1}{4}(t^2 - 4t - 12) = 0$
$\frac{1}{4}(t + 4)(t - 6) = 0$
$t + 4 = 0$ or $t - 6 = 0$
$t = -4$ $t = 6$
Since time must be positive, it takes 6 s for the paper airplane to hit the ground.

Chapter 4 Review Page 259 Question 11

a) Let x represent the width of the rectangular prism. Then, the length is $x + 2$.
$V = \ell wh$
$V = (x + 2)(x)(15)$
$V = 15x^2 + 30x$

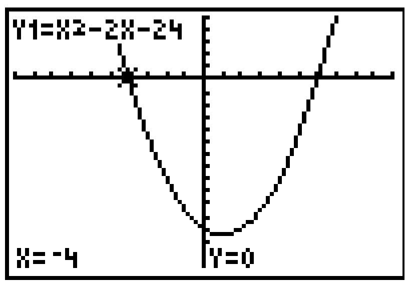
b) $2145 = 15x^2 + 30x$

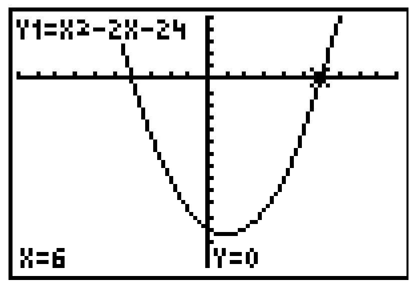
c) $2145 = 15x^2 + 30x$
$0 = 15x^2 + 30x - 2145$
$0 = 15(x^2 + 2x - 143)$
$0 = 15(x - 11)(x + 13)$
$x - 11 = 0$ or $x + 13 = 0$
$x = 11$ $\quad$ $x = -13$
Dimensions must be positive. So, the dimensions of the base of the rectangular prism are 11 m by 13 m.

Chapter 4 Review Page 259 Question 12

Solve by factoring:
$x^2 - 2x - 24 = 0$
$(x - 6)(x + 4) = 0$
$x - 6 = 0$ or $x + 4 = 0$
$x = 6$ $\quad$ $x = -4$

Solve by graphing:





Example: I prefer factoring to graphing, because this method is quicker for a quadratic equation that can be factored.

Chapter 4 Review Page 259 Question 13

For each expression to be a perfect square, the value of c must be half the square of the coefficient of the x-term.
a) For $x^2 + 4x + k$, the coefficient of the x-term is 4. So, $k = 2^2 = 4$.

b) For $x^2 + 3x + k$, the coefficient of the x-term is 3. So, $k = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$.

Chapter 4 Review Page 259 Question 14

a) $2x^2 - 98 = 0$
$2x^2 = 98$
$x^2 = 49$
$x = \pm 7$

b) $(x + 3)^2 = 25$
$x + 3 = \pm 5$
$x = -3 \pm 5$
$x = -3 + 5$ or $x = -3 - 5$
$x = 2$ $\quad$ $x = -8$

c) $(x - 5)^2 = 24$

$$x - 5 = \pm\sqrt{24}$$
$$x = 5 \pm \sqrt{24}$$
$$x = 5 \pm 2\sqrt{6}$$

d) $(x - 1)^2 = \frac{5}{9}$

$$x - 1 = \pm\sqrt{\frac{5}{9}}$$
$$x = 1 \pm \frac{\sqrt{5}}{3}$$
$$x = \frac{3 \pm \sqrt{5}}{3}$$

Chapter 4 Review Page 259 Question 15

a) $-2x^2 + 16x - 3 = 0$

$$x^2 - 8x = -\frac{3}{2}$$
$$x^2 - 8x + 16 = -\frac{3}{2} + 16$$
$$(x - 4)^2 = \frac{29}{2}$$
$$x - 4 = \pm\sqrt{\frac{29}{2}}$$
$$x = 4 \pm \frac{\sqrt{58}}{2}$$
$$x = \frac{8 \pm \sqrt{58}}{2}$$

b) $5y^2 + 20y + 1 = 0$

$$y^2 + 4y = -\frac{1}{5}$$
$$y^2 + 4y + 4 = -\frac{1}{5} + 4$$
$$(y + 2)^2 = \frac{19}{5}$$
$$y + 2 = \pm\sqrt{\frac{19}{5}}$$
$$y = -2 \pm \frac{\sqrt{95}}{5}$$
$$y = \frac{-10 \pm \sqrt{95}}{5}$$

c) $4p^2 + 2p = -5$

$$p^2 + \frac{1}{2}p = -\frac{5}{4}$$
$$p^2 + \frac{1}{2}y + \frac{1}{16} = -\frac{5}{4} + \frac{1}{16}$$
$$\left(p + \frac{1}{4}\right)^2 = -\frac{19}{4}$$

Since a square of an expression must be positive,
this quadratic equation has no real roots.

Chapter 4 Review Page 259 Question 16

$$-5t^2 + 200t + 9750 = 0$$
$$t^2 - 40t = 1950$$
$$t^2 - 40t + 400 = 1950 + 400$$
$$(t - 20)^2 = 2350$$
$$t - 20 = \pm\sqrt{2350}$$
$$t = 20 \pm \sqrt{2350}$$

$t = 20 + \sqrt{2350}$ or $t = 20 - \sqrt{2350}$

$t \approx 68.5$ $t = -28.5$

Since time must be positive, the aircraft takes approximately 68.5 s to return to the ground.

