Chapter 6 Rational Expressions and Equations

Section 6.1 Rational Expressions

Section 6.1 Page 317 Question 1

a) $\frac{3}{5} = \frac{3(6)}{5(6)} = \frac{18}{30}$

b) $\frac{2}{5} = \frac{2(7x)}{5(7x)} = \frac{14x}{35x}$, $x \neq 0$

c) $\frac{4}{7} = \frac{44}{77}$

d) $\frac{x+2}{x-3} = \frac{4(x+2)}{4(x-3)} = \frac{4x+8}{4x-12}$

e) $\frac{3(6)}{8(6)} = \frac{3}{8}$

f) $\frac{1}{y-2} = \frac{\square}{(y-2)(y+2)}$

$\frac{1}{y-2} = \frac{y+2}{y^2-2}$

Section 6.1 Page 317 Question 2

a) Divide the numerator and the denominator by *pq*.

$$\frac{\frac{3p^2q}{pq}}{\frac{pq^2}{pq}} = \frac{3p}{q}$$

b) Multiply the numerator and the denominator by (x – 4).

$$\frac{2}{x+4} = \frac{2(x-4)}{(x+4)(x-4)}$$
$$= \frac{2x-8}{x^2-16}$$

c) Divide the numerator and the denominator by (m – 3).

$$\frac{-4(m-3)}{m^2-9} = \frac{\frac{-4(m-3)}{(m-3)}}{\frac{(m-3)(m+3)}{(m-3)}}$$
$$= \frac{-4}{m+3}$$

d) Multiply the numerator and the denominator by $(y^2 + y)$.

$$\begin{aligned}\frac{1}{y-1} &= \frac{1(y^2+y)}{(y-1)(y^2+y)}\\ &= \frac{y^2+y}{y^3-y}\end{aligned}$$

Section 6.1 Page 317 Question 3

a) The denominator of $\frac{-4}{x}$ is zero when $x = 0$.

b) The denominator of $\frac{3c-1}{c-1}$ is zero when $c - 1 = 0$. That is, when $c = 1$.

c) The denominator of $\frac{y}{y+5}$ is zero when $y + 5 = 0$. That is, when $y = -5$.

d) The denominator of $\frac{m+3}{5}$ is never zero. There is no value of *m* that makes the denominator zero.

e) The denominator of $\frac{1}{d^2-1}$ is zero when $d^2 - 1 = 0$. That is, when $d^2 = 1$, or $d = \pm 1$.

f) The denominator of $\frac{x-1}{x^2+1}$ is zero if $x^2 + 1 = 0$. This never occurs, because $x^2 + 1 \geq 1$ for all values of *x*. There is no value of *x* that makes the denominator zero.

Section 6.1 Page 317 Question 4

In each the denominator cannot be zero, as division by zero is not defined.

a) For $\frac{3a}{4-a}$, $4 - a \neq 0$. So, the non-permissible value is $a = 4$.

b) For $\frac{2e+8}{e}$, $e \neq 0$. So, the non-permissible value is $e = 0$.

c) For $\frac{3(y+7)}{(y-4)(y+2)}$, $(y - 4)(y + 2) \neq 0$. So, the non-permissible values are $y = 4$ and $y = -2$.

d) For $\dfrac{-7(r-1)}{(r-1)(r+3)}$, $(r-1)(r+3) \neq 0$. So, the non-permissible values are $r = 1$ and $r = -3$.

e) For $\dfrac{2k+8}{k^2}$, $k^2 \neq 0$. So, the non-permissible value is $k = 0$.

f) For $\dfrac{6x-8}{(3x-4)(2x+5)}$, $(3x-4)(2x+5) \neq 0$. So, the non-permissible values are $x = \dfrac{4}{3}$ and $x = -\dfrac{5}{2}$.

