Chapter 5 Radical Expressions and Equations

Section 5.1 Working With Radicals

Section 5.1 Page 278 Question 1

Mixed Radical Form	Entire Radical Form
$4\sqrt{7}$	$4\sqrt{7} = \sqrt{4^2(7)} = \sqrt{112}$
$\sqrt{50} = \sqrt{25(2)} = 5\sqrt{2}$	$\sqrt{50}$
$-11\sqrt{8}$	$-11\sqrt{8} = -\sqrt{11^2(8)} = -\sqrt{968}$
$-\sqrt{200} = -\sqrt{100(2)} = -10\sqrt{2}$	$-\sqrt{200}$

Section 5.1 Page 278 Question 2

a) $\sqrt{56} = \sqrt{4(14)}$
$= 2\sqrt{14}$

b) $3\sqrt{75} = 3\sqrt{25(3)}$
$= 15\sqrt{3}$

c) $\sqrt[3]{24} = \sqrt[3]{8(3)}$
$= \sqrt[3]{2^3(3)}$
$= 2\sqrt[3]{3}$

d) $\sqrt{c^3d^2} = \sqrt{c^2(c)(d^2)}$
$= cd\sqrt{c}$

Section 5.1 Page 278 Question 3

a) $3\sqrt{8m^4} = 3\sqrt{4(2)(m^2)(m^2)}$
$= 6m^2\sqrt{2}, \quad m \in \mathrm{R}$

b) $\sqrt[3]{24q^5} = \sqrt[3]{2^3(3)(q^3)(q^2)}$
$= 2q\sqrt[3]{3q^2}, \quad q \in \mathrm{R}$

c) $-2\sqrt[5]{160s^5t^6} = -2\sqrt[5]{2^5(5)(s^5)(t^5)(t)}$
$= -4st\sqrt[5]{5t}, \quad s, t \in \mathrm{R}$

Section 5.1 Page 279 Question 4

Mixed Radical Form	Entire Radical Form
$3n\sqrt{5}$	$3n\sqrt{5} = \sqrt{(3n)^2(5)}$ $= \sqrt{45n^2}, n \geq 0$ or $-\sqrt{45n^2}, n < 0$
$\sqrt[3]{-432} = \sqrt[3]{2(-6)^3} = -6\sqrt[3]{2}$	$\sqrt[3]{-432}$
$\frac{1}{2a}\sqrt[3]{7a}$	$\frac{1}{2a}\sqrt[3]{7a} = \sqrt[3]{\left(\frac{1}{2a}\right)^3(7a)}$ $= \sqrt[3]{\frac{1}{8a^3}(7a)}$ $= \sqrt[3]{\frac{7}{8a^2}}, a \neq 0$
$\sqrt[3]{128x^4} = \sqrt[3]{4^3(2)x^3(x)}$ $= 4x\sqrt[3]{2x}$	$\sqrt[3]{128x^4}$

Section 5.1 Page 279 Question 5

a) For $15\sqrt{5}$ and $8\sqrt{125}$, express the second radical in terms of $\sqrt{5}$.

$15\sqrt{5}$

$$8\sqrt{125} = 8\sqrt{25(5)}$$
$$= 40\sqrt{5}$$

b) For $8\sqrt{112z^8}$ and $48\sqrt{7z^4}$, express both radicals in terms of $\sqrt{7}$.

$$8\sqrt{112z^8} = 8\sqrt{16(7)z^8}$$
$$= 32z^4\sqrt{7}$$

$$48\sqrt{7z^4} = 48z^2\sqrt{7}$$

c) For $-35\sqrt[4]{w^2}$ and $3\sqrt[4]{81w^{10}}$, express the second radical in terms of $\sqrt{w^2}$.

$-35\sqrt[4]{w^2}$

$$3\sqrt[4]{81w^{10}} = 3\sqrt[4]{3^4(w^8)(w^2)}$$
$$= 9w^2\sqrt[4]{w^2}$$

d) For $6\sqrt[3]{2}$ and $6\sqrt[3]{54}$, express the second radical in terms of $\sqrt[3]{2}$.

$6\sqrt[3]{2}$

$$6\sqrt[3]{54} = 6\sqrt[4]{3^3(2)}$$
$$= 18\sqrt[3]{2}$$

Section 5.1 Page 279 Question 6

a) $3\sqrt{6} = \sqrt{9(6)}$ $= \sqrt{54}$ $10 = \sqrt{100}$ $7\sqrt{2} = \sqrt{49(2)}$ $= \sqrt{98}$

The numbers from least to greatest are $3\sqrt{6}$, $7\sqrt{2}$, and 10.

b) $-2\sqrt{3} = -\sqrt{4(3)}$ $= -\sqrt{12}$ $-4 = -\sqrt{16}$ $-3\sqrt{2} = -\sqrt{9(2)}$ $= -\sqrt{18}$ $-2\sqrt{\frac{7}{2}} = -\sqrt{4\left(\frac{7}{2}\right)}$ $= -\sqrt{14}$

The numbers from least to greatest are $-3\sqrt{2}$, –4, $-2\sqrt{\frac{7}{2}}$, and $-2\sqrt{3}$.

c) $\sqrt[3]{21}$ $3\sqrt[3]{2} = \sqrt[3]{27(2)}$ $= \sqrt[3]{54}$ $2.8 = \sqrt[3]{2.8^3}$ $= \sqrt[3]{21.952}$ $2\sqrt[3]{5} = \sqrt[3]{8(5)}$ $= \sqrt[3]{40}$

The numbers from least to greatest are $\sqrt[3]{21}$, 2.8, $2\sqrt[3]{5}$, and $3\sqrt[3]{2}$.

Section 5.1 Page 279 Question 7

$-2\sqrt{3} = -3.464...$ –4 $-3\sqrt{2} = -4.242...$ $-2\sqrt{\frac{7}{2}} = -3.741...$

The numbers from least to greatest are $-3\sqrt{2}$, –4, $-2\sqrt{\frac{7}{2}}$, and $-2\sqrt{3}$.

Section 5.1 Page 279 Question 8

a) $-\sqrt{5} + 9\sqrt{5} - 4\sqrt{5} = 4\sqrt{5}$

b) $1.4\sqrt{2} + 9\sqrt{2} - 7 = 10.4\sqrt{2} - 7$

c) $\sqrt[4]{11} - 1 - 5\sqrt[4]{11} + 15 = -4\sqrt[4]{11} + 14$

d) $-\sqrt{6} + \frac{9}{2}\sqrt{10} - \frac{5}{2}\sqrt{10} + \frac{1}{3}\sqrt{6} = -\frac{2}{3}\sqrt{6} + 2\sqrt{10}$

Section 5.1 Page 279 Question 9

a)
$$\begin{aligned} 3\sqrt{75} - \sqrt{27} &= 3\sqrt{25(3)} - \sqrt{9(3)} \\ &= 15\sqrt{3} - 3\sqrt{3} \\ &= 12\sqrt{3} \end{aligned}$$

b) $2\sqrt{18}+9\sqrt{7}-\sqrt{63}=2\sqrt{9(2)}+9\sqrt{7}-\sqrt{9(7)}$
$=6\sqrt{2}+9\sqrt{7}-3\sqrt{7}$
$=6\sqrt{2}+6\sqrt{7}$

c) $-8\sqrt{45}+5.1-\sqrt{80}+17.4=-8\sqrt{9(5)}-\sqrt{16(5)}+22.5$
$=-24\sqrt{5}-4\sqrt{5}+22.5$
$=-28\sqrt{5}+22.5$

d) $\frac{2}{3}\sqrt[3]{81}+\frac{\sqrt[3]{375}}{4}-4\sqrt{99}+5\sqrt{11}=\frac{2}{3}\sqrt[3]{27(3)}+\frac{\sqrt[3]{125(3)}}{4}-4\sqrt{9(11)}+5\sqrt{11}$
$=2\sqrt[3]{3}+\frac{5\sqrt[3]{3}}{4}-12\sqrt{11}+5\sqrt{11}$
$=\frac{13\sqrt[3]{3}}{4}-7\sqrt{11}$

Section 5.1 Page 279 Question 10

a) $2\sqrt{a^3}+6\sqrt{a^3}=8\sqrt{a^3}$
$=8a\sqrt{a},\ a\geq 0$

b) $3\sqrt{2x}+3\sqrt{8x}-\sqrt{x}=3\sqrt{2x}+3\sqrt{4(2)(x)}-\sqrt{x}$
$=3\sqrt{2x}+6\sqrt{2x}-\sqrt{x}$
$=9\sqrt{2x}-\sqrt{x},\ \ x\geq 0$

c) $-4\sqrt[3]{625r}+\sqrt[3]{40r^4}=-4\sqrt[3]{125(5)(r)}+\sqrt[3]{8(5)(r^3)(r)}$
$=-20\sqrt[3]{5r}+2r\sqrt[3]{5r}$
$=(2r-20)\sqrt[3]{5r}$
$=2(r-10)\sqrt[3]{5r},\ r\in \mathrm{R}$

d) $\frac{w}{5}\sqrt[3]{-64}+\frac{\sqrt[3]{512w^3}}{5}-\frac{2}{5}\sqrt{50w}-4\sqrt{2w}=-\frac{4w}{5}+\frac{8w}{5}-\frac{2}{5}\sqrt{25(2)(w)}-4\sqrt{2w}$
$=\frac{4w}{5}-2\sqrt{2w}-4\sqrt{2w}$
$=\frac{4w}{5}-6\sqrt{2w},\ w\geq 0$

Section 5.1 Page 279 Question 11

$w = 6.3\sqrt{1013 - p}$

$w = 6.3\sqrt{1013 - 965}$

$w = 6.3\sqrt{48}$

$w = 25.2\sqrt{3}$

The exact wind speed of a hurricane if the air pressure is 965 mbar is $25.2\sqrt{3}$ m/s.

Section 5.1 Page 279 Question 12

$c^2 = 12^2 + 12^2$
$c^2 = 144 + 144$
$c^2 = 288$
$c = \sqrt{288}$
$c = 12\sqrt{2}$

The length of the hypotenuse is $12\sqrt{2}$ cm.

Section 5.1 Page 280 Question 13

Distance to Mars:

$d = \sqrt[3]{25n^2}$

$d = \sqrt[3]{25(704)^2}$

$d = \sqrt[3]{25[64(11)]^2}$

$d = 16\sqrt[3]{25(11)^2}$

$d = 16\sqrt[3]{3025}$

Distance to Mercury:

$d = \sqrt[3]{25n^2}$

$d = \sqrt[3]{25(88)^2}$

$d = \sqrt[3]{25[8(11)]^2}$

$d = 4\sqrt[3]{25(11)^2}$

$d = 4\sqrt[3]{3025}$

The difference between the distances of Mars and Mercury to the Sun is $16\sqrt[3]{3025} - 4\sqrt[3]{3025}$, or $12\sqrt[3]{3025}$ million kilometres.

Section 5.1 Page 280 Question 14

$s = \sqrt{10d}$

$s = \sqrt{10(12)}$

$s = \sqrt{120}$

$s = 2\sqrt{30}$

The speed of a tsunami with depth 12 m is $2\sqrt{30}$ m/s, or 11 m/s to the nearest metre per second.

Section 5.1 Page 280 Question 15

a) Since the given area of the circle (πr^2) is 38π m^2, the radius is $\sqrt{38}$ m. So, the diagonal of the square, which is also the diameter of the circle is $2\sqrt{38}$ m.

b) First, find the side length of the square.

$$s^2 + s^2 = \left(2\sqrt{38}\right)^2$$
$$2s^2 = 152$$
$$s^2 = 76$$
$$s = \sqrt{76}$$
$$s = 2\sqrt{19}$$

The perimeter of the square is $4\left(2\sqrt{19}\right)$, or $8\sqrt{19}$ m.

Section 5.1 Page 280 Question 16

First, find the half-perimeter for a triangle with side lengths 8 mm, 10 mm, and 12 mm.
0.5(8 + 10 + 12) = 15
Find the area of the triangle using Heron's formula.

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$
$$A = \sqrt{15(15-8)(15-10)(15-12)}$$
$$A = \sqrt{15(7)(5)(3)}$$
$$A = \sqrt{1575}$$
$$A = 15\sqrt{7}$$

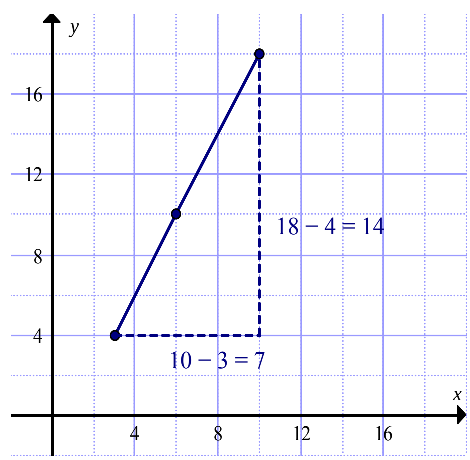
The area of the triangle is $\sqrt{1575}$ mm^2 or $15\sqrt{7}$ mm^2.

Section 5.1 Page 280 Question 17

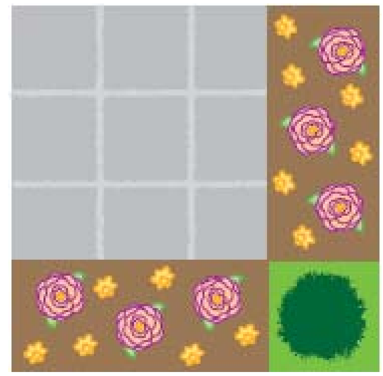
Using a diagram, the points lie on the same straight line.
Use the Pythagorean theorem to find the distance between the starting and ending points.

$$c^2 = 7^2 + 14^2$$
$$c^2 = 49 + 196$$
$$c^2 = 245$$
$$c = \sqrt{245}$$
$$c = 7\sqrt{5}$$

The ant travels $7\sqrt{5}$ units.