Chapter 4 Review Page 259 Question 17

a) Solve $0 = -\frac{1}{2}d^2 + 2d + 1$ to find the horizontal distance the snowboarder has travelled.

b) $-\frac{1}{2}d^2 + 2d + 1 = 0$

$$d^2 - 4d = 2$$
$$d^2 - 4d + 4 = 2 + 4$$
$$(d - 2)^2 = 6$$
$$d - 2 = \pm\sqrt{6}$$
$$d = 2 \pm \sqrt{6}$$

$d = 2 + \sqrt{6}$ or $d = 2 - \sqrt{6}$

$d \approx 4.4$ $d = -0.4$

Since distance must be positive, the horizontal distance travelled by the snowboarder is 4.4 m, to the nearest tenth of a metre.

Chapter 4 Review Page 260 Question 18

a) For $2x^2 + 11x + 5 = 0$, $a = 2$, $b = 11$, and $c = 5$.

$b^2 - 4ac = 11^2 - 4(2)(5)$

$b^2 - 4ac = 121 - 40$

$b^2 - 4ac = 81$

Since the value of the discriminant is positive, there are two distinct real roots.

b) For $4x^2 - 4x + 1 = 0$, $a = 4$, $b = -4$, and $c = 1$.
$b^2 - 4ac = (-4)^2 - 4(4)(1)$
$b^2 - 4ac = 16 - 16$
$b^2 - 4ac = 0$
Since the value of the discriminant is zero, there is one distinct real root.

c) For $3p^2 + 6p + 24 = 0$, $a = 3$, $b = 6$, and $c = 24$.
$b^2 - 4ac = 6^2 - 4(3)(24)$
$b^2 - 4ac = 36 - 288$
$b^2 - 4ac = -252$
Since the value of the discriminant is negative, there are no real roots.

d) For $4x^2 + 4x - 7 = 0$, $a = 4$, $b = 4$, and $c = -7$.
$b^2 - 4ac = 4^2 - 4(4)(-7)$
$b^2 - 4ac = 16 + 112$
$b^2 - 4ac = 128$
Since the value of the discriminant is positive, there are two distinct real roots.

Chapter 4 Review Page 260 Question 19

a) For $-3x^2 - 2x + 5 = 0$, $a = -3$, $b = -2$, and $c = 5$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(-3)(5)}}{2(-3)}$$

$$x = \frac{2 \pm \sqrt{4 + 60}}{-6}$$

$$x = \frac{2 \pm \sqrt{64}}{-6}$$

$$x = \frac{2 \pm 8}{-6}$$

$$x = \frac{2 + 8}{-6} \qquad \text{or} \qquad x = \frac{2 - 8}{-6}$$

$$x = -\frac{10}{6} \qquad\qquad x = \frac{-6}{-6}$$

$$x = -\frac{5}{3} \qquad\qquad x = 1$$

The roots are $-\frac{5}{3}$ and 1.

b) For $5x^2 + 7x + 1 = 0$, $a = 5$, $b = 7$, and $c = 1$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-7 \pm \sqrt{7^2 - 4(5)(1)}}{2(5)}$$

$$x = \frac{-7 \pm \sqrt{29}}{10}$$

The roots are $\frac{-7+\sqrt{29}}{10}$ and $\frac{-7-\sqrt{29}}{10}$.

c) For $3x^2 - 4x - 1 = 0$, $a = 3$, $b = -4$, and $c = -1$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-1)}}{2(3)}$$

$$x = \frac{4 \pm \sqrt{28}}{6}$$

$$x = \frac{4 \pm 2\sqrt{7}}{6}$$

$$x = \frac{2 \pm \sqrt{7}}{3}$$

The roots are $\frac{2+\sqrt{7}}{3}$ and $\frac{2-\sqrt{7}}{3}$.

d) For $25x^2 + 90x + 81 = 0$, $a = 25$, $b = 90$, and $c = 81$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-90 \pm \sqrt{90^2 - 4(25)(81)}}{2(25)}$$

$$x = \frac{-90 \pm \sqrt{8100 - 8100}}{50}$$

$$x = -\frac{9}{5}$$

The root is $-\frac{9}{5}$.

Chapter 4 Review   Page 260   Question 20

a) Solve $0 = -2x^2 + 6x + 1$ to find the maximum horizontal distance the water jet can reach.

b) Substitute into the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(-2)(1)}}{2(-2)}$$

$$x = \frac{-6 \pm \sqrt{36 + 8}}{-4}$$

$$x = \frac{-6 \pm \sqrt{44}}{-4}$$

$$x = \frac{-6 + \sqrt{44}}{-4} \quad \text{or} \quad x = \frac{-6 - \sqrt{44}}{-4}$$

$$x \approx -0.2 \qquad\qquad x \approx 3.2$$

Since distance is positive, the maximum horizontal distance the water jet can reach is 3.2 m, to the nearest tenth of a metre.

Chapter 4 Review   Page 260   Question 21

a) Let x represent the number of fare decreases. The new fare is $3.70 - 0.05x$.

b) The new number of people that use the ferry per day is $2480 + 40x$.

c) $R = (\text{number of people})(\text{fare})$
$R = (2480 + 40x)(3.70 - 0.05x)$
$R = -2x^2 + 24x + 9176$

d) $9246 = -2x^2 + 24x + 9176$
$0 = -2x^2 + 24x - 70$
$0 = x^2 - 12x + 35$
$0 = (x - 7)(x - 5)$
$x - 7 = 0$ or $x - 5 = 0$
$x = 7$ $\qquad$ $x = 5$
Either 5 or 7 fare decreases will result in a revenue of $9246.

Chapter 4 Review Page 260 Question 22

The correct order of the steps is D, C, E, B, A, and F.

Algebraic Steps	Explanations
$ax^2 + bx = -c$	Subtract c from both sides.
$x^2 + \frac{b}{a}x = -\frac{c}{a}$	Divide both sides by a.
$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{c}{a}$	Complete the square.
$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$	Factor the perfect square trinomial.
$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$	Take the square root of both sides.
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Solve for x.

Chapter 4 Practice Test

Chapter 4 Practice Test Page 261 Question 1

The x-intercepts of the graph represent the locations of the zeros of the function.
Choice **C**, (1, 0) and (5, 0).

Chapter 4 Practice Test Page 261 Question 2

$x^2 - 3x - 10 = (x - 5)(x + 2)$
One factor is choice **B**, $x - 5$.

Chapter 4 Practice Test Page 261 Question 3

For $2x^2 + kx - 1$ to be factorable, k must represent the sum of the factors of ac, or 2(–1), or –2.
Choice **D,** –1 and 1.

Chapter 4 Practice Test Page 261 Question 4

$$-\frac{1}{2}x^2 + x + \frac{7}{2} = 0$$
$$x^2 - 2x = 7$$
$$x^2 - 2x + 1 = 7 + 1$$
$$(x - 1)^2 = 8$$

$x - 1 = \pm\sqrt{8}$

$x = 1 \pm \sqrt{8}$

$x = 1 + \sqrt{8}$ or $x = 1 - \sqrt{8}$

$x \approx 3.83$ $x = -1.83$

Choice **B,** –1.83 and 3.83.

Chapter 4 Practice Test Page 261 Question 5

$15 = \dfrac{n^2 - n}{2}$

$30 = n^2 - n$

$0 = n^2 - n - 30$

$0 = (n - 6)(n + 5)$

$n - 6 = 0$ or $n + 5 = 0$

$n = 6$ $n = -5$

Since the number of teams must be positive, there are 6 teams.

Choice **B,** 6.

Chapter 4 Practice Test Page 261 Question 6

a) $0 = x^2 - 4x + 3$

$0 = (x - 1)(x - 3)$

$x - 1 = 0$ or $x - 3 = 0$

$x = 1$ $x = 3$

b) $0 = 2x^2 - 7x - 15$

$0 = (2x + 3)(x - 5)$

$2x + 3 = 0$ or $x - 5 = 0$

$x = -\dfrac{3}{2}$ $x = 5$

c) $0 = -x^2 - 2x + 3$

$0 = x^2 + 2x - 3$

$0 = (x - 1)(x + 3)$

$x - 1 = 0$ or $x + 3 = 0$

$x = 1$ $x = -3$

Chapter 4 Practice Test Page 261 Question 7

$3x^2 + 5x - 1 = 0$

$x^2 + \dfrac{5}{3}x = \dfrac{1}{3}$

$x^2 + \dfrac{5}{3}x + \dfrac{25}{36} = \dfrac{1}{3} + \dfrac{25}{36}$

$\left(x + \dfrac{5}{6}\right)^2 = \dfrac{37}{36}$

$$x + \frac{5}{6} = \pm\sqrt{\frac{37}{36}}$$

$$x = -\frac{5}{6} \pm \frac{\sqrt{37}}{6}$$

$$x = \frac{-5 \pm \sqrt{37}}{6}$$

The roots are $\frac{-5+\sqrt{37}}{6}$ and $\frac{-5-\sqrt{37}}{6}$.

Chapter 4 Practice Test Page 261 Question 8

For $x^2 + 4x - 7 = 0$, $a = 1$, $b = 4$, and $c = -7$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-7)}}{2(1)}$$

$$x = \frac{-4 \pm \sqrt{44}}{2}$$

$$x = \frac{-4 \pm 2\sqrt{11}}{2}$$

$$x = -2 \pm \sqrt{11}$$

The roots are $-2+\sqrt{11}$ and $-2-\sqrt{11}$.

Chapter 4 Practice Test Page 261 Question 9

a) For $x^2 + 10x + 25 = 0$, $a = 1$, $b = 10$, and $c = 25$.
$b^2 - 4ac = 10^2 - 4(1)(25)$
$b^2 - 4ac = 100 - 100$
$b^2 - 4ac = 0$
Since the value of the discriminant is zero, there is one distinct real root.

b) For $2x^2 + x - 5 = 0$, $a = 2$, $b = 1$, and $c = -5$.
$b^2 - 4ac = 1^2 - 4(2)(-5)$
$b^2 - 4ac = 41$
Since the value of the discriminant is positive, there are two distinct real roots.

c) For $2x^2 - 4x + 6 = 0$, $a = 2$, $b = -4$, and $c = 6$.
$b^2 - 4ac = (-4)^2 - 4(2)(6)$
$b^2 - 4ac = -32$
Since the value of the discriminant is negative, there are no real roots.

d) For $\frac{2}{3}x^2 + \frac{1}{2}x - 3 = 0$, $a = \frac{2}{3}$, $b = \frac{1}{2}$, and $c = -3$.

$b^2 - 4ac = \left(\frac{1}{2}\right)^2 - 4\left(\frac{2}{3}\right)(-3)$

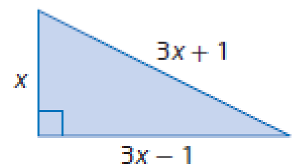
$b^2 - 4ac = \frac{1}{4} + 8$

$b^2 - 4ac = 8.25$

Since the value of the discriminant is positive, there are two distinct real roots.

Chapter 4 Practice Test Page 262 Question 10

a) Let x represent the length of the shorter leg.



b) Use the Pythagorean Theorem.

$(3x + 1)^2 = x^2 + (3x - 1)^2$

c) $(3x + 1)^2 = x^2 + (3x - 1)^2$

$9x^2 + 6x + 1 = x^2 + 9x^2 - 6x + 1$

$0 = x^2 - 12x$

$0 = x(x - 12)$

$x = 0$ or $x - 12 = 0$

$x = 12$

Since the length must be positive, the sides of the triangle are 12 cm, 35 cm, and 37 cm.