Section 5.1 Page 280 Question 18



Since the area of the entire square backyard is 98 m^2, the side length is $\sqrt{98}$ m.
Since the area of the green square is 8 m^2, the side length is $\sqrt{8}$ m.
Find the perimeter of one of the rectangular flowerbeds.

$$P = 2\sqrt{8} + 2\left(\sqrt{98} - \sqrt{8}\right)$$
$$P = 2\sqrt{98}$$
$$P = 14\sqrt{2}$$

The perimeter is $14\sqrt{2}$ m.

Section 5.1 Page 280 Question 19

Brady is correct. Kristen's final radical, $5y\sqrt{4y^3}$, is not in simplest form.
Express the radicand as a product of prime factors and combine identical pairs.

$$\begin{aligned} 5y\sqrt{4y^3} &= 5y\sqrt{2(2)(y)(y)(y)} \\ &= 10y^2\sqrt{y} \end{aligned}$$

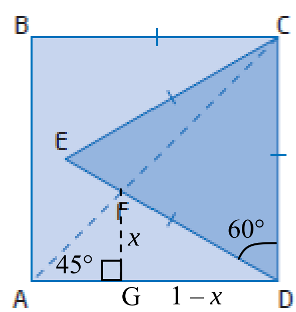
Section 5.1 Page 280 Question 20

$$\begin{aligned} 2\sqrt{216} &= 2\sqrt{36(6)} \\ &= 12\sqrt{6} \end{aligned} \qquad \begin{aligned} 3\sqrt{96} &= 3\sqrt{16(6)} \\ &= 12\sqrt{6} \end{aligned} \qquad 4\sqrt{58} \qquad \begin{aligned} 6\sqrt{24} &= 6\sqrt{4(6)} \\ &= 12\sqrt{6} \end{aligned}$$

The expression $4\sqrt{58}$ is not equivalent to $12\sqrt{6}$.

Section 5.1 Page 281 Question 21

From the given information,
- square ABCD has perimeter 4 m, so CD = 1 m
- △CDE is an equilateral triangle, so ∠CDE = 60°
- CA is a diagonal, so ∠DAC = 45°



In △ADF, draw the perpendicular from F to AD, meeting AD at point G. Let the height of FG be x.
Then, in △FDG, ∠FDG = 30° and DG = $1 - x$.

$$\tan 30° = \frac{x}{1-x}$$

$$\frac{1}{\sqrt{3}} = \frac{x}{1-x}$$

$$1 - x = \sqrt{3}x$$

$$1 = x(\sqrt{3}+1)$$

$$x = \frac{1}{\sqrt{3}+1}$$

$$x = \frac{\sqrt{3}-1}{2}$$

Next, in △AFG, FG = GA = x.

$AF^2 = x^2 + x^2$

$$AF^2 = \left(\frac{\sqrt{3}-1}{2}\right)^2 + \left(\frac{\sqrt{3}-1}{2}\right)^2$$

$$= 2\left(\frac{4-2\sqrt{3}}{4}\right)$$

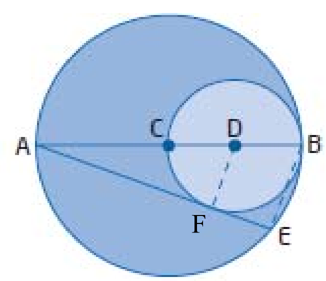
$$= 2 - \sqrt{3}$$

$$AF = \sqrt{2-\sqrt{3}}$$

The exact length of AF is $\sqrt{2-\sqrt{3}}$ m.

Section 5.1 Page 281 Question 22

Since AB = 18 cm, AC = 9 cm and CD = 4.5 cm.
Consider right △ADF, where DF = 4.5 cm and AD = 13.5 cm.
Use the Pythagorean theorem.



$(AF)^2 = (AD)^2 - (DF)^2$

$(AF)^2 = 13.5^2 - 4.5^2$

$(AF)^2 = 162$

$AF = \sqrt{162}$

$AF = 9\sqrt{2}$

Consider similar triangles △ADF and △ABE.

$$\frac{AE}{AF} = \frac{AB}{AD}$$

$$\frac{AE}{9\sqrt{2}} = \frac{18}{13.5}$$

$$AE = \frac{18}{13.5}(9\sqrt{2})$$

$$AE = 12\sqrt{2}$$

The exact length of AE is $12\sqrt{2}$ cm.

Section 5.1 Page 281 Question 23

Given $t_1 = \sqrt{27}$ and $t_4 = 9\sqrt{3}$.

Use the formula for the general term of an arithmetic sequence with $t_1 = \sqrt{27}$, $t_4 = 9\sqrt{3}$, and $n = 4$.

$$t_n = t_1 + (n - 1)d$$

$$9\sqrt{3} = \sqrt{27} + (4-1)d$$

$$9\sqrt{3} - \sqrt{27} = 3d$$

$$9\sqrt{3} - 3\sqrt{3} = 3d$$

$$6\sqrt{3} = 3d$$

$$d = 2\sqrt{3}$$

Find the two missing terms, t_2 and t_3.

$t_2 = t_1 + d$	$t_3 = t_2 + d$
$t_2 = \sqrt{27} + 2\sqrt{3}$	$t_3 = 5\sqrt{3} + 2\sqrt{3}$
$t_2 = 3\sqrt{3} + 2\sqrt{3}$	$t_3 = 7\sqrt{3}$
$t_2 = 5\sqrt{3}$	

The common difference is $2\sqrt{3}$ and the missing terms are $5\sqrt{3}$ and $7\sqrt{3}$.

Section 5.1 Page 281 Question 24

Examples:

a) To find the greatest sum of two of the radicals, simplify the two positive radicals and then add.

$$2\sqrt{75} + 108^{\frac{1}{2}} = 10\sqrt{3} + 6\sqrt{3}$$
$$= 16\sqrt{3}$$

b) To find the greatest difference of two of the radicals, simplify the greatest positive radical and the least negative radical and then subtract.

$$2\sqrt{75} - (-3\sqrt{12}) = 10\sqrt{3} - (-6\sqrt{3})$$
$$= 16\sqrt{3}$$

Section 5.1 Page 281 Question 25

Examples:

a) Let $x = 2$ and $x = -2$.

$(-x)^2 = x^2$ $\qquad$ $(-x)^2 = x^2$

$(-2)^2 = 2^2$ $\qquad$ $(-(-2))^2 = (-2)^2$

$4 = 4$ $\qquad$ $4 = 4$

b) Let $x = 2$ and $x = -2$.

$\sqrt{x^2} \neq \pm x$ $\qquad$ $\sqrt{x^2} \neq \pm x$

$\sqrt{2^2} \neq \pm 2$ $\qquad$ $\sqrt{(-2)^2} \neq \pm(-2)$

$+2 \neq \pm 2$ $\qquad$ $+2 \neq \pm 2$

Section 5.2 Multiplying and Dividing Radical Expressions

Section 5.2 Page 289 Question 1

a) $$\begin{aligned} 2\sqrt{5}\left(7\sqrt{3}\right) &= 2(7)\sqrt{5(3)} \\ &= 14\sqrt{15} \end{aligned}$$

b) $$\begin{aligned} -\sqrt{32}\left(7\sqrt{2}\right) &= -1(7)\sqrt{32(2)} \\ &= -7\sqrt{64} \\ &= -56 \end{aligned}$$

c) $$\begin{aligned} 2\sqrt[4]{48}\left(\sqrt[4]{5}\right) &= 2\sqrt[4]{48(5)} \\ &= 2\sqrt[4]{240} \\ &= 4\sqrt[4]{15} \end{aligned}$$

d) $$\begin{aligned} 4\sqrt{19x}\left(\sqrt{2x^2}\right) &= 4x\sqrt{19x(2)} \\ &= 4x\sqrt{38x} \end{aligned}$$

e) $$\begin{aligned} \sqrt[3]{54y^7}\left(\sqrt[3]{6y^4}\right) &= 3y^2\sqrt[3]{2y}\left(y\sqrt[3]{6y}\right) \\ &= 3y^3\sqrt[3]{12y^2} \end{aligned}$$

f) $\sqrt{6t}\left(3t^2\sqrt{\frac{t}{4}}\right) = \frac{3t^2}{2}\sqrt{6t^2}$

$= \frac{3t^3}{2}\sqrt{6}$

Section 5.2 Page 289 Question 2

a) $\sqrt{11}(3-4\sqrt{7}) = \sqrt{11}(3) - \sqrt{11}\left(4\sqrt{7}\right)$

$= 3\sqrt{11} - 4\sqrt{77}$

b) $-\sqrt{2}\left(14\sqrt{5}+3\sqrt{6}-\sqrt{13}\right)) = -\sqrt{2}\left(14\sqrt{5}\right) - \sqrt{2}\left(3\sqrt{6}\right) - \sqrt{2}\left(-\sqrt{13}\right)$

$= -14\sqrt{10} - 3\sqrt{12} + \sqrt{26}$

$= -14\sqrt{10} - 6\sqrt{3} + \sqrt{26}$

c) $\sqrt{y}\left(2\sqrt{y}+1\right) = \sqrt{y}\left(2\sqrt{y}\right) + \sqrt{y}\left(1\right)$

$= 2y + \sqrt{y}$

d) $z\sqrt{3}\left(z\sqrt{12} - 5z + 2\right) = z\sqrt{3}\left(z\sqrt{12}\right) - z\sqrt{3}\left(5z\right) + z\sqrt{3}\left(2\right)$

$= z^2\sqrt{36} - 5z^2\sqrt{3} + 2z\sqrt{3}$

$= 6z^2 - 5z^2\sqrt{3} + 2z\sqrt{3}$

Section 5.2 Page 289 Question 3

a) $-3\left(\sqrt{2}-4\right) + 9\sqrt{2} = -3\sqrt{2} + 12 + 9\sqrt{2}$

$= 6\sqrt{2} + 12$

b) $7\left(-1-2\sqrt{6}\right) + 5\sqrt{6} + 8 = -7 - 14\sqrt{6} + 5\sqrt{6} + 8$

$= 1 - 9\sqrt{6}$

c) $4\sqrt{5}\left(\sqrt{3j}+8\right) - 3\sqrt{15j} + \sqrt{5} = 4\sqrt{15j} + 32\sqrt{5} - 3\sqrt{15j} + \sqrt{5}$

$= \sqrt{15j} + 33\sqrt{5},\ j \geq 0$

d) $3 - \sqrt[3]{4k}\left(12 + 2\sqrt[3]{8}\right) = 3 - 12\sqrt[3]{4k} - 4\sqrt[3]{4k}$

$= 3 - 16\sqrt[3]{4k}$

Section 5.2 Page 290 Question 4

a) $\left(8\sqrt{7}+2\right)\left(\sqrt{2}-3\right)=8\sqrt{7}\left(\sqrt{2}\right)-8\sqrt{7}\left(3\right)+2\sqrt{2}-6$
$=8\sqrt{14}-24\sqrt{7}+2\sqrt{2}-6$

b) $\left(4-9\sqrt{5}\right)\left(4+9\sqrt{5}\right)=4(4)+4\left(9\sqrt{5}\right)-9\sqrt{5}\left(4\right)-9\sqrt{5}\left(9\sqrt{5}\right)$
$=16+36\sqrt{5}-36\sqrt{5}-81(5)$
$=-389$

c) $\left(\sqrt{3}+2\sqrt{15}\right)\left(\sqrt{3}-\sqrt{15}\right)=\sqrt{3}\left(\sqrt{3}\right)-\sqrt{3}\left(\sqrt{15}\right)+2\sqrt{15}\left(\sqrt{3}\right)-2\sqrt{15}\left(\sqrt{15}\right)$
$=3-\sqrt{45}+2\sqrt{45}-30$
$=-27+\sqrt{45}$
$=-27+3\sqrt{5}$

d) $\left(6\sqrt[3]{2}-4\sqrt{13}\right)^2=\left(6\sqrt[3]{2}-4\sqrt{13}\right)\left(6\sqrt[3]{2}-4\sqrt{13}\right)$
$=6\sqrt[3]{2}\left(6\sqrt[3]{2}\right)-6\sqrt[3]{2}\left(4\sqrt{13}\right)-4\sqrt{13}\left(6\sqrt[3]{2}\right)+4\sqrt{13}\left(4\sqrt{13}\right)$
$=36\sqrt[3]{4}-24\sqrt[3]{2}\left(\sqrt{13}\right)-24\sqrt[3]{2}\left(\sqrt{13}\right)+208$
$=36\sqrt[3]{4}-48\sqrt[3]{2}\left(\sqrt{13}\right)+208$

Section 5.2 Page 290 Question 5

a) $\left(15\sqrt{c}+2\right)\left(\sqrt{2c}-6\right)=15\sqrt{c}\left(\sqrt{2c}\right)-15\sqrt{c}\left(6\right)+2\sqrt{2c}-2(6)$
$=15c\sqrt{2}-90\sqrt{c}+2\sqrt{2c}-12, c\geq 0$

b) $\left(1-10\sqrt{8x^3}\right)\left(2+7\sqrt{5x}\right)=1(2)+1\left(7\sqrt{5x}\right)-10\sqrt{8x^3}\left(2\right)-10\sqrt{8x^3}\left(7\sqrt{5x}\right)$
$=2+7\sqrt{5x}-20\sqrt{8x^3}-70\sqrt{40x^4}$
$=2+7\sqrt{5x}-40x\sqrt{2x}-140x^2\sqrt{10}, x\geq 0$

c) $\left(9\sqrt{2m}-4\sqrt{6m}\right)^2=\left(9\sqrt{2m}-4\sqrt{6m}\right)\left(9\sqrt{2m}-4\sqrt{6m}\right)$
$=9\sqrt{2m}\left(9\sqrt{2m}\right)-9\sqrt{2m}\left(4\sqrt{6m}\right)-4\sqrt{6m}\left(9\sqrt{2m}\right)+4\sqrt{6m}\left(4\sqrt{6m}\right)$
$=162m-36\sqrt{12m^2}-36\sqrt{12m^2}+96m$
$=258m-144m\sqrt{3}, m\geq 0$

d)

$$\left(10r-4\sqrt[3]{4r}\right)\left(2\sqrt[3]{6r^2}+3\sqrt[3]{12r}\right)=10r\left(2\sqrt[3]{6r^2}\right)+10r\left(3\sqrt[3]{12r}\right)-4\sqrt[3]{4r}\left(2\sqrt[3]{6r^2}\right)-4\sqrt[3]{4r}\left(3\sqrt[3]{12r}\right)$$
$$=20r\sqrt[3]{6r^2}+30r\sqrt[3]{12r}-8\sqrt[3]{24r^3}-12\sqrt[3]{48r^2}$$
$$=20r\sqrt[3]{6r^2}+30r\sqrt[3]{12r}-16r\sqrt[3]{3}-24\sqrt[3]{6r^2}$$

Section 5.2 Page 290 Question 6

a) $$\frac{\sqrt{80}}{\sqrt{10}}=\sqrt{\frac{80}{10}}$$
$$=\sqrt{8}$$
$$=2\sqrt{2}$$

b) $$\frac{-2\sqrt{12}}{4\sqrt{3}}=-\frac{1}{2}\sqrt{\frac{12}{3}}$$
$$=-\frac{1}{2}\sqrt{4}$$
$$=-1$$

c) $$\frac{3\sqrt{22}}{\sqrt{11}}=3\sqrt{\frac{22}{11}}$$
$$=3\sqrt{2}$$

d)

$$\frac{3\sqrt{135m^5}}{\sqrt{21m^3}}=3\sqrt{\frac{135m^5}{21m^3}}$$
$$=3\sqrt{\frac{45m^2}{7}}$$
$$=9m\frac{\sqrt{5}}{\sqrt{7}}\left(\frac{\sqrt{7}}{\sqrt{7}}\right)$$
$$=\frac{9m\sqrt{35}}{7}$$

Section 5.2 Page 290 Question 7

a)

$$\frac{9\sqrt{432p^5}-7\sqrt{27p^5}}{\sqrt{33p^4}}=\frac{108p^2\sqrt{3p}-21p^2\sqrt{3p}}{p^2\sqrt{33}}$$
$$=\frac{87\sqrt{3p}}{\sqrt{33}}\left(\frac{\sqrt{33}}{\sqrt{33}}\right)$$
$$=\frac{261\sqrt{11p}}{33}$$
$$=\frac{87\sqrt{11p}}{11}$$

b)

$$\frac{6\sqrt[3]{4v^7}}{\sqrt[3]{14v}}=6\sqrt[3]{\frac{4v^7}{14v}}$$
$$=6v^2\sqrt[3]{\frac{2}{7}}$$
$$=\frac{6v^2\sqrt[3]{2}}{\sqrt[3]{7}}\left(\frac{\left(\sqrt[3]{7}\right)^2}{\left(\sqrt[3]{7}\right)^2}\right)$$
$$=\frac{6v^2\sqrt[3]{98}}{7}$$

Section 5.2 Page 290 Question 8

a) $$\frac{20}{\sqrt{10}}=\frac{20}{\sqrt{10}}\left(\frac{\sqrt{10}}{\sqrt{10}}\right)$$
$$=\frac{20\sqrt{10}}{10}$$
$$=2\sqrt{10}$$

b) $$\frac{-\sqrt{21}}{\sqrt{7m}}=\frac{-\sqrt{21}}{\sqrt{7m}}\left(\frac{\sqrt{7m}}{\sqrt{7m}}\right)$$
$$=\frac{-\sqrt{147m}}{7m}$$
$$=\frac{-7\sqrt{3m}}{7m}$$
$$=\frac{-\sqrt{3m}}{m}$$

c)

$$-\frac{2}{3}\sqrt{\frac{5}{12u}} = -\frac{2}{3}\left(\frac{\sqrt{5}}{\sqrt{12u}}\right)\left(\frac{\sqrt{12u}}{\sqrt{12u}}\right)$$
$$= -\frac{2}{3}\left(\frac{\sqrt{60u}}{12u}\right)$$
$$= -\frac{2}{3}\left(\frac{2\sqrt{15u}}{12u}\right)$$
$$= -\frac{\sqrt{15u}}{9u}$$

d) $20\sqrt[3]{\frac{6t}{5}} = 20\frac{\sqrt[3]{6t}}{\sqrt[3]{5}}\left(\frac{\left(\sqrt[3]{5}\right)^2}{\left(\sqrt[3]{5}\right)^2}\right)$

$$= \frac{20\sqrt[3]{150t}}{5}$$
$$= 4\sqrt[3]{150t}$$

Section 5.2 Page 290 Question 9

a) The conjugate for $2\sqrt{3}+1$ is $2\sqrt{3}-1$.

$$\left(2\sqrt{3}+1\right)\left(2\sqrt{3}-1\right) = \left(2\sqrt{3}\right)^2 - 1^2$$
$$= 12 - 1$$
$$= 11$$

b) The conjugate for $7-\sqrt{11}$ is $7+\sqrt{11}$.

$$\left(7-\sqrt{11}\right)\left(7+\sqrt{11}\right) = 7^2 - \left(\sqrt{11}\right)^2$$
$$= 49 - 11$$
$$= 38$$

c) The conjugate for $8\sqrt{z}-3\sqrt{7}$ is $8\sqrt{z}+3\sqrt{7}$.

$$\left(8\sqrt{z}-3\sqrt{7}\right)\left(8\sqrt{z}+3\sqrt{7}\right) = \left(8\sqrt{z}\right)^2 - \left(3\sqrt{7}\right)^2$$
$$= 64z - 63$$

d) The conjugate for $19\sqrt{h}+4\sqrt{2h}$ is $19\sqrt{h}-4\sqrt{2h}$.

$$\left(19\sqrt{h}+4\sqrt{2h}\right)\left(19\sqrt{h}+4\sqrt{2h}\right)=\left(19\sqrt{h}\right)^2-\left(4\sqrt{2h}\right)^2$$
$$=361h-32h$$
$$=329h$$

Section 5.2 Page 290 Question 10

a) $$\frac{5}{2-\sqrt{3}}=\frac{5}{2-\sqrt{3}}\left(\frac{2+\sqrt{3}}{2+\sqrt{3}}\right)$$
$$=\frac{5\left(2+\sqrt{3}\right)}{2^2-\left(\sqrt{3}\right)^2}$$
$$=\frac{5\left(2+\sqrt{3}\right)}{4-3}$$
$$=10+5\sqrt{3}$$

b)
$$\frac{7\sqrt{2}}{\sqrt{6}+8}=\frac{7\sqrt{2}}{\sqrt{6}+8}\left(\frac{\sqrt{6}-8}{\sqrt{6}-8}\right)$$
$$=\frac{7\sqrt{2}\left(\sqrt{6}-8\right)}{\left(\sqrt{6}\right)^2-8^2}$$
$$=\frac{7\sqrt{2}\left(\sqrt{6}-8\right)}{6-64}$$
$$=\frac{7\sqrt{12}-56\sqrt{2}}{-58}$$
$$=\frac{14\sqrt{3}-56\sqrt{2}}{-58}$$
$$=\frac{-7\sqrt{3}+28\sqrt{2}}{29}$$

c)

$$\frac{-\sqrt{7}}{\sqrt{5}-2\sqrt{2}} = \frac{-\sqrt{7}}{\sqrt{5}-2\sqrt{2}}\left(\frac{\sqrt{5}+2\sqrt{2}}{\sqrt{5}+2\sqrt{2}}\right)$$
$$= \frac{-\sqrt{7}\left(\sqrt{5}+2\sqrt{2}\right)}{\left(\sqrt{5}\right)^2-\left(2\sqrt{2}\right)^2}$$
$$= \frac{-\sqrt{7}\left(\sqrt{5}+2\sqrt{2}\right)}{5-8}$$
$$= \frac{\sqrt{35}+2\sqrt{14}}{3}$$

d)

$$\frac{\sqrt{3}+\sqrt{13}}{\sqrt{3}-\sqrt{13}} = \frac{\sqrt{3}+\sqrt{13}}{\sqrt{3}-\sqrt{13}}\left(\frac{\sqrt{3}+\sqrt{13}}{\sqrt{3}+\sqrt{13}}\right)$$
$$= \frac{\left(\sqrt{3}\right)^2+2\sqrt{3}\left(\sqrt{13}\right)+\left(\sqrt{13}\right)^2}{\left(\sqrt{3}\right)^2-\left(\sqrt{13}\right)^2}$$
$$= \frac{3+2\sqrt{39}+13}{3-13}$$
$$= \frac{16+2\sqrt{39}}{-10}$$
$$= \frac{-8-\sqrt{39}}{5}$$

Section 5.2 Page 290 Question 11

a) $$\frac{4r}{\sqrt{6}r+9} = \frac{4r}{\sqrt{6}r+9}\left(\frac{\sqrt{6}r-9}{\sqrt{6}r-9}\right)$$
$$= \frac{4r\left(\sqrt{6}r-9\right)}{\left(\sqrt{6}r\right)^2-9^2}$$
$$= \frac{4r^2\sqrt{6}-36r}{6r^2-81},\ r \neq \frac{\pm 3\sqrt{6}}{2}$$

b) $\frac{18\sqrt{3n}}{\sqrt{24n}} = \frac{18\sqrt{3n}}{\sqrt{24n}}\left(\frac{\sqrt{24n}}{\sqrt{24n}}\right)$

$$= \frac{18\sqrt{72n^2}}{24n}$$

$$= \frac{108n\sqrt{2}}{24n}$$

$$= \frac{9\sqrt{2}}{2},\ n > 0$$

c)

$$\frac{8}{4-\sqrt{6t}} = \frac{8}{4-\sqrt{6t}}\left(\frac{4+\sqrt{6t}}{4+\sqrt{6t}}\right)$$

$$= \frac{8\left(4+\sqrt{6t}\right)}{4^2-\left(\sqrt{6t}\right)^2}$$

$$= \frac{32+8\sqrt{6t}}{16-6t}$$

$$= \frac{16+4\sqrt{6t}}{8-3t},\ t \geq 0, t \neq \frac{8}{3}$$

d)

$$\frac{5\sqrt{3y}}{\sqrt{10}+2} = \frac{5\sqrt{3y}}{\sqrt{10}+2}\left(\frac{\sqrt{10}-2}{\sqrt{10}-2}\right)$$

$$= \frac{5\sqrt{3y}\left(\sqrt{10}-2\right)}{\left(\sqrt{10}\right)^2-2^2}$$

$$= \frac{5\sqrt{30y}-10\sqrt{3y}}{10-4}$$

$$= \frac{5\sqrt{30y}-10\sqrt{3y}}{6},\ y \geq 0$$

Section 5.2 Page 290 Question 12

$$\left(c+c\sqrt{c}\right)\left(c+7\sqrt{3c}\right) = c(c)+c\left(7\sqrt{3c}\right)+c\sqrt{c}(c)+c\sqrt{c}\left(7\sqrt{3c}\right)$$

$$= c^2+7c\sqrt{3c}+c^2\sqrt{c}+7c^2\sqrt{3}$$

Section 5.2 Page 290 Question 13

a) When applying the distributive property in line 2, Malcolm distributed the 4 to both the whole number and the root of the second term in the numerator, $2\sqrt{2}$. The correct numerator is $12 + 8\sqrt{2}$.

$$\begin{aligned}\frac{4}{3-2\sqrt{2}} &= \frac{4}{3-2\sqrt{2}}\left(\frac{3+2\sqrt{2}}{3+2\sqrt{2}}\right)\\ &= \frac{12+8\sqrt{2}}{9-8}\\ &= 12+8\sqrt{2}\end{aligned}$$

b) Check: $\frac{4}{3-2\sqrt{2}}$ = 23.313… and $12+8\sqrt{2}$ = 23.313….

Section 5.2 Page 291 Question 14

$$\begin{aligned}\frac{2}{\sqrt{5}-1} &= \frac{2}{\sqrt{5}-1}\left(\frac{\sqrt{5}+1}{\sqrt{5}+1}\right)\\ &= \frac{2\left(\sqrt{5}+1\right)}{\left(\sqrt{5}\right)^2-1^2}\\ &= \frac{2\left(\sqrt{5}+1\right)}{5-1}\\ &= \frac{2\left(\sqrt{5}+1\right)}{4}\\ &= \frac{\sqrt{5}+1}{2}\end{aligned}$$

Section 5.2 Page 291 Question 15

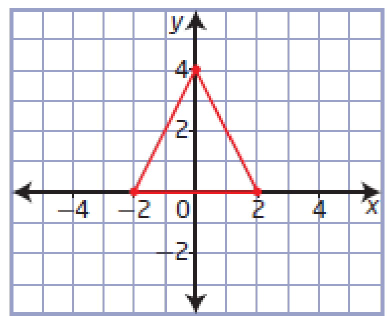
a) $$\begin{aligned}T &= 2\pi\sqrt{\frac{L}{10}}\\ &= 2\pi\frac{\sqrt{L}}{\sqrt{10}}\left(\frac{\sqrt{10}}{\sqrt{10}}\right)\\ &= \frac{2\pi\sqrt{10L}}{10}\\ &= \frac{\pi\sqrt{10L}}{5}\end{aligned}$$

b) Find the period for $L = 27$.

$$T = \frac{\pi\sqrt{10L}}{5}$$
$$= \frac{\pi\sqrt{10(27)}}{5}$$
$$= \frac{3\pi\sqrt{30}}{5}$$

The time to complete 3 cycles is $3\left(\frac{3\pi\sqrt{30}}{5}\right)$, or $\frac{9\pi\sqrt{30}}{5}$ s.