Chapter 4 Practice Test Page 262 Question 11

a) $-5t^2 + 10t + 35 = 0$

$t^2 - 2t = 7$

$t^2 - 2t + 1 = 7 + 1$

$(t - 1)^2 = 8$

$t - 1 = \pm\sqrt{8}$

$t = 1 \pm \sqrt{8}$

$t = 1 + \sqrt{8}$ or $t = 1 - \sqrt{8}$

$t \approx 3.8$ $t = -1.8$

Time must be positive. So, the pebble hits the river after 3.8 s, to the nearest tenth of a second.

b) Substitute $t = 0$ to find the height of the scenic lookout.
$h(t) = -5t^2 + 10t + 35$
$h(0) = -5(0)^2 + 10(0) + 35$
$h(0) = 35$
The scenic lookout is 35 m above the river.

c) Example: I chose to complete the square to solve the quadratic equation. The equation cannot be factored but the coefficients are "nice."

Chapter 4 Practice Test Page 262 Question 12

Let x represent the length cut from each rod.
Use the Pythagorean Theorem.

$$(44 - x)^2 = (41 - x)^2 + (20 - x)^2$$
$$x^2 - 88x + 1936 = x^2 - 82x + 1681 + x^2 - 40x + 400$$
$$0 = x^2 - 34x + 145$$
$$0 = (x - 5)(x - 29)$$

$x - 5 = 0$ or $x - 29 = 0$
$x = 5$ $\quad$ $x = 29$

The length cut from each rod cannot be greater than the shortest rod, 21 cm.
So, the length cut from each rod is 5 cm.

Chapter 4 Practice Test Page 262 Question 13

Let x represent the width of the paper. Then, $50 - x$ represents the length.

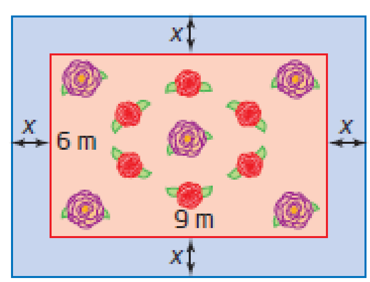
$$A = \ell w$$
$$616 = (50 - x)x$$
$$x^2 - 50x + 616 = 0$$
$$(x - 28)(x - 22) = 0$$

$x - 28 = 0$ or $x - 22 = 0$
$x = 28$ $\quad$ $x = 22$

The dimensions of the paper are 28 cm by 22 cm.

Chapter 4 Practice Test Page 262 Question 14

a) $(9 + 2x)(6 + 2x) = 2(54)$
$4x^2 + 30x + 54 = 108$
$4x^2 + 30x - 54 = 0$



b) $4x^2 + 30x - 54 = 0$
$2(2x^2 + 15x - 28) = 0$
$2(2x - 3)(x + 9) = 0$

$2x - 3 = 0$ or $x + 9 = 0$

$x = \frac{3}{2}$ $\quad$ $x = -9$

Since width must be positive, the grass strip will be 1.5 m in width.
Example: Factoring is a quick method.

c) The outside dimensions are 12 m by 9 m. So, the perimeter of the outside of the path is 42 m.

Cumulative Review, Chapters 3–4

Cumulative Review Page 264 Question 1

a) For a vertex in quadrant III, both the x-coordinate and the y-coordinate must be negative. The quadratic function $y = 2(x + 2)^2 - 3$ meets this requirement. Choose **C**.

b) For a parabola that opens downward, $a < 0$.. The quadratic function $y = -5(x - 2)^2 - 3$ meets this requirement. Choose **A**.

c) For an axis of symmetry of $x = 3$, the value of p is 3. The quadratic function $y = 3(x - 3)^2 - 5$ meets this requirement. Choose **D**.

d) For a range of $\{y \mid y \geq 5, y \in \mathrm{R}\}$, $a > 0$ and $q = 5$. The quadratic function $y = 3(x + 3)^2 + 5$ meets this requirement. Choose **B**.

Cumulative Review Page 264 Question 2

a) The function $y = (x - 6) - 1$ has degree 1. It is not a quadratic function.

b) The function $y = -5(x + 1)^2$ has degree 2. It is a quadratic function.

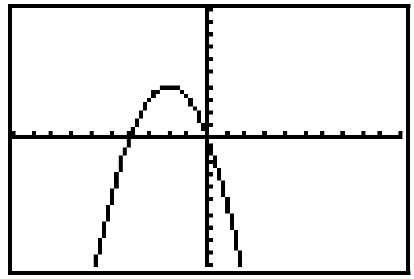
c) The function $y = \sqrt{(x+2)^2} + 7$ has degree 1. It is not a quadratic function.

d) The function $y + 8 = x^2$ has degree 2. It is a quadratic function.

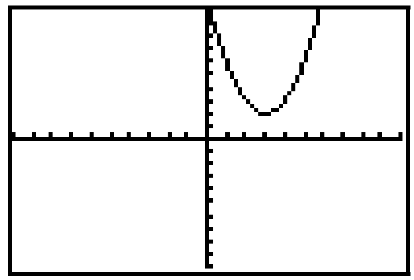
Cumulative Review Page 264 Question 3

Examples:

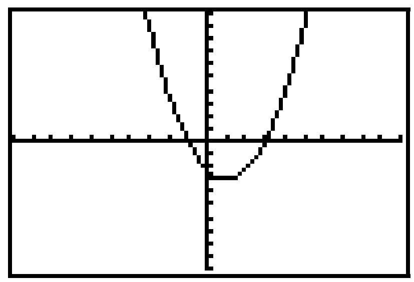
a) For an axis of symmetry with equation $x = -2$ and a range of $\{y \mid y \le 4, y \in R\}$, the graph must have a vertex at (–2, 4) and open downward.



b) For an axis of symmetry with equation $x = 3$ and a range of $\{y \mid y \ge 2, y \in R\}$, the graph must have a vertex at (3, 2) and open upward.



c) For a parabola that opens upward with vertex at (1, –3) and an x-intercept at (3, 0), the other x-intercept must be at (–1, 0) and have a range of $\{y \mid y \ge -3, y \in R\}$.