Section 5.2 Page 291 Question 16



The triangular course is in the shape of an isosceles triangle.
The length of the base is 4 units.
Use the Pythagorean theorem to find the length of one of the equal sides.

$c^2 = 4^2 + 2^2$
$c^2 = 16 + 4$
$c^2 = 20$
$c = \sqrt{20}$
$c = 2\sqrt{5}$

Find the perimeter of the triangular course.

$P = 4 + 2\left(2\sqrt{5}\right)$
$P = 4 + 4\sqrt{5}$

Find the exact length of the track.

$$\sqrt{9245}\left(4+4\sqrt{5}\right) = 4\sqrt{9245} + 4\sqrt{46225}$$
$$= 4\left(43\sqrt{5}\right) + 4(215)$$
$$= 172\sqrt{5} + 860$$

The exact length of the track is $(172\sqrt{5} + 860)$ m.

Section 5.2 Page 291 Question 17

$$\left(\frac{1+\sqrt{5}}{2-\sqrt{3}}\right)\left(\frac{1-\sqrt{5}}{2-\sqrt{3}}\right)=\frac{1^2-\left(\sqrt{5}\right)^2}{2^2-2\left(\sqrt{3}\right)-\sqrt{3}(2)+\left(\sqrt{3}\right)^2}$$
$$=\frac{1-5}{4-4\sqrt{3}+3}$$
$$=\frac{-4}{7-4\sqrt{3}}\left(\frac{7+4\sqrt{3}}{7+4\sqrt{3}}\right)$$
$$=\frac{-4\left(7+4\sqrt{3}\right)}{7^2-\left(4\sqrt{3}\right)^2}$$
$$=-28-16\sqrt{3}$$

Section 5.2 Page 291 Question 18

a) $\sqrt[3]{192}=4\sqrt[3]{3}$

The edge length of the actual cube $4\sqrt[3]{3}$ mm.

b) $\sqrt[3]{\frac{192}{4}}=\sqrt[3]{48}$
$$=2\sqrt[3]{6}$$

c) $4\sqrt[3]{3}:2\sqrt[3]{6}$

$2\sqrt[3]{3}:\sqrt[3]{6}$

Section 5.2 Page 291 Question 19

a) Lev forgot to reverse the direction of the inequality sign when he divided both sides by –5.

The corrected calculation follows:

$3-5x>0$

$-5x>-3$

$x<\frac{3}{5}$

b) Variables involved in radical expressions sometimes have restrictions on their values to ensure a non-negative value. This is the case for radicands with an index that is an even number.

c) Example: $\frac{2x\sqrt[3]{3-5x}}{\sqrt[3]{14}}$

The expression does not have a variable in the denominator and radicands with an odd number index may be any real number.

Section 5.2 Page 292 Question 20

Olivia made an error in line 3. She incorrectly evaluated $\sqrt{25}$ as ±5 instead of 5.
The corrected solution follows:

$$\begin{aligned}\frac{2c-c\sqrt{25}}{\sqrt{3}} &= \frac{\left(2c-c\sqrt{25}\right)}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right)\\ &= \frac{\sqrt{3}\left(2c-c\sqrt{25}\right)}{3}\\ &= \frac{\sqrt{3}\left(2c-5c\right)}{3}\\ &= \frac{\sqrt{3}\left(-3c\right)}{3}\\ &= -c\sqrt{3}\end{aligned}$$

Section 5.2 Page 292 Question 21

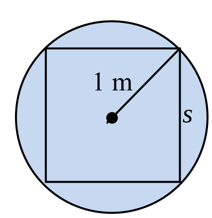
For the right triangular prism, $h = 5\sqrt{7}$, $b = 3\sqrt{2}$, and $l = 7\sqrt{14}$.

$$V = \frac{1}{2}bhl$$
$$V = \frac{1}{2}\left(3\sqrt{2}\right)\left(5\sqrt{7}\right)\left(7\sqrt{14}\right)$$
$$V = \frac{105}{2}\sqrt{2(7)(14)}$$
$$V = \frac{105}{2}(14)$$
$$V = 735$$

The volume of the right triangular prism is 735 cm^3.

Section 5.2 Page 292 Question 22

First, determine the side length of the cube inscribed in a sphere of radius 1 m.



$$\begin{aligned}s^2 + s^2 &= 2^2\\ 2s^2 &= 4\\ s^2 &= 2\\ s &= \sqrt{2}\end{aligned}$$

Determine the surface area of the cube.

$SA = 6s^2$

$SA = 6\left(\sqrt{2}\right)^2$

$SA = 12$

The surface area of the cube is 12 m^2.

Section 5.2 Page 292 Question 23

$$\begin{aligned}\text{Midpoint AB} &= \left(\frac{\sqrt{27}+3\sqrt{48}}{2}, \frac{-\sqrt{50}+2\sqrt{98}}{2}\right)\\ &= \left(\frac{3\sqrt{3}+12\sqrt{3}}{2}, \frac{-5\sqrt{2}+14\sqrt{2}}{2}\right)\\ &= \left(\frac{15\sqrt{3}}{2}, \frac{9\sqrt{2}}{2}\right)\end{aligned}$$

Section 5.2 Page 292 Question 24

$$\begin{aligned}\left(3\left(\sqrt{x}\right)^{-1}-5\right)^{-2} &= \left(\frac{3}{\sqrt{x}}-5\right)^{-2}\\ &= \left(\frac{3-5\sqrt{x}}{\sqrt{x}}\right)^{-2}\\ &= \left(\frac{\sqrt{x}}{3-5\sqrt{x}}\right)^{2}\\ &= \frac{x}{9+25x-30\sqrt{x}}\left(\frac{9+25x+30\sqrt{x}}{9+25x+30\sqrt{x}}\right)\\ &= \frac{9x+25x^2+30x\sqrt{x}}{81-450x-625x^2}\end{aligned}$$

Section 5.2 Page 292 Question 25

a) Use the quadratic formula with $a = 1$, $b = 6$, and $c = 3$.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-6\pm\sqrt{6^2-4(1)(3)}}{2(1)}$$
$$x=\frac{-6\pm\sqrt{24}}{2}$$
$$x=\frac{-6\pm2\sqrt{6}}{2}$$
$$x=-3\pm\sqrt{6}$$
The exact roots are $x=-3+\sqrt{6}$ and $x=-3-\sqrt{6}$.

b) $-3+\sqrt{6}+\left(-3-\sqrt{6}\right)=-6$

c) $\left(-3+\sqrt{6}\right)\left(-3-\sqrt{6}\right)=9-6$
$=3$

d) Example: The answer to part b) is equal to $-\frac{b}{a}$. The answer to part c) is equal to $\frac{c}{a}$.

Section 5.2 Page 292 Question 26

$$\frac{\sqrt[c]{a}}{\sqrt[n]{r}}=\frac{\sqrt[c]{a}}{\sqrt[n]{r}}\left(\frac{\sqrt[n-1]{r}}{\sqrt[n-1]{r}}\right)$$
$$=\frac{\sqrt[c]{a}\sqrt[n-1]{r}}{r}$$

Section 5.2 Page 292 Question 27

First, find the length of the hypotenuse of the triangular faces.
$$c^2=\left(5\sqrt{7}\right)^2+\left(3\sqrt{2}\right)^2$$
$$c^2=175+18$$
$$c^2=193$$
$$c=\sqrt{193}$$

$$SA = 5\sqrt{7}\left(3\sqrt{2}\right) + 5\sqrt{7}\left(7\sqrt{14}\right) + 3\sqrt{2}\left(7\sqrt{14}\right) + \sqrt{193}\left(7\sqrt{14}\right)$$
$$= 15\sqrt{14} + 35\sqrt{98} + 21\sqrt{28} + 7\sqrt{2702}$$
$$= 15\sqrt{14} + 245\sqrt{2} + 42\sqrt{7} + 7\sqrt{2702}$$

The exact surface area of the right triangular prism is
$(15\sqrt{14} + 245\sqrt{2} + 42\sqrt{7} + 7\sqrt{2702})$ cm^2.

Section 5.2 Page 292 Question 28

Example: You can multiply and divide polynomial expressions with the same variables, and you can multiply and divide radicals with the same index.

Section 5.2 Page 292 Question 29

To rationalize a square-root binomial denominator, multiply the numerator and denominator by the conjugate of the denominator. The product of a pair of conjugates is a difference of squares.

$$\frac{5}{2-\sqrt{3}} = \frac{5}{2-\sqrt{3}}\left(\frac{2+\sqrt{3}}{2+\sqrt{3}}\right)$$
$$= \frac{5\left(2+\sqrt{3}\right)}{2^2-\left(\sqrt{3}\right)^2}$$
$$= \frac{5\left(2+\sqrt{3}\right)}{4-3}$$
$$= 10 + 5\sqrt{3}$$

Section 5.2 Page 292 Question 30

a) Substitute $t = 0$.
$h(t) = –5t^2 + 10t + 3$
$h(0) = –5(0)^2 + 10(0) + 3$
$h(0) = 3$
The snowboarder's height above the landing area at the beginning of the jump is 3 m.

b) $h(t) = –5t^2 + 10t + 3$
$h(t) = –5(t^2 – 2t) + 3$
$h(t) = –5(t^2 – 2t + 1 – 1) + 3$
$h(t) = –5(t – 1)^2 + 5 + 3$
$h(t) = –5(t – 1)^2 + 8$

Isolate t.
$h(t) = –5(t – 1)^2 + 8$
$h(t) – 8 = –5(t – 1)^2$

$$\frac{h(t)-8}{-5}=(t-1)^2$$
$$\sqrt{\frac{8-h(t)}{5}}=t-1$$
$$t=1+\sqrt{\frac{8-h(t)}{5}}$$

c) First, determine the time to complete the jump by using the result from part b) with $h(t) = 0$.

$$t=1+\sqrt{\frac{8-h(t)}{5}}$$
$$t=1+\sqrt{\frac{8-0}{5}}$$
$$t=1+\frac{\sqrt{8}}{\sqrt{5}}$$
$$t=\frac{\sqrt{5}+\sqrt{8}}{\sqrt{5}}\left(\frac{\sqrt{5}}{\sqrt{5}}\right)$$
$$t=\frac{5+\sqrt{40}}{5}$$
$$t=\frac{5+2\sqrt{10}}{5}$$

So, the time halfway through the jump is $\frac{5+2\sqrt{10}}{10}$ s.

Substitute $t=\frac{5+2\sqrt{10}}{10}$ into $h(t) = -5t^2 + 10t + 3$.

$$h\left(\frac{5+2\sqrt{10}}{10}\right) = -5\left(\frac{5+2\sqrt{10}}{10}\right)^2 + 10\left(\frac{5+2\sqrt{10}}{10}\right) + 3$$

$$= -5\left(\frac{25+20\sqrt{10}+40}{100}\right) + 5 + 2\sqrt{10} + 3$$

$$= \frac{-65-20\sqrt{10}}{20} + 8 + 2\sqrt{10}$$

$$= \frac{-13-4\sqrt{10}}{4} + 8 + 2\sqrt{10}$$

$$= \frac{-13-4\sqrt{10}+32+8\sqrt{10}}{4}$$

$$= \frac{19+4\sqrt{10}}{4}$$

The exact height of the snowboarder halfway through the jump is $\frac{19+4\sqrt{10}}{4}$ m.

Section 5.2 Page 293 Question 31

Use the quadratic formula with $a = 3$, $b = 5$, and $c = 1$.

$$m = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$m = \frac{-5 \pm \sqrt{5^2-4(3)(1)}}{2(3)}$$

$$m = \frac{-5 \pm \sqrt{13}}{6}$$

The solutions $m = \frac{-5+\sqrt{13}}{6}$ and $m = \frac{-5-\sqrt{13}}{6}$ are correct.

Section 5.2 Page 293 Question 32

a) $\dfrac{\sqrt[3]{\dfrac{3V}{2\pi}}}{\sqrt[3]{\dfrac{V-1}{4\pi}}} = \sqrt[3]{\dfrac{3V}{2\pi}\left(\dfrac{4\pi}{V-1}\right)}$

$$= \frac{\sqrt[3]{6V}}{\sqrt[3]{V-1}} \frac{\left(\sqrt[3]{V-1}\right)^2}{\left(\sqrt[3]{V-1}\right)^2}$$

$$= \frac{\sqrt[3]{6V(V-1)^2}}{V-1}$$

b) For volumes greater than 1 the ratio is a real number.

Section 5.2 Page 293 Question 33

Step 1 $y = \sqrt{x}$

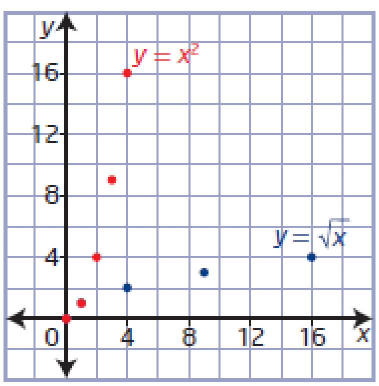
x	y
0	0
1	1
4	2
9	3
16	4

$y = x^2$

x	y
0	0
1	1
2	4
3	9
4	16

Step 2 Example: The values of *x* and *y* have been interchanged.

Step 3



Example: The restrictions on the radical function produce the right half of the parabola.

Section 5.3 Radical Equations

Section 5.3 Page 300 Question 1

a) $\left(\sqrt{3z}\right)^2 = 3z$

b) $\left(\sqrt{x-4}\right)^2 = x-4$

c) $\left(2\sqrt{x+7}\right)^2 = 4(x+7)$
$= 4x+28$

d) $\left(-4\sqrt{9-2y}\right)^2 = 16(9-2y)$
$= 144-32y$

Section 5.3 Page 300 Question 2

Isolate the radical expression. Square both sides of the equation.

$$\begin{aligned} \sqrt{x}+5 &= 11,\ x \ge 0 \\ \sqrt{x} &= 6 \\ \left(\sqrt{x}\right)^2 &= 6^2 \\ x &= 36 \end{aligned}$$

Section 5.3 Page 300 Question 3

a)
$$\begin{aligned} \sqrt{2x} &= 3,\ x \ge 0 \\ \left(\sqrt{2x}\right)^2 &= 3^2 \\ 2x &= 9 \\ x &= \frac{9}{2} \end{aligned}$$

Check for $x = \dfrac{9}{2}$.

Left Side	Right Side
$\sqrt{2x}$	3

$= \sqrt{2\left(\frac{9}{2}\right)}$

$= \sqrt{9}$

$= 3$

Left Side = Right Side

The solution is $x = 3$.

b) $\sqrt{-8x} = 4, x \le 0$

$\left(\sqrt{-8x}\right)^2 = 4^2$

$-8x = 16$

$x = -2$

Check for $x = -2$.

Left Side	Right Side
$\sqrt{-8x}$	4
$= \sqrt{-8(-2)}$	
$= \sqrt{16}$	
$= 4$	

Left Side = Right Side

The solution is $x = -2$.

c) $7 = \sqrt{5 - 2x}, x \le \frac{5}{2}$

$7^2 = \left(\sqrt{5 - 2x}\right)^2$

$49 = 5 - 2x$

$2x = -44$

$x = -22$

Check for $x = -22$.

Left Side	Right Side
7	$\sqrt{5 - 2x}$
	$= \sqrt{5 - 2(-22)}$
	$= \sqrt{49}$
	$= 7$

Left Side = Right Side

The solution is $x = -22$.

Section 5.3 Page 300 Question 4

a) $\sqrt{z}+8=13,\ z\ge 0$

$$\begin{aligned}\sqrt{z}&=5\\ \left(\sqrt{z}\right)^2&=5^2\\ z&=25\end{aligned}$$

Check for $z = 25$.

Left Side	Right Side
$\sqrt{z}+8$	13
$=\sqrt{25}+8$	
$=5+8$	
$=13$	

Left Side = Right Side

The solution is $z = 25$.

b) $2-\sqrt{y}=-4,\ y\ge 0$

$$\begin{aligned}-\sqrt{y}&=-6\\ \sqrt{y}&=6\\ \left(\sqrt{y}\right)^2&=6^2\\ y&=36\end{aligned}$$

Check for $y = 36$.

Left Side	Right Side
$2-\sqrt{y}$	–4
$=2-\sqrt{36}$	
$=2-6$	
$=-4$	

Left Side = Right Side

The solution is $y = 36$.

c) $\sqrt{3x}-8=-6,\ x\ge 0$

$$\begin{aligned}\sqrt{3x}&=2\\ \left(\sqrt{3x}\right)^2&=2^2\\ 3x&=4\\ x&=\frac{4}{3}\end{aligned}$$

Check for $x = \frac{4}{3}$.

Left Side   Right Side

$\sqrt{3x} - 8$ -6

$= \sqrt{3\left(\frac{4}{3}\right)} - 8$

$= \sqrt{4} - 8$

$= -6$

Left Side = Right Side

The solution is $x = \frac{4}{3}$.

d) $-5 = 2 - \sqrt{-6m},\ m \le 0$

$$\begin{aligned}
\sqrt{-6m} &= 7 \\
\left(\sqrt{-6m}\right)^2 &= 7^2 \\
-6m &= 49 \\
m &= -\frac{49}{6}
\end{aligned}$$

Check for $m = -\frac{49}{6}$.

Left Side   Right Side

-5 $2 - \sqrt{-6m}$

$= 2 - \sqrt{-6\left(-\frac{49}{6}\right)}$

$= 2 - \sqrt{49}$

$= -5$

Left Side = Right Side

The solution is $m = -\frac{49}{6}$.

Section 5.3 Page 300 Question 5

$$\begin{aligned}
k + 4 &= \sqrt{-2k},\ k \le 0 \\
(k+4)^2 &= \left(\sqrt{-2k}\right)^2 \\
k^2 + 8k + 16 &= -2k \\
k^2 + 10k + 16 &= 0 \\
(k+8)(k+2) &= 0
\end{aligned}$$

$k + 8 = 0$ or $k + 2 = 0$

$k = -8$ $k = -2$

Check for $k = -8$.

Left Side	Right Side
$k + 4$	$\sqrt{-2k}$
$= -8 + 4$	$= \sqrt{-2(-8)}$
$= -4$	$= \sqrt{16}$
	$= 4$

Left Side $\neq$ Right Side

$k = -8$ is an extraneous root.

Check for $k = -2$.

Left Side	Right Side
$k + 4$	$\sqrt{-2k}$
$= -2 + 4$	$= \sqrt{-2(-2)}$
$= 2$	$= \sqrt{4}$
	$= 2$

Left Side = Right Side

Section 5.3 Page 300 Question 6

a)

$$-3\sqrt{n-1} + 7 = -14,\ n \geq 1$$
$$-3\sqrt{n-1} = -21$$
$$\sqrt{n-1} = 7$$
$$\left(\sqrt{n-1}\right)^2 = 7^2$$
$$n - 1 = 49$$
$$n = 50$$

b) $-7 - 4\sqrt{2x-1} = 17,\ x \geq \frac{1}{2}$

$$-4\sqrt{2x-1} = 24$$
$$\sqrt{2x-1} = -6$$

There is no solution.

c) $12 = -3 + 5\sqrt{8-x},\ x \leq 8$

$$15 = 5\sqrt{8-x}$$
$$3 = \sqrt{8-x}$$
$$3^2 = \left(\sqrt{8-x}\right)^2$$
$$9 = 8 - x$$
$$x = -1$$

Section 5.3 Page 301 Question 7

a) $\sqrt{m^2-3}=5,\ m \le -\sqrt{3}$ or $m \ge \sqrt{3}$

$$\left(\sqrt{m^2-3}\right)^2 = 5^2$$
$$m^2-3=25$$
$$m^2=28$$
$$m=\pm\sqrt{28}$$
$$m=\pm 2\sqrt{7}$$

b) $\sqrt{x^2+12x}=8,\ x \le -12$ or $x \ge 0$

$$\left(\sqrt{x^2+12x}\right)^2 = 8^2$$
$$x^2+12x=64$$
$$x^2+12x-64=0$$
$$(x+16)(x-4)=0$$

$x + 16 = 0$ or $x - 4 = 0$

$x = -16$ $x = 4$

c)

$$\sqrt{\frac{q^2}{2}+11}=q-1$$
$$\left(\sqrt{\frac{q^2}{2}+11}\right)^2=(q-1)^2$$
$$\frac{q^2}{2}+11=q^2-2q+1$$
$$q^2+22=2q^2-4q+2$$
$$0=q^2-4q-20$$

Use the quadratic formula with $a = 1$, $b = -4$, and $c = -20$.

$$q = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$q = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-20)}}{2(1)}$$
$$q = \frac{4 \pm \sqrt{96}}{2}$$
$$q = \frac{4 \pm 4\sqrt{6}}{2}$$
$$q = 2 \pm 2\sqrt{6}$$

d)

$$2n + 2\sqrt{n^2 - 7} = 14,\ n \leq -\sqrt{7} \text{ or } x \geq \sqrt{7}$$
$$2\sqrt{n^2 - 7} = 14 - 2n$$
$$\sqrt{n^2 - 7} = 7 - n$$
$$\left(\sqrt{n^2 - 7}\right)^2 = (7 - n)^2$$
$$n^2 - 7 = 49 - 14n + n^2$$
$$14n = 56$$
$$n = \frac{56}{14}$$
$$n = 4$$

Section 5.3 Page 301 Question 8

a)

$$5 + \sqrt{3x - 5} = x,\ x \geq \frac{5}{3}$$
$$\sqrt{3x - 5} = x - 5$$
$$\left(\sqrt{3x - 5}\right)^2 = (x - 5)^2$$
$$3x - 5 = x^2 - 10x + 25$$
$$0 = x^2 - 13x + 30$$
$$0 = (x - 10)(x - 3)$$

$x - 10 = 0$ or $x - 3 = 0$

$x = 10$ $\quad$ $x = 3$

Since $x = 3$ is an extraneous root, the solution is $x = 10$.

b)

$$\sqrt{x^2+30x}=8,\ x\le -30 \text{ or } x\ge 0$$
$$\left(\sqrt{x^2+30x}\right)^2=8^2$$
$$x^2+30x=64$$
$$x^2+30x-64=0$$
$$(x+32)(x-2)=0$$

$x + 32 = 0$ or $x - 2 = 0$

$x = -32$ $\quad$ $x = 2$

c)

$$\sqrt{d+5}=d-1,\ d\ge -5$$
$$\left(\sqrt{d+5}\right)^2=(d-1)^2$$
$$d+5=d^2-2d+1$$
$$0=d^2-3d-4$$
$$0=(d-4)(d+1)$$

$d - 4 = 0$ or $d + 1 = 0$

$d = 4$ $\quad$ $d = -1$

Since $d = -1$ is an extraneous root, the solution is $d = 4$.

d)

$$\sqrt{\frac{j+1}{3}}+5j=3j-1,\ j\ge -1$$
$$\sqrt{\frac{j+1}{3}}=-2j-1$$
$$\left(\sqrt{\frac{j+1}{3}}\right)^2=(-2j-1)^2$$
$$\frac{j+1}{3}=4j^2+4j+1$$
$$j+1=12j^2+12j+3$$
$$0=12j^2+11j+2$$
$$0=(3j+2)(4j+1)$$

$3j + 2 = 0$ or $4j + 1 = 0$

$j=-\frac{2}{3}$ $\quad$ $j=-\frac{1}{4}$

Since $j=-\frac{1}{4}$ is an extraneous root, the solution is $j=-\frac{2}{3}$.

Section 5.3 Page 301 Question 9

a) $\sqrt{2k} = \sqrt{8},\ k \geq 0$

$$\left(\sqrt{2k}\right)^2 = \left(\sqrt{8}\right)^2$$
$$2k = 8$$
$$k = 4$$

b) $\sqrt{-3m} = \sqrt{-7m},\ m \leq 0$

$$\left(\sqrt{-3m}\right)^2 = \left(\sqrt{-7m}\right)^2$$
$$-3m = -7m$$
$$4m = 0$$
$$m = 0$$

c) $5\sqrt{\frac{j}{2}} = \sqrt{200},\ j \geq 0$

$$\left(5\sqrt{\frac{j}{2}}\right)^2 = \left(\sqrt{200}\right)^2$$
$$\frac{25j}{2} = 200$$
$$25j = 400$$
$$j = 16$$

d)

$$\begin{aligned}
5+\sqrt{n} &= \sqrt{3n},\ n \ge 0 \\
5 &= \sqrt{3n}-\sqrt{n} \\
5^2 &= \left(\sqrt{3n}-\sqrt{n}\right)^2 \\
25 &= 3n-2\sqrt{3n^2}+n \\
25 &= 4n-2n\sqrt{3} \\
25 &= n\left(4-2\sqrt{3}\right) \\
n &= \frac{25}{4-2\sqrt{3}} \\
n &= \frac{25}{4-2\sqrt{3}}\left(\frac{4+2\sqrt{3}}{4+2\sqrt{3}}\right) \\
n &= \frac{100+50\sqrt{3}}{4} \\
n &= \frac{50+25\sqrt{3}}{2}
\end{aligned}$$

Section 5.3 Page 301 Question 10

a) $\sqrt{z+5} = \sqrt{2z-1},\ z \ge \frac{1}{2}$

$$\begin{aligned}
\left(\sqrt{z+5}\right)^2 &= \left(\sqrt{2z-1}\right)^2 \\
z+5 &= 2z-1 \\
-z &= -6 \\
z &= 6
\end{aligned}$$

b) $\sqrt{6y-1} = \sqrt{-17+y^2},\ y \ge \sqrt{17}$

$$\begin{aligned}
\left(\sqrt{6y-1}\right)^2 &= \left(\sqrt{-17+y^2}\right)^2 \\
6y-1 &= -17+y^2 \\
0 &= y^2-6y-16 \\
0 &= (y-8)(y+2)
\end{aligned}$$

$y - 8 = 0$ or $y + 2 = 0$

$y = 8$ $\quad$ $y = -2$

Since $y = -2$ is an extraneous root, the solution is $y = 8$.

c)

$$\sqrt{5r-9}-3=\sqrt{r+4}-2,\ r\geq\frac{9}{5}$$
$$\sqrt{5r-9}-1=\sqrt{r+4}$$
$$\left(\sqrt{5r-9}-1\right)^2=\left(\sqrt{r+4}\right)^2$$
$$5r-9-2\left(\sqrt{5r-9}\right)+1=r+4$$
$$4r-12=2\left(\sqrt{5r-9}\right)$$
$$(2r-6)^2=\left(\sqrt{5r-9}\right)^2$$
$$4r^2-24r+36=5r-9$$
$$4r^2-29r+45=0$$
$$(4r-9)(r-5)=0$$

$4r-9=0$ or $r-5=0$

$r=\frac{9}{4}$ $\qquad$ $r=5$

Since $r=\frac{9}{4}$ is an extraneous root, the solution is $r = 5$.

d)

$$\sqrt{x+19}+\sqrt{x-2}=7,\ x\geq 2$$
$$\sqrt{x+19}-7=-\sqrt{x-2}$$
$$\left(\sqrt{x+19}-7\right)^2=\left(-\sqrt{x-2}\right)^2$$
$$x+19-14\left(\sqrt{x+19}\right)+49=x-2$$
$$-14\left(\sqrt{x+19}\right)=-70$$
$$\sqrt{x+19}=5$$
$$\left(\sqrt{x+19}\right)^2=5^2$$
$$x+19=25$$
$$x=6$$

Section 5.3 Page 301 Question 11

Isolating the radical in the equations $\sqrt{3y-1}-2=5$ and $4-\sqrt{m+6}=-9$ results in it being equated to a positive value, which has a solution.