Cumulative Review Page 264 Question 4

a) For $f(x) = (x + 4)^2 - 3$, $a = 1$, $p = -4$, and $q = -3$.
The vertex is located at (–4, –3).
The domain is $\{x \mid x \in R\}$ and the range is $\{y \mid y \ge -3, y \in R\}$.
The equation of the axis of symmetry is $x = -4$.
The x-intercepts are approximately –5.7 and –2.3.
The y-intercept is $(0 + 4)^2 - 3$, or 13.

b) For $f(x) = -(x - 2)^2 + 1$, $a = -1$, $p = 2$, and $q = 1$.
The vertex is located at (2, 1).
The domain is $\{x \mid x \in R\}$ and the range is $\{y \mid y \le 1, y \in R\}$.
The equation of the axis of symmetry is $x = 2$.
The x-intercepts are 1 and 3.
The y-intercept is $-(0 - 2)^2 + 1$, or –3.

c) For $f(x) = -2x^2 - 6$, $a = -2$, $p = 0$, and $q = -6$.
The vertex is located at (0, –6).
The domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \le -6, y \in \mathrm{R}\}$.
The equation of the axis of symmetry is $x = 0$.
Since the graph opens downward **(**$a < 0$) and has a maximum value of –6, which is below the x-axis, there are no x-intercepts.
The y-intercept is $-2(0)^2 - 6$, or –6.

d) For $f(x) = \frac{1}{2}(x + 8)^2 + 6$, $a = \frac{1}{2}$, $p = -8$, and $q = 6$.
The vertex is located at (–8, 6).
The domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \ge 6, y \in \mathrm{R}\}$.
The equation of the axis of symmetry is $x = -8$.
Since the graph opens upward **(**$a > 0$) and has a minimum value of 6, which is above the x-axis, there are no x-intercepts.
The y-intercept is $\frac{1}{2}(0 + 8)^2 + 6$, or 38.

Cumulative Review Page 264 Question 5

a) Complete the square to write $y = x^2 - 10x + 18$ in vertex form.
$y = x^2 - 10x + 18$
$y = (x^2 - 10x) + 18$
$y = (x^2 - 10x + 25 - 25) + 18$
$y = (x^2 - 10x + 25) - 25 + 18$
$y = (x - 5)^2 - 7$
The graph of $y = (x - 5)^2 - 7$ will have the same shape as the graph of $y = x^2$, since $a = 1$. Since $p = 5$ and $q = -7$, this represents a horizontal translation of 5 units to the right and a vertical translation of 7 units down relative to the graph of $y = x^2$.

b) Complete the square to write $y = -x^2 + 4x - 7$ in vertex form.
$y = -x^2 + 4x - 7$
$y = -(x^2 - 4x) - 7$
$y = -(x^2 - 4x + 4 - 4) - 7$
$y = -(x^2 - 4x + 4) + 4 - 7$
$y = -(x - 2)^2 - 3$
The graph of $y = -(x - 2)^2 - 3$ will have the same shape as the graph of $y = x^2$ but be reflected in the x-axis, since $a = -1$. Since $p = 2$ and $q = -3$, this represents a horizontal translation of 2 units to the right and a vertical translation of 3 units down relative to the graph of $y = x^2$.

c) Complete the square to write $y = 3x^2 - 6x + 5$ in vertex form.

$y = 3x^2 - 6x + 5$
$y = 3(x^2 - 2x) + 5$
$y = 3(x^2 - 2x + 1 - 1) + 5$
$y = 3(x^2 - 2x + 1) - 3 + 5$
$y = 3(x - 1)^2 + 2$

The graph of $y = 3(x - 1)^2 + 2$ will be narrower than the graph of $y = x^2$, since $a > 1$. Since $p = 1$ and $q = 2$, this represents a horizontal translation of 1 unit to the right and a vertical translation of 2 units up relative to the graph of $y = x^2$.

d) Complete the square to write $y = \frac{1}{4}x^2 + 4x + 20$ in vertex form.

$y = \frac{1}{4}x^2 + 4x + 20$

$y = \frac{1}{4}(x^2 + 16x) + 20$

$y = \frac{1}{4}(x^2 + 16x + 64 - 64) + 20$

$y = \frac{1}{4}(x^2 + 16x + 64) - 16 + 20$

$y = \frac{1}{4}(x + 8)^2 + 4$

The graph of $y = \frac{1}{4}(x + 8)^2 + 4$ will be wider than the graph of $y = x^2$, since $0 < a < 1$. Since $p = -8$ and $q = 4$, this represents a horizontal translation of 8 units to the left and a vertical translation of 4 units up relative to the graph of $y = x^2$.

Cumulative Review Page 264 Question 6

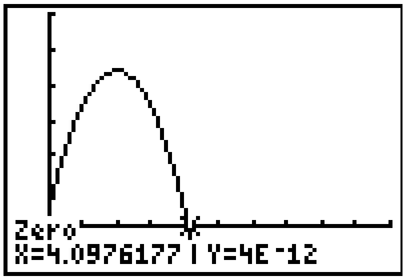
a) Complete the square to find the maximum.

$h(t) = -5t^2 + 20t + 2$
$h(t) = -5(t^2 - 4t) + 2$
$h(t) = -5(t^2 - 4t + 4 - 4) + 2$
$h(t) = -5(t^2 - 4t + 4) + 20 + 2$
$h(t) = -5(t - 2)^2 + 22$

The maximum height reached by the arrow is 22 m.

b) The arrow was shot from a height of 2 m.

c) Use a graphing calculator to graph the function with window settings of x: [–1, 10, 1] and y: [–5, 30, 5]. The arrow hits the ground in 4 s, to the nearest second.



Cumulative Review Page 264 Question 7

When solving quadratic equations, you may consider the relationship among the **roots** of a quadratic equation, the **zeros** of the corresponding quadratic function, and the ***x*-intercepts** of the graph of the quadratic function.

Cumulative Review Page 264 Question 8

a) $9x^2 + 6x - 8 = (3x + 4)(3x - 2)$

b) $16r^2 - 81s^2 = (4r - 9s)(4r + 9s$

c) Let $r = x + 1$.

$2(x + 1)^2 + 11(x + 1) + 14$
$= 2r^2 + 11r + 14$
$= (2r + 7)(r + 2)$
$= (2(x + 1) + 7)(x + 1 + 2)$
$= (2x + 9)(x + 3)$

d) Let $r = xy$.

$x^2y^2 - 5xy - 36$
$= r^2 - 5r - 36$
$= (r - 9)(r + 4)$
$= (xy - 9)(xy + 4)$

e) Use the pattern for factoring a difference of squares.

$9(3a + b)^2 - 4(2a - b)^2$
$= [3(3a + b) - 2(2a - b)][3(3a + b) + 2(2a - b)]$
$= (5a + 5b)(13a + b)$
$= 5(a + b)(13a + b)$

f) $121r^2 - 400 = (11r - 20)(11r + 20)$

Cumulative Review Page 264 Question 9

Let x, $x + 1$, and $x + 2$ represent the three consecutive integers. For a sum of squares of the integers equal to 194,

$194 = x^2 + (x + 1)^2 + (x + 2)^2$
$194 = x^2 + x^2 + 2x + 1 + x^2 + 4x + 4$
$0 = 3x^2 + 6x - 189$
$0 = 3(x^2 + 2x - 63)$
$0 = 3(x - 7)(x + 9)$

$x - 7 = 0$ or $x + 9 = 0$

$x = 7$ $x = -9$

The three consecutive integers are 7, 8, and 9 or –9, –8, and –7.

Cumulative Review Page 265 Question 10

Let x represent the number of seats in each row. Then, the number of rows is $x + 4$. For a total of 285 seats,

$285 = x(x + 4)$

$0 = x^2 + 4x - 285$

$0 = (x + 19)(x - 15)$

$x + 19 = 0$ or $x - 15 = 0$

$x = -19$ $x = 15$

Since the number seats must be positive, $x = -19$ is an extraneous root.
There are 15 seats in each row and 19 rows.

Cumulative Review Page 265 Question 11

Let x represent the width of the deck. Then, the radius of the deck and hot tub is $x + 1$. For a total area of 63.6 m^2,

$\pi(x + 1)^2 = 63.6$

$(x + 1)^2 = \dfrac{63.6}{\pi}$

$x + 1 = \pm\sqrt{\dfrac{63.6}{\pi}}$

$x = -1 \pm \sqrt{\dfrac{63.6}{\pi}}$

$x = -1 + \sqrt{\dfrac{63.6}{\pi}}$ or $x = -1 - \sqrt{\dfrac{63.6}{\pi}}$

$x \approx 3.5$ $x \approx -5.5$

Since the width must be positive, $x = -5.5$ is an extraneous root.
The deck is 3.5 m wide, to the nearest tenth of a metre.

Cumulative Review Page 265 Question 12

Example: Dallas forgot to factor out 2 from the coefficient of the x-term in line 1. In line 2, Dallas should have added 2 times the value added in the brackets to the right side. The correct solution is shown.

$2(x^2 - 6x) = 7$

$2(x^2 - 6x + 9) = 7 + 18$

$$2(x-3)^2 = 25$$
$$x = 3 \pm \sqrt{\frac{25}{2}}$$
$$x = 3 \pm \frac{5\sqrt{2}}{2}$$
$$x = \frac{6 \pm 5\sqrt{2}}{2}$$

Doug should have substituted –12 as the first value in the numerator in line 1. In line 2, Doug miscalculated the result under the radical. In line 3, he incorrectly simplified $\frac{\sqrt{80}}{4}$.

The correct solution is shown.

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(2)(-7)}}{2(2)}$$
$$x = \frac{12 \pm \sqrt{200}}{4}$$
$$x = \frac{12 \pm 10\sqrt{2}}{4}$$
$$x = \frac{6 \pm 5\sqrt{2}}{2}$$

Cumulative Review Page 265 Question 13

a) Solve $3x^2 - 6 = 0$ by taking the square root.

$$3x^2 - 6 = 0$$
$$3x^2 = 6$$
$$x = \pm\sqrt{2}$$

For $x = \sqrt{2}$:

Left Side	Right Side
$3x^2 - 6$	0
$= 3(\sqrt{2})^2 - 6$	
$= 6 - 6$	
$= 0$	

Left Side = Right Side

For $x = -\sqrt{2}$:

Left Side	Right Side
$3x^2 - 6$	0
$= 3(-\sqrt{2})^2 - 6$	
$= 6 - 6$	
$= 0$	

Left Side = Right Side

The roots are $\pm\sqrt{2}$.

b) Solve $m^2 - 15m = -26$ by factoring.