The equation $\sqrt{x+8}+9=2$ will have an extraneous root. Isolating the radical results in it being equated to a negative value, which has no solution.

Section 5.3 Page 301 Question 12

Jerry forgot to note the restriction in the first step, and he made a mistake in the third line when he incorrectly squared the expression $x - 3$. The corrected solution is shown:

$$3+\sqrt{x+17}=x,\ x\geq -17$$
$$\sqrt{x+17}=x-3$$
$$\left(\sqrt{x+17}\right)^2=(x-3)^2$$
$$x+17=x^2-6x+9$$
$$0=x^2-7x-8$$
$$0=(x-8)(x+1)$$

$x - 8 = 0$ or $x + 1 = 0$

$x = 8$ $\quad$ $x = -1$

Since $x = -1$ is an extraneous root, the solution is $x = 8$.

Section 5.3 Page 301 Question 13

Substitute $v = 50$ and solve for l.

$$v=12.6\sqrt{l}+8$$
$$50=12.6\sqrt{l}+8$$
$$\frac{42}{12.6}=\sqrt{l}$$
$$\left(\frac{42}{12.6}\right)^2=\left(\sqrt{l}\right)^2$$
$$11.111...=l$$

The length of skid mark expected is 11.1 m, to the nearest tenth of a metre.

Section 5.3 Page 301 Question 14

a) Substitute $v = 40$.

$$B=1.33\sqrt{v+10.0}-3.49$$
$$B=1.33\sqrt{40+10.0}-3.49$$
$$B=1.33\sqrt{50.0}-3.49$$
$$B=5.914...$$

The wind scale is 6.

b) Substitute $B = 3$.

$$B = 1.33\sqrt{v + 10.0} - 3.49$$
$$3 = 1.33\sqrt{v + 10.0} - 3.49$$
$$6.49 = 1.33\sqrt{v + 10.0}$$
$$\frac{6.49}{1.33} = \sqrt{v + 10.0}$$
$$\left(\frac{6.49}{1.33}\right)^2 = \left(\sqrt{v + 10.0}\right)^2$$
$$\left(\frac{6.49}{1.33}\right)^2 = v + 10.0$$
$$\left(\frac{6.49}{1.33}\right)^2 - 10.0 = v$$
$$13.811... = v$$

The wind velocity is approximately 13.8 km/h.

Section 5.3 Page 302 Question 15

Substitute $t = 4$.

$$t = \frac{1}{5}\sqrt{\frac{m}{3}}$$
$$4 = \frac{1}{5}\sqrt{\frac{m}{3}}$$
$$20 = \sqrt{\frac{m}{3}}$$
$$20^2 = \left(\sqrt{\frac{m}{3}}\right)^2$$
$$400 = \frac{m}{3}$$
$$1200 = m$$

It can support a mass of 1200 kg.

Section 5.3 Page 302 Question 16

$$
\begin{aligned}
\sqrt{n}+2&=n,\ n\geq 0\\
\sqrt{n}&=n-2\\
\left(\sqrt{n}\right)^2&=(n-2)^2\\
n&=n^2-4n+4\\
0&=n^2-5n+4\\
0&=(n-4)(n-1)
\end{aligned}
$$

$n - 4 = 0$ or $n - 1 = 0$

$n = 4$ $\quad$ $n = 1$

Since $n = 1$ is an extraneous root, the solution is $n = 4$.

Section 5.3 Page 302 Question 17

a) $v=\sqrt{2h(9.8)},\ h\geq 0$

$v=\sqrt{19.6h}$

b) Substitute $v = 30$ and solve for h.

$$
\begin{aligned}
v&=\sqrt{19.6h}\\
30&=\sqrt{19.6h}\\
30^2&=\left(\sqrt{19.6h}\right)^2\\
900&=19.6h\\
\frac{900}{19.6}&=h\\
45.918...&=h
\end{aligned}
$$

The spray is expected to reach a height of approximately 45.9 m.

c) Substitute $v = 35$ and solve for h.

$$
\begin{aligned}
v&=\sqrt{19.6h}\\
35&=\sqrt{19.6h}\\
35^2&=\left(\sqrt{19.6h}\right)^2\\
900&=19.6h\\
\frac{1225}{19.6}&=h\\
62.5&=h
\end{aligned}
$$

The pump can reach a maximum height of 62.5 m, which meets the 60-m minimum requirement if only just.

Section 5.3 Page 302 Question 18

Substitute h = 200 and d = 1609 and solve for r.

$$d = \sqrt{2rh + h^2}$$
$$1609 = \sqrt{2r(200) + 200^2}$$
$$1609 = \sqrt{400r + 40\,000}$$
$$1609 = \sqrt{400(r + 100)}$$
$$1609 = 20\sqrt{r + 100}$$
$$\frac{1609}{20} = \sqrt{r + 100}$$
$$80.45^2 = \left(\sqrt{r + 100}\right)^2$$
$$6472.2025 = r + 100$$
$$6372.2025 = r$$

The radius of Earth is approximately 6372.2 km.

Section 5.3 Page 302 Question 19

$$\sqrt{3x} = \sqrt{ax} + 2$$
$$\sqrt{3x} - 2 = \sqrt{ax}$$
$$\left(\sqrt{3x} - 2\right)^2 = \left(\sqrt{ax}\right)^2$$
$$3x - 4\sqrt{3x} + 4 = ax$$
$$\frac{3x - 4\sqrt{3x} + 4}{x} = a$$

Section 5.3 Page 302 Question 20

a) Example: A radical equated to a negative value will result in no solution.

$$\sqrt{1 - x} + 2 = 0$$
$$\sqrt{1 - x} = -2$$
$$\left(\sqrt{1 - x}\right)^2 = (-2)^2$$
$$1 - x = 4$$
$$-3 = x$$

The result $x = -3$ is an extraneous solution and no solutions are valid.

b) Example: One extraneous root can occur when the squaring of both sides results in a quadratic expression.

$$x-2=\sqrt{x+10}$$
$$(x-2)^2=\left(\sqrt{x+10}\right)^2$$
$$x^2-4x+4=x+10$$
$$x^2-5x-6=0$$
$$(x-6)(x+1)=0$$

$x - 6 = 0$ or $x + 1 = 0$

$x = 6$ $\quad$ $x = -1$

Since $x = -1$ is an extraneous root, the solution is $x = 6$.

Section 5.3 Page 302 Question 21

$$t_m-t_E=0.5$$
$$\sqrt{\frac{h}{1.8}}-\sqrt{\frac{h}{4.9}}=0.5$$
$$\left(\sqrt{\frac{h}{1.8}}-\sqrt{\frac{h}{4.9}}\right)^2=0.5^2$$
$$\frac{h}{1.8}-2\sqrt{\frac{h}{1.8}}\left(\sqrt{\frac{h}{4.9}}\right)+\frac{h}{4.9}=0.25$$
$$\frac{6.7h}{8.82}-2\sqrt{\frac{h^2}{8.82}}=0.25$$
$$\frac{6.7h}{8.82}-2h\sqrt{\frac{1}{8.82}}=0.25$$
$$\frac{6.7h}{8.82}-2h\frac{\sqrt{8.82}}{8.82}=0.25$$
$$h\left(\frac{6.7-2\sqrt{8.82}}{8.82}\right)=0.25$$
$$h=0.25\left(\frac{8.82}{6.7-2\sqrt{8.82}}\right)$$
$$h=2.900...$$

The object was dropped from a height of 2.9 m.

Section 5.3 Page 302 Question 22

Substitute $r = 1740$ and $d = 610$ and solve for h.

$$d = \sqrt{2rh + h^2}$$
$$610 = \sqrt{2(1740)h + h^2}$$
$$610 = \sqrt{3480h + h^2}$$
$$610^2 = \left(\sqrt{3480h + h^2}\right)^2$$
$$372\,100 = 3480h + h^2$$
$$0 = h^2 + 3480h - 372\,100$$
Use the quadratic formula with $a = 1$, $b = 3480$, and $c = –372\ 100$.
$$h = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$h = \frac{-3480 \pm \sqrt{3480^2 - 4(1)(-372\,100)}}{2(1)}$$
$$h = \frac{-3480 \pm \sqrt{13\,598\,800}}{2}$$
$$h = \frac{-3480 \pm 20\sqrt{33\,997}}{2}$$
$$h = -1740 \pm 10\sqrt{33\,997}$$
$h = -1740 + 10\sqrt{33\,997}$ or $h = -1740 - 10\sqrt{33\,997}$
$h = 103.827...$ $\quad$ $h = -3583.857...$
Since h must be positive, the distance to the horizon is 104 km, to the nearest kilometre.

Section 5.3 Page 303 Question 23

a) Complete the square to find the vertex.
$P = –n^2 + 200n$
$P = –(n^2 – 200n)$
$P = –(n^2 – 200n + 10\ 000 – 10\ 000)$
$P = –(n – 100)^2 + 10\ 000$
The maximum profit is \$10 000 with 100 employees.

b)
$$P = –(n – 100)^2 +$$
$$P – 10\ 000 = –(n – 100)^2$$
$$–P + 10\ 000 = (n – 100)^2$$
$$\sqrt{-P + 10\,000} = n – 100$$
$$\sqrt{-P + 10\,000} + 100 = n$$

c) $P \leq 10\ 000$

d) The original function has domain $\{n \mid 0 \le n \le 200,\ n \in \mathrm{W}\}$ and range $\{P \mid 0 \le P \le 10\ 000,\ n \in \mathrm{W}\}$. The restriction in part c) is similar to the range of this function.

Section 5.3 Page 303 Question 24

Example: Both types of equations may involve rearranging and factoring. Solving a radical involves squaring both sides while solving a quadratic equation by completing the square involves taking a square root.

Section 5.3 Page 303 Question 25

Example: Extraneous roots may occur because squaring both sides and solving the quadratic equation may result in roots that do not satisfy the original equation.
For example,

$$x-2=\sqrt{x+10}$$
$$(x-2)^2=\left(\sqrt{x+10}\right)^2$$
$$x^2-4x+4=x+10$$
$$x^2-5x-6=0$$
$$(x-6)(x+1)=0$$

$x-6=0$ or $x+1=0$
$x=6$ $\quad x=-1$

Since $x = -1$ is an extraneous root, the solution is $x = 6$.

The extraneous root is introduced in line 2. The expression $x - 2$ could have a positive or negative value, but when squared the result is the same positive number. However, $\sqrt{x+10}$ only represents a positive value.

Section 5.3 Page 303 Question 26

a) Substitute $P_i = 320$ and $P_f = 390$.

$$r=-1+\sqrt[3]{\frac{P_f}{P_i}}$$
$$r=-1+\sqrt[3]{\frac{390}{320}}$$
$$r=0.068...$$

The annual growth rate is 6.8% to the nearest tenth of a percent.

b)

$$r = -1 + \sqrt[3]{\frac{P_f}{P_i}}$$

$$r + 1 = \sqrt[3]{\frac{P_f}{P_i}}$$

$$(r+1)^3 = \left(\sqrt[3]{\frac{P_f}{P_i}}\right)^3$$

$$(r+1)^3 = \frac{P_f}{P_i}$$

$$P_i(r+1)^3 = P_f$$

c) The initial population is 320 moose. After 1 year, the population is 320(1.068), or about 342 moose. After 2 years, the population is $320(1.068)^2$, or about 365 moose. After 3 years, the final population is 390 moose.

d) The set of populations in part c) represent a geometric sequence.

Section 5.3 Page 303 Question 27

Step 1 The table shows the number of nested radicals, the expression, and the decimal approximation.

1	$\sqrt{6+\sqrt{6}}$	2.906 800 603
2	$\sqrt{6+\sqrt{6+\sqrt{6}}}$	2.984 426 344
3	$\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6}}}}$	2.997 403 267
4	$\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6}}}}}$	2.999 567 18
5	$\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6}}}}}}$	2.999 927 862
6	$\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6}}}}}}}$	2.999 987 977
7	$\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6}}}}}}}}$	2.999 997 996
8	$\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6}}}}}}}}}$	2.999 999 666
9	$\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6}}}}}}}}}}$	2.999 999 944

Step 2 The predicted value for $\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\ldots}}}}$ is 3.

Step 3

$$x = \sqrt{6 + x},\ x \ge -6$$
$$x^2 = \left(\sqrt{6 + x}\right)^2$$
$$x^2 = 6 + x$$
$$x^2 - x - 6 = 0$$
$$(x - 3)(x + 2) = 0$$

$x - 3 = 0$ or $x + 2 = 0$
$x = 3$ $\quad$ $x = -2$

Step 4 Since $x = -2$ is an extraneous root, the solution is $x = 3$.

Step 5 Example: $\sqrt{12 + \sqrt{12 + \sqrt{12 + \sqrt{12 + \ldots}}}}$ results in a rational root of 4.

Step 6 Example:

$$x = \sqrt{12 + x},\ x \ge -12$$
$$x^2 = \left(\sqrt{12 + x}\right)^2$$
$$x^2 = 12 + x$$
$$x^2 - x - 12 = 0$$
$$(x - 4)(x + 3) = 0$$

$x - 4 = 0$ or $x + 3 = 0$
$x = 4$ $\quad$ $x = -3$

Since $x = -3$ is an extraneous root, the solution is $x = 4$.