$m^2 - 15m = -26$

$m^2 - 15m + 26 = 0$

$(m - 13)(m - 2) = 0$

$m - 13 = 0$ or $m - 2 = 0$

$m = 13$ $\quad$ $m = 2$

For $m = 13$:

Left Side	Right Side
$m^2 - 15m$	-26
$= 13^2 - 15(13)$	
$= -26$	

Left Side = Right Side

For $m = 2$:

Left Side	Right Side
$m^2 - 15m$	-26
$= 2^2 - 15(2)$	
$= -26$	

Left Side = Right Side

The roots are 13 and 2.

c) Solve $s^2 - 2s - 35 = 0$ by factoring.

$s^2 - 2s - 35 = 0$

$(s - 7)(s + 5) = 0$

$s - 7 = 0$ or $s + 5 = 0$

$s = 7$ $\quad$ $s = -5$

For $s = 7$:

Left Side	Right Side
$s^2 - 2s - 35$	0
$= 7^2 - 2(7) - 35$	
$= 49 - 14 - 35$	
$= 0$	

Left Side = Right Side

For $s = -5$:

Left Side	Right Side
$s^2 - 2s - 35$	0
$= (-5)^2 - 2(-5) - 35$	
$= 25 + 10 - 35$	
$= 0$	

Left Side = Right Side

The roots are 13 and 2.

d) Solve $-16x^2 + 47x + 3 = 0$ by factoring.

$-16x^2 + 47x + 3 = 0$

$-(16x^2 - 47x - 3) = 0$

$-(16x + 1)(x - 3) = 0$

$16x + 1 = 0$ or $x - 3 = 0$

$x = -\frac{1}{16}$ $\quad$ $x = 3$

For $x = -\frac{1}{16}$:

Left Side	Right Side
$-16x^2 + 47x + 3$	0

For $x = 3$:

Left Side	Right Side
$-16x^2 + 47x + 3$	0

$= -16\left(-\frac{1}{16}\right)^2 + 47\left(-\frac{1}{16}\right) + 3$

$= -\frac{1}{16} - \frac{47}{16} + \frac{48}{16}$

$= 0$

Left Side = Right Side

$= -16(3)^2 + 47(3) + 3$

$= -144 + 141 + 3$

$= 0$

Left Side = Right Side

Cumulative Review Page 265 Question 14

a) For $x^2 - 6x + 3 = 0$, $a = 1$, $b = -6$, and $c = 3$.
$b^2 - 4ac = (-6)^2 - 4(1)(3)$
$b^2 - 4ac = 36 - 12$
$b^2 - 4ac = 24$
Since the value of the discriminant is positive, there are two distinct real roots.

b) For $x^2 + 22x + 121 = 0$, $a = 1$, $b = 22$, and $c = 121$.
$b^2 - 4ac = 22^2 - 4(1)(121)$
$b^2 - 4ac = 484 - 484$
$b^2 - 4ac = 0$
Since the value of the discriminant is zero, there is one distinct real root.

c) For $-x^2 + 3x - 5 = 0$, $a = -1$, $b = 3$, and $c = -5$.
$b^2 - 4ac = 3^2 - 4(-1)(-5)$
$b^2 - 4ac = 9 - 20$
$b^2 - 4ac = -11$
Since the value of the discriminant is negative, there are no real roots.

Cumulative Review Page 265 Question 15

a) Let x represent the side length of the bottom square. Then, $x + 1$ represents the side length of the top square. For a total area of 85 in.^{2},
$85 = x^2 + (x + 1)^2$

b) Solve by factoring.
$85 = x^2 + (x + 1)^2$
$0 = 2x^2 + 2x - 84$
$0 = 2(x^2 + x - 42)$
$0 = 2(x + 7)(x - 6)$
$x + 7 = 0$ or $x - 6 = 0$
$x = -7$ $\qquad$ $x = 6$

c) The side length of the bottom of the box is 6 in. and the side length of the the top of the box is 7 in..

d) Since the side length must be positive, $x = –7$ is an extraneous root.

Unit 2 Test

Unit 2 Test Page 266 Question 1

The graph of the function that is congruent to the graph of $f(x) = x^2 + 3$ but translated vertically 2 units down is $f(x) = x^2 + 1$. Choice **A**.

Unit 2 Test Page 266 Question 2

For a quadratic function with a vertex at (–1, –2) and passing through the point (1, 6), the equation is of the form $y = a(x + 1)^2 – 2$. Use the given point to find a.
$y = a(x + 1)^2 – 2$
$6 = a(1 + 1)^2 – 2$
$8 = 4a$
$a = 2$
The equation of the quadratic function is $y = 2(x + 1)^2 – 2$. Choice **D**.

Unit 2 Test Page 266 Question 3

The graph of $y = ax^2 + q$ intersects the x-axis in two places when $a > 0$ and the vertex is below the x-axis, or $q < 0$, or when $a < 0$ and the vertex is above the x-axis, or $q > 0$. Choice **D**.

Unit 2 Test Page 266 Question 4

Change $y = 2x^2 – 8x + 2$ to vertex form.
$y = 2x^2 – 8x + 2$
$y = 2(x^2 – 4x) + 2$
$y = 2(x^2 – 4x + 4 – 4) + 2$
$y = 2[(x^2 – 4x + 4) – 4] + 2$
$y = 2(x – 2)^2 – 8 + 2$
$y = 2(x – 2)^2 – 6$
So, $p = 2$ and $q = –6$. Choice **B**.

Unit 2 Test Page 266 Question 5

Michelle made her first error in Step 1. She did not correctly factor out –3 from the *x*-term.
Find the correct coordinates of the vertex.

$$y = -3\left(x^2 - \frac{5}{3}x\right) - 2$$

$$y = -3\left(x^2 - \frac{5}{3}x + \frac{25}{36}\right) - 2 + (-3)\left(-\frac{25}{36}\right)$$

$$y = -3\left(x - \frac{5}{6}\right)^2 + \frac{1}{12}$$

The vertex is $\left(\frac{5}{6}, \frac{1}{12}\right)$. Choice **B**.

Unit 2 Test Page 266 Question 6

For $y = -3x^2 - 4x + 5$, $a = -3$, $b = -4$, and $c = 5$.
$b^2 - 4ac = (-4)^2 - 4(-3)(5)$
$b^2 - 4ac = 16 + 60$
$b^2 - 4ac = 76$
The value of the discriminant for the quadratic function $y = -3x^2 - 4x + 5$ is **76**.

Unit 2 Test Page 266 Question 7

Let n represent the number of rent increases. The new price is $200 + 20n$.
The new number of units rented is $80 - n$.
Revenue = (price)(number of sessions)
$R = (200 + 20n)(80 - n)$
$R = -20n^2 + 1400n + 16\,000$
$R = -20(n^2 - 70n) + 16\,000$
$R = -20(n^2 - 70n + 1225 - 1225) + 16\,000$
$R = -20[(n^2 - 70n + 1225) - 1225] + 16\,000$
$R = -20(n - 35)^2 + 24\,500 + 16\,000$
$R = -20(n - 35)^2 + 40\,500$
For maximum revenue, the new price is 200 + 20(35), or $900.
The manager of an 80-unit apartment complex is trying to decide what rent to charge. At a rent of $200 per week, all the units will be full. For each increase in rent of $20 per week, one more unit will become vacant. The manager should charge **$900** per week to maximize the revenue of the apartment complex.

Unit 2 Test Page 266 Question 8

For $9x^2 + 4x - 1 = 0$, $a = 9$, $b = 4$, and $c = -1$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(9)(-1)}}{2(9)}$$

$$x = \frac{-4 \pm \sqrt{16 + 36}}{18}$$

$$x = \frac{-4 \pm \sqrt{52}}{18}$$

$$x = \frac{-4 + \sqrt{52}}{18} \quad \text{or} \quad x = \frac{-4 - \sqrt{52}}{18}$$

$$x \approx 0.18 \qquad\qquad x \approx -0.62$$

The greater solution to the quadratic equation $9x^2 + 4x - 1 = 0$, rounded to the nearest hundredth, is **0.18**.

Unit 2 Test Page 267 Question 9

a) $A = \pi r^2$

$9000 = \pi r^2$

$$\frac{9000}{\pi} = r^2$$

$$r = \sqrt{\frac{9000}{\pi}}$$

$r \approx 53.5$

The radius of the largest circle you can paint is 53.5 cm, to the nearest tenth of a centimetre.

b) Two cans of paint can cover 18 000 cm^2.

$$r = \sqrt{\frac{18\,000}{\pi}}$$

$r \approx 75.7$

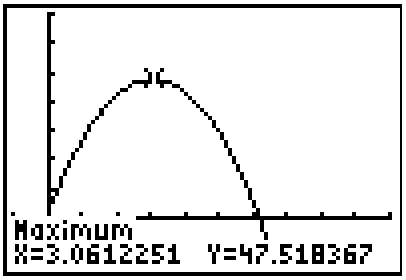
The radius of the largest circle you can paint is 75.7 cm, to the nearest tenth of a centimetre.

c) No. The radius does not double when the amount of paint doubles.

$2(53.5) = 107 \neq 75.7$

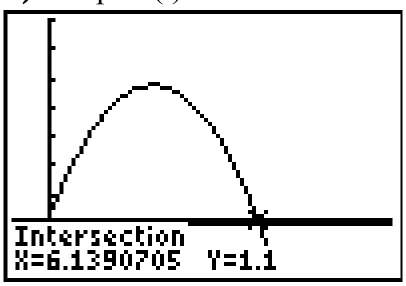
Unit 2 Test Page 267 Question 10

a) Graph $h(t) = -4.9t^2 + 30t + 1.6$ to find the maximum height of the ball.



The maximum height that the ball reaches is 47.5 m, to the nearest tenth of a metre.

b) Graph $h(t) = -4.9t^2 + 30t + 1.6$ and $h(t) = 1.1$ and find the point of intersection.



The pitcher catches the ball after 6.1 s, to the nearest tenth of a second.

Unit 2 Test Page 267 Question 11

Let x represent the side length of the square base of the box. Then, for a volume of 128 cm^2,

$$V = 2x^2$$
$$128 = 2x^2$$
$$64 = x^2$$
$$x = \pm 8$$

Since length must be positive, the side length is 8 cm. So, the size of cardboard needed is 12 cm by 12 cm.

Unit 2 Test Page 267 Question 12

a) Let the three consecutive integers be x, $x + 1$, and $x + 2$. For a sum of squares of the integers equal to 677,

$$677 = x^2 + (x + 1)^2 + (x + 2)^2$$
$$677 = x^2 + x^2 + 2x + 1 + x^2 + 4x + 4$$
$$0 = 3x^2 + 6x - 672$$

b) Solve.

$0 = 3(x^2 + 2x - 224)$

$0 = 3(x - 14)(x + 16)$

$x - 14 = 0$ or $x + 16 = 0$

$x = 14$ $x = -16$

The roots are 14 and –16.

c) The side lengths are 14 in., 15 in., and 16 in..

d) Since the three consecutive integers represent side lengths, the values must be positive.