Chapter 5 Review

Chapter 5 Review Page 304 Question 1

a) $8\sqrt{5} = \sqrt{8^2(5)}$
$= \sqrt{320}$

b) $-2\sqrt[5]{3} = \sqrt[5]{(-2)^5(3)}$
$= \sqrt[5]{-96}$

c) $3y^3\sqrt{7} = \sqrt{(3y^3)^2(7)}$
$= \sqrt{63y^6}$

d) $-3z\sqrt[3]{4z} = \sqrt[3]{(-3z)^3(4z)}$
$= \sqrt[3]{-108z^4}$

Chapter 5 Review Page 304 Question 2

a) $\sqrt{72} = \sqrt{36(2)}$
$= 6\sqrt{2}$

b) $3\sqrt{40} = 3\sqrt{4(10)}$
$= 6\sqrt{10}$

c) $\sqrt{27m^2} = \sqrt{9(3)(m^2)}$
$= 3m\sqrt{3}$

d) $\sqrt[3]{80x^5y^6} = \sqrt[3]{8(10)(x^3)(x^2)(y^6)}$
$= 2xy^2\sqrt[3]{10x^2}$

Chapter 5 Review Page 304 Question 3

a) $-\sqrt{13} + 2\sqrt{13} = \sqrt{13}$

b) $4\sqrt{7} - 2\sqrt{112} = 4\sqrt{7} - 2\sqrt{16(7)}$
$= 4\sqrt{7} - 8\sqrt{7}$
$= -4\sqrt{7}$

c) $-\sqrt[3]{3} + \sqrt[3]{24} = -\sqrt[3]{3} + \sqrt[3]{8(3)}$
$= -\sqrt[3]{3} + 2\sqrt[3]{3}$
$= \sqrt[3]{3}$

Chapter 5 Review Page 304 Question 4

a) $4\sqrt{45x^3} - \sqrt{27x} + 17\sqrt{3x} - 9\sqrt{125x^3}$, $x \geq 0$
$= 12x\sqrt{5x} - 3\sqrt{3x} + 17\sqrt{3x} - 45x\sqrt{5x}$
$= -33x\sqrt{5x} + 14\sqrt{3x}$

b) $\frac{2}{5}\sqrt{44a}+\sqrt{144a^3}-\frac{\sqrt{11a}}{2}, a \geq 0$

$=\frac{4}{5}\sqrt{11a}+12a\sqrt{a}-\frac{\sqrt{11a}}{2}$

$=\frac{3}{10}\sqrt{11a}+12a\sqrt{a}$

Chapter 5 Review Page 304 Question 5

$2\sqrt{112}=8\sqrt{7}$ $\sqrt{448}=8\sqrt{7}$ $3\sqrt{42}$ $4\sqrt{28}=8\sqrt{7}$

The expression $3\sqrt{42}$ is not equivalent to $8\sqrt{7}$.

Chapter 5 Review Page 304 Question 6

$3\sqrt{7}=\sqrt{63}$ $\sqrt{65}$ $2\sqrt{17}=\sqrt{68}$ $8=\sqrt{64}$

The numbers from least to greatest are $3\sqrt{7}$, 8, $\sqrt{65}$, and $2\sqrt{17}$.

Chapter 5 Review Page 304 Question 7

a) $v=\sqrt{169d}$

$v=13\sqrt{d}$

b) Substitute d = 13.4.

$v=13\sqrt{d}$

$v=13\sqrt{13.4}$

$v=47.587...$

The speed of the car is 48 km/h, to the nearest kilometre per hour.

Chapter 5 Review Page 304 Question 8

Determine the perimeter of a square with side length of $s=\sqrt{24.0}$.

$P = 4s$

$P=4\sqrt{24.0}$

$P=8\sqrt{6}$

The perimeter of the city is $8\sqrt{6}$ km.

Chapter 5 Review Page 304 Question 9

a) The equation $-3^2 = \pm 9$ is not true. The left side is equivalent to –9 only.

b) The equation $(-3)^2 = 9$ is true. The left side is equivalent to 9 only.

c) The equation $\sqrt{9} = \pm 3$ is not true. The left side is equivalent to 3 only.

Chapter 5 Review Page 304 Question 10

a) $\sqrt{2}\left(\sqrt{6}\right) = \sqrt{12}$
$= 2\sqrt{3}$

b) $\left(-3f\sqrt{15}\right)\left(2f^3\sqrt{5}\right) = -6f^4\sqrt{75}$
$= -30f^4\sqrt{3}$

c) $\left(\sqrt[4]{8}\right)\left(3\sqrt[4]{18}\right) = 3\sqrt[4]{144}$
$= 6\sqrt[4]{9}$

Chapter 5 Review Page 304 Question 11

a) $\left(2-\sqrt{5}\right)\left(2+\sqrt{5}\right) = 2^2 - \left(\sqrt{5}\right)^2$
$= 4 - 5$
$= -1$

b) $\left(5\sqrt{3}-\sqrt{8}\right)^2 = \left(5\sqrt{3}-\sqrt{8}\right)\left(5\sqrt{3}-\sqrt{8}\right)$
$= 75 - 10\left(\sqrt{3}\right)\left(\sqrt{8}\right) + 8$
$= 83 - 10\sqrt{24}$
$= 83 - 20\sqrt{6}$

c) $\left(a+3\sqrt{a}\right)\left(a+7\sqrt{4a}\right) = a^2 + 7a\sqrt{4a} + 3a\sqrt{a} + 21\sqrt{4a^2}$
$= a^2 + 14a\sqrt{a} + 3a\sqrt{a} + 42a$
$= a^2 + 17a\sqrt{a} + 42a, a \ge 0$

Chapter 5 Review Page 304 Question 12

$$\frac{5+\sqrt{17}}{2}\left(\frac{5-\sqrt{17}}{2}\right) = \frac{1}{4}\left(5^2 - \left(\sqrt{17}\right)^2\right)$$

Since the product is the difference of two squares, $x = \frac{5+\sqrt{17}}{2}$ and $x = \frac{5-\sqrt{17}}{2}$ are a conjugate pair.

Solving $x^2 - 5x + 2 = 0$ using the quadratic formula yields $x = \frac{5 \pm \sqrt{17}}{2}$. So these are solutions to the equation.

Chapter 5 Review Page 305 Question 13

a)
$$\begin{aligned}\frac{\sqrt{6}}{\sqrt{12}} &= \frac{\sqrt{6}}{\sqrt{12}}\left(\frac{\sqrt{12}}{\sqrt{12}}\right)\\ &= \frac{\sqrt{72}}{12}\\ &= \frac{6\sqrt{2}}{12}\\ &= \frac{\sqrt{2}}{2}\end{aligned}$$

b)
$$\begin{aligned}\frac{-1}{\sqrt[3]{25}} &= \frac{-1}{\sqrt[3]{25}}\left(\frac{\sqrt[3]{25}}{\sqrt[3]{25}}\right)^2\\ &= \frac{-\sqrt[3]{625}}{25}\\ &= \frac{-5\sqrt[3]{5}}{25}\\ &= \frac{-\sqrt[3]{5}}{5}\end{aligned}$$

c)
$$\begin{aligned}-4\sqrt{\frac{2a^2}{9}} &= -4\left(\frac{\sqrt{2a^2}}{\sqrt{9}}\right)\\ &= \frac{-4a\sqrt{2}}{3}\end{aligned}$$

Chapter 5 Review Page 305 Question 14

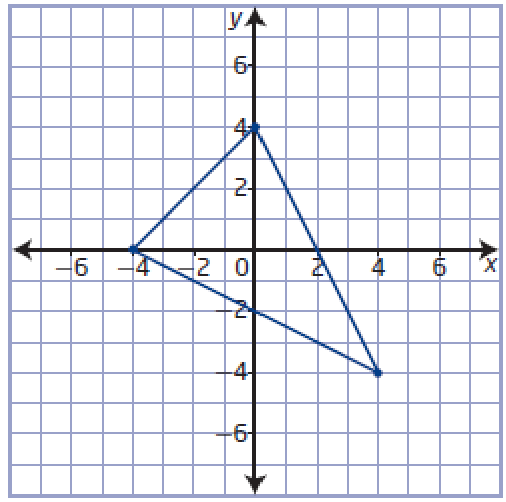
a) $$\frac{-2}{4-\sqrt{3}} = \frac{-2}{4-\sqrt{3}}\left(\frac{4+\sqrt{3}}{4+\sqrt{3}}\right)$$
$$= \frac{-2\left(4+\sqrt{3}\right)}{16-3}$$
$$= \frac{-8-2\sqrt{3}}{13}$$

b) $$\frac{\sqrt{7}}{2\sqrt{5}-\sqrt{7}} = \frac{\sqrt{7}}{2\sqrt{5}-\sqrt{7}}\left(\frac{2\sqrt{5}+\sqrt{7}}{2\sqrt{5}+\sqrt{7}}\right)$$
$$= \frac{2\sqrt{35}+7}{20-7}$$
$$= \frac{2\sqrt{35}+7}{13}$$

c) $$\frac{18}{6+\sqrt{27m}} = \frac{18}{6+\sqrt{27m}}\left(\frac{6-\sqrt{27m}}{6-\sqrt{27m}}\right)$$
$$= \frac{108-18\sqrt{27m}}{36-27m}$$
$$= \frac{12-2\sqrt{27m}}{4-3m}$$
$$= \frac{12-6\sqrt{3m}}{4-3m},\ m \geq 0, m \neq \frac{4}{3}$$

d) $$\frac{a+\sqrt{b}}{a-\sqrt{b}} = \frac{a+\sqrt{b}}{a-\sqrt{b}}\left(\frac{a+\sqrt{b}}{a+\sqrt{b}}\right)$$
$$= \frac{a^2+2a\sqrt{b}+b}{a^2-b},\ b \geq 0, a \neq \pm\sqrt{b}$$

Chapter 5 Review   Page 305   Question 15



Use the Pythagorean theorem to find the lengths of sides of the triangle.
For side from (–4, 0) to (0, 4):
$c^2 = 4^2 + 4^2$
$c^2 = 32$
$c = \sqrt{32}$
For side from (0, 4) to (4, –4):
$c^2 = 8^2 + 4^2$
$c^2 = 80$
$c = \sqrt{80}$
The third side is the same length as the one above.
Find the perimeter.
$P = \sqrt{32} + 2\sqrt{80}$
$P = 4\sqrt{2} + 8\sqrt{5}$

Chapter 5 Review   Page 305   Question 16

a)

$$\left(\frac{-5\sqrt{3}}{\sqrt{6}}\right)\left(\frac{-\sqrt{7}}{3\sqrt{21}}\right) = \frac{5\sqrt{21}}{3\sqrt{126}}$$
$$= \frac{5\sqrt{21}}{3\sqrt{21(6)}}$$
$$= \frac{5}{3\sqrt{6}}\left(\frac{\sqrt{6}}{\sqrt{6}}\right)$$
$$= \frac{5\sqrt{6}}{18}$$

b)

$$\left(\frac{2a\sqrt{a^3}}{9}\right)\left(\frac{12}{-\sqrt{8a}}\right)=\frac{24a\sqrt{a^3}}{-9\sqrt{8a}}$$
$$=\frac{24a^2\sqrt{a}}{-18\sqrt{2a}}$$
$$=\frac{4a^2}{-3\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$$
$$=\frac{4a^2\sqrt{2}}{-6}$$
$$=\frac{2a^2\sqrt{2}}{-3}$$

Chapter 5 Review Page 305 Question 17

Substitute $A = 12$ and $w = 4-\sqrt{2}$ and solve for l.

$$A = lw$$
$$12 = l\left(4-\sqrt{2}\right)$$
$$l=\frac{12}{4-\sqrt{2}}$$
$$l=\frac{12}{4-\sqrt{2}}\left(\frac{4+\sqrt{2}}{4+\sqrt{2}}\right)$$
$$l=\frac{48+12\sqrt{2}}{16-2}$$
$$l=\frac{48+12\sqrt{2}}{14}$$
$$l=\frac{24+6\sqrt{2}}{7}$$

An expression for the length is $\frac{24+6\sqrt{2}}{7}$ units.

Chapter 5 Review Page 305 Question 18

a) $-\sqrt{x}=-7,\ x\geq 0$
$$\sqrt{x}=7$$
$$\left(\sqrt{x}\right)^2=7^2$$
$$x=49$$

Check for $x = 49$.

Left Side   Right Side

$-\sqrt{x}$ -7

$= -\sqrt{49}$

$= -7$

Left Side = Right Side

The solution is $x = 49$.

b) $\sqrt{4-x} = -2,\ x \le 4$

$$\left(\sqrt{4-x}\right)^2 = (-2)^2$$
$$4 - x = 4$$
$$x = 0$$

Check for $x = 0$.

Left Side   Right Side

$\sqrt{4-x}$ -2

$= \sqrt{4-0}$

$= 2$

Left Side ≠ Right Side

There is no solution.

c) $5 - \sqrt{2x} = -1,\ x \ge 0$

$$6 = \sqrt{2x}$$
$$6^2 = \left(\sqrt{2x}\right)^2$$
$$36 = 2x$$
$$18 = x$$

Check for $x = 18$.

Left Side   Right Side

$5 - \sqrt{2x}$ -1

$= 5 - \sqrt{2(18)}$

$= -1$

Left Side = Right Side

The solution is $x = 18$.

d)

$$1+\sqrt{\frac{7x}{3}}=8,\ x\ge 0$$

$$\sqrt{\frac{7x}{3}}=7$$

$$\left(\sqrt{\frac{7x}{3}}\right)^2=7^2$$

$$\frac{7x}{3}=49$$

$$7x=147$$

$$x=21$$

Check for $x=21$.

Left Side	Right Side
$1+\sqrt{\frac{7x}{3}}$	8
$=1+\sqrt{\frac{7(21)}{3}}$	
$=1+\sqrt{49}$	
$=8$	

Left Side = Right Side

The solution is $x=21$.

Chapter 5 Review Page 305 Question 19

a) $\sqrt{5x-3}=\sqrt{7x-12},\ x\ge\frac{12}{7}$

$$\left(\sqrt{5x-3}\right)^2=\left(\sqrt{7x-12}\right)^2$$

$$5x-3=7x-12$$

$$-2x=-9$$

$$x=\frac{9}{2}$$

b) $\sqrt{y-3}=y-3,\ y\ge 3$

$$\left(\sqrt{y-3}\right)^2=\left(y-3\right)^2$$

$$y-3=y^2-6y+9$$

$$0=y^2-7y+12$$

$$0=(y-4)(y-3)$$

$y - 4 = 0$ or $y - 3 = 0$

$y = 4$ $y = 3$

c) $\sqrt{7n+25} - n = 1, n \geq -\frac{25}{7}$

$$\sqrt{7n+25} = 1 + n$$
$$\left(\sqrt{7n+25}\right)^2 = (1+n)^2$$
$$7n + 25 = n^2 + 2n + 1$$
$$0 = n^2 - 5n - 24$$
$$0 = (n-8)(n+3)$$

$n - 8 = 0$ or $n + 3 = 0$

$n = 8$ $n = -3$

Since $n = -3$ is an extraneous root, the solution is $n = 8$.

d)

$$\sqrt{8 - \frac{m}{3}} = \sqrt{3m} - 4, 0 \leq m \leq 24$$
$$\sqrt{8 - \frac{m}{3}} - \sqrt{3m} = -4$$
$$\left(\sqrt{8 - \frac{m}{3}}\right)^2 = \left(\sqrt{3m} - 4\right)^2$$
$$8 - \frac{m}{3} = 3m - 8\sqrt{3m} + 16$$
$$-8 - \frac{10m}{3} = -8\sqrt{3m}$$
$$1 + \frac{5m}{12} = \sqrt{3m}$$
$$\left(1 + \frac{5m}{12}\right)^2 = \left(\sqrt{3m}\right)^2$$
$$1 + \frac{10m}{12} + \frac{25m^2}{144} = 3m$$
$$144 + 120m + 25m^2 = 432m$$
$$25m^2 - 312m + 144 = 0$$
$$(m-12)(25m-12) = 0$$

$m - 12 = 0$ or $25m - 12 = 0$

$m = 12$ $m = \frac{12}{25}$

Since $m = \frac{12}{25}$ is an extraneous root, the solution is $m = 12$.

e)
$$\begin{aligned} \sqrt[3]{3x-1}+7 &= 3 \\ \sqrt[3]{3x-1} &= -4 \\ \left(\sqrt[3]{3x-1}\right)^3 &= (-4)^3 \\ 3x-1 &= -64 \\ 3x &= -63 \\ x &= -21 \end{aligned}$$

Chapter 5 Review Page 305 Question 20

Example: Isolate the radical. Next, square both sides. Then, expand and simplify. Factor the quadratic equation. Possible solutions are $n = –3$ and $n = 8$. The root $n = –3$ is an extraneous root because when it is substituted into the original equation a false statement is reached.

Chapter 5 Review Page 305 Question 21

Substitute $d = 7.1$ and solve for h.

$$\begin{aligned} d &= \sqrt{\frac{3h}{2}} \\ 7.1 &= \sqrt{\frac{3h}{2}} \\ 7.1^2 &= \left(\sqrt{\frac{3h}{2}}\right)^2 \\ 50.41 &= \frac{3h}{2} \\ 100.82 &= 3h \\ h &= 33.606... \end{aligned}$$

The height of the crew is about 33.6 m.

Chapter 5 Practice Test

Chapter 5 Practice Test Page 306 Question 1

$$-3\left(\sqrt[3]{2}\right) = \sqrt[3]{(-3)^3 2}$$
$$= \sqrt[3]{-54}$$

Choice **B**.

Chapter 5 Practice Test Page 306 Question 2

The condition on the variable in $2\sqrt{-7n}$ is $-7n \geq 0$, or $n \leq 0$. Choice **D**.

Chapter 5 Practice Test Page 306 Question 3

$$-2x\sqrt{6x} + 5x\sqrt{6x} = 3x\sqrt{6x}$$

Choice **C**.

Chapter 5 Practice Test Page 306 Question 4

$$\sqrt{540}\left(\sqrt{6y}\right) = 6\sqrt{15}\left(\sqrt{6y}\right)$$
$$= 6\sqrt{90y}$$
$$= 18\sqrt{10y}$$

Choice **D**.

Chapter 5 Practice Test Page 306 Question 5

$$x + 7 = \sqrt{23 - x}$$
$$(x+7)^2 = \left(\sqrt{23 - x}\right)^2$$
$$x^2 + 14x + 49 = 23 - x$$
$$x^2 + 15x + 26 = 0$$
$$(x+13)(x+2) = 0$$

$x + 13 = 0$ or $x + 2 = 0$

$x = -13$ $\quad$ $x = -2$

Since $x = -3$ is an extraneous root, the solution is $x = -2$. Choice **B**.

Chapter 5 Practice Test Page 306 Question 6

$$\frac{5}{7}\sqrt{\frac{3}{2}} = \frac{5}{7}\left(\frac{\sqrt{3}}{\sqrt{2}}\right)\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$$
$$= \frac{5}{7}\left(\frac{\sqrt{6}}{2}\right)$$
$$= \frac{5}{14}\sqrt{6}$$

Choice **C**.

Chapter 5 Practice Test Page 306 Question 7

$3\sqrt{11} = \sqrt{99}$ $5\sqrt{6} = \sqrt{150}$ $9\sqrt{2} = \sqrt{162}$ $\sqrt{160}$

The numbers from least to greatest are $3\sqrt{11}$, $5\sqrt{6}$, $\sqrt{160}$, and $9\sqrt{2}$.

Chapter 5 Practice Test Page 306 Question 8

$$\frac{\left(2\sqrt{5n}\right)\left(3\sqrt{8n}\right)}{1-12\sqrt{2}} = \frac{6\sqrt{40n^2}}{1-12\sqrt{2}}$$
$$= \frac{12n\sqrt{10}}{1-12\sqrt{2}}\left(\frac{1+12\sqrt{2}}{1+12\sqrt{2}}\right)$$
$$= \frac{12n\sqrt{10}+144n\sqrt{20}}{1-288}$$
$$= \frac{12n\sqrt{10}+288n\sqrt{5}}{-287}$$

Chapter 5 Practice Test Page 306 Question 9

$$3-x = \sqrt{x^2-5},\ x \le -\sqrt{5} \text{ or } x \ge \sqrt{5}$$
$$(3-x)^2 = \left(\sqrt{x^2-5}\right)^2$$
$$9-6x+x^2 = x^2-5$$
$$-6x = -14$$
$$x = \frac{14}{6}$$
$$x = \frac{7}{3}$$

Chapter 5 Practice Test Page 306 Question 10

$$\sqrt{9y+1} = 3 + \sqrt{4y-2}$$
$$\left(\sqrt{9y+1}\right)^2 = \left(3 + \sqrt{4y-2}\right)^2$$
$$9y+1 = 9 + 6\sqrt{4y-2} + 4y - 2$$
$$5y - 6 = 6\sqrt{4y-2}$$
$$(5y-6)^2 = \left(6\sqrt{4y-2}\right)^2$$
$$25y^2 - 60y + 36 = 36(4y-2)$$
$$25y^2 - 204y + 108 = 0$$

Use the quadratic formula with $a = 25$, $b = -204$, and $c = 108$.

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$y = \frac{-(-204) \pm \sqrt{(-204)^2 - 4(25)(108)}}{2(25)}$$
$$y = \frac{204 \pm \sqrt{30\,816}}{50}$$
$$y = \frac{204 \pm 12\sqrt{214}}{50}$$
$$y = \frac{102 \pm 6\sqrt{214}}{25}$$

Check for $y = \dfrac{102 + 6\sqrt{214}}{25}$.

Left Side	Right Side
$\sqrt{9y+1}$	$3 + \sqrt{4y-2}$
$= \sqrt{9\left(\frac{102 + 6\sqrt{214}}{25}\right) + 1}$	$= 3 + \sqrt{4\left(\frac{102 + 6\sqrt{214}}{25}\right) - 2}$
$= 8.3257...$	$= 8.3257...$

Left side = Right Side

Check for $y = \dfrac{102 - 6\sqrt{214}}{25}$.

Left Side	Right Side
$\sqrt{9y+1}$	$3 + \sqrt{4y-2}$

$= \sqrt{9\left(\frac{102-6\sqrt{214}}{25}\right)+1}$ $\quad$ $= 3+\sqrt{4\left(\frac{102-6\sqrt{214}}{25}\right)-2}$

$= 2.4742...$ $\quad$ $= 3.5257...$

Left side ≠ Right Side

Since $y = \frac{102-6\sqrt{214}}{25}$ is an extraneous root, the solution is $y = \frac{102+6\sqrt{214}}{25}$.

Chapter 5 Practice Test Page 306 Question 11

Combine pairs of identical factors.

$$\begin{aligned}\sqrt{450} &= \sqrt{2(3)(3)(5)(5)}\\ &= 3(5)\sqrt{2}\\ &= 15\sqrt{2}\end{aligned}$$

Chapter 5 Practice Test Page 306 Question 12

Use the Pythagorean theorem to find the lengths of sides of the two right isosceles triangle.

For triangle with side lengths of 4 km:

$c^2 = 4^2 + 4^2$

$c^2 = 32$

$c = \sqrt{32}$

For triangle with side lengths of 5 km:

$c^2 = 5^2 + 5^2$

$c^2 = 50$

$c = \sqrt{50}$

Find the sum of the hypotenuses.

$d = \sqrt{32} + \sqrt{50}$

$d = 4\sqrt{2} + 5\sqrt{2}$

$d = 9\sqrt{2}$

The boat is $9\sqrt{2}$ km from its starting point.

Chapter 5 Practice Test Page 306 Question 13

a) To rationalize the denominator of $\frac{4}{\sqrt{6}}$, multiply the numerator and denominator by $\sqrt{6}$ because $\sqrt{6}\left(\sqrt{6}\right) = 6$.

b) To rationalize the denominator of $\frac{22}{\sqrt{y-3}}$, multiply the numerator and denominator by $\sqrt{y-3}$ because $\sqrt{y-3}\left(\sqrt{y-3}\right) = y-3$.

c) To rationalize the denominator of $\frac{2}{\sqrt[3]{7}}$, multiply the numerator and denominator by $\left(\sqrt[3]{7}\right)^2$ because $\sqrt[3]{7}\left(\sqrt[3]{7}\right)^2 = 7$.

Chapter 5 Practice Test Page 307 Question 14

Substitute $m = 3$ and solve for C.

$$m = \sqrt{\frac{C}{700}}$$

$$3 = \sqrt{\frac{C}{700}}$$

$$3^2 = \left(\sqrt{\frac{C}{700}}\right)^2$$

$$9 = \frac{C}{700}$$

$$C = 6300$$

The cost of a 3-carat diamond in $6300.

Chapter 5 Practice Test Page 307 Question 15

Teya's solution is correct.

Chapter 5 Practice Test Page 307 Question 16

a) Let x represent the length of the other leg. Then, an expression for the hypotenuse, c, is

$$c^2 = 1^2 + x^2$$

$$c^2 = 1 + x^2$$

$$c = \sqrt{1 + x^2}$$

b) Substitute $c = 11$ and solve for x.

$$c = \sqrt{1 + x^2}$$
$$11 = \sqrt{1 + x^2}$$
$$11^2 = \left(\sqrt{1 + x^2}\right)^2$$
$$121 = 1 + x^2$$
$$120 = x^2$$
$$x = \sqrt{120}$$
$$x = 2\sqrt{30}$$
The length of the unknown leg is $2\sqrt{30}$ units.

Chapter 5 Practice Test Page 307 Question 17

a) $I = \sqrt{\frac{P}{R}}$
$$I^2 = \left(\sqrt{\frac{P}{R}}\right)^2$$
$$I^2 = \frac{P}{R}$$
$$R = \frac{P}{I^2}$$

b) Substitute $I = 0.5$ and $P = 100$.
$$R = \frac{P}{I^2}$$
$$R = \frac{100}{0.5^2}$$
$$R = 400$$
The resistance in the bulb is 400 Ω.

Chapter 5 Practice Test Page 307 Question 18

a) Let s represent the edge length of the cube.
$$SA = 6s^2$$
$$\frac{SA}{6} = s^2$$
$$s = \sqrt{\frac{SA}{6}}$$

b) Substitute $SA = 33$.

$s = \sqrt{\frac{SA}{6}}$

$s = \sqrt{\frac{33}{6}}$

$s = \frac{\sqrt{11}}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$

$s = \frac{\sqrt{22}}{2}$

The edge length is $\frac{\sqrt{22}}{2}$ cm.

c) Let s_{new} represent the edge length of the new cube.

$s_{\text{new}} = \sqrt{\frac{2SA}{6}}$

$s_{\text{new}} = 4\sqrt{\frac{SA}{6}}$

$s_{\text{new}} = 4s$

The edge length will change by a scale factor of 4.

Chapter 5 Practice Test Page 307 Question 19

a) Substitute $P = 3500$, $n = 2$, and $A = 3713.15$.

$A = P(1 + i)^n$

$3713.15 = 3500(1 + i)^2$

b) $3713.15 = 3500(1 + i)^2$

$\frac{3713.15}{3500} = (1+i)^2$

$1.0609 = (1+i)^2$

$\sqrt{1.0609} = 1+i$

$i = \sqrt{1.0609} - 1$

$i = 0.03$

The interest rate is 3%.