Section 6.1 Page 318 Question 5

a) For $\dfrac{2\pi r^2}{8\pi r^3}$, $8\pi r^3 \neq 0$. So, the excluded value is $r \neq 0$.

b) For $\dfrac{2t+t^2}{t^2-1}$, $t^2 - 1 \neq 0$. So, the excluded values are $t \neq \pm 1$.

c) For $\dfrac{x-2}{10-5x}$, $10 - 5x \neq 0$. So, the excluded value is $x \neq 2$.

d) For $\dfrac{3g}{g^3-9g}$, $g^3 - 9g \neq 0$. Then, $g^3 - 9g = g(g^2 - 9) = g(g-3)(g+3)$. So, the excluded values are $g \neq 0$ and $g \neq \pm 3$.

Section 6.1 Page 318 Question 6

a) $$\frac{2\overset{1}{\cancel{c}}\,\overset{1}{\cancel{(c-5)}}}{3\underset{1}{\cancel{c}}\,\underset{1}{\cancel{(c-5)}}} = \frac{2}{3},\ c \neq 0, 5$$

b) $$\frac{3\overset{1}{\cancel{w}}(2w+3)}{2\underset{1}{\cancel{w}}(3w+2)} = \frac{3(2w+3)}{2(3w+2)},\ w \neq -\frac{2}{3}, 0$$

c) $$\frac{\overset{1}{\cancel{(x-7)}}\,(x+7)}{(2x-1)\,\underset{1}{\cancel{(x-7)}}} = \frac{x+7}{2x-1},\ x \neq \frac{1}{2}, 7$$

d) $\dfrac{\overset{1}{\cancel{5}}\ \overset{-1}{\cancel{(a-3)}}\ \overset{1}{\cancel{(a+2)}}}{\underset{2}{\cancel{10}}\ \underset{1}{\cancel{(3-a)}}\ \underset{1}{\cancel{(a+2)}}} = -\dfrac{1}{2},\ a \neq 3, -2$

Section 6.1 Page 318 Question 7

a) x^2 is not a factor, it is a term. Dividing out the x^2 would be like reducing $\dfrac{25}{210}$ to $\dfrac{5}{10}$: you cannot just strike out part of each number.

b) Determine what values of the variable make the denominator zero. Factor the denominator, then set each factor equal to zero and solve.

$x^2 + 2x - 3 = 0$
$(x + 3)(x - 1) = 0$
$x + 3 = 0$ or $x - 1 = 0$
$x = -3$ $\quad$ $x = 1$

The non-permissible values are 1 and –3.

c) If possible, factor the numerator and the denominator. Then, divide both the numerator and the denominator by any common factors.

$$\begin{aligned}\frac{x^2-1}{x^2+2x-3} &= \frac{\overset{1}{\cancel{(x-1)}}(x+1)}{(x+3)\underset{1}{\cancel{(x-1)}}}\\ &= \frac{x+1}{x+3}, \quad x \neq 1, -3\end{aligned}$$

Section 6.1 Page 318 Question 8

a) $\dfrac{\overset{3}{\cancel{6}}\ \overset{r}{\cancel{r^2}}\ \overset{1}{\cancel{p^3}}}{\underset{2}{\cancel{4}}\ \underset{1}{\cancel{r}}\ \underset{p}{\cancel{p^4}}} = \dfrac{3r}{2p},\ p \neq 0,\ r \neq 0$

b) $$\begin{aligned}\frac{3x-6}{10-5x} &= \frac{3\ \overset{-1}{\cancel{(x-2)}}}{5\ \underset{1}{\cancel{(2-x)}}}\\ &= -\frac{3}{5}, \quad x \neq 2\end{aligned}$$

c) $$\begin{aligned}\frac{b^2+2b-24}{2b^2-72} &= \frac{(b+6)(b-4)}{2(b^2-36)}\\ &= \frac{\overset{1}{\cancel{(b+6)}}(b-4)}{2(b-6)\underset{1}{\cancel{(b+6)}}}\\ &= \frac{b-4}{2(b-6)}, \quad b \neq \pm 6\end{aligned}$$

d) $$\frac{10k^2+55k+75}{20k^2-10k-150}=\frac{5(2k^2+11k+15)}{10(2k^2-k-15)}$$
$$=\frac{\overset{1}{\cancel{5}}\ \overset{1}{\cancel{(2k+5)}}(k+3)}{\underset{2}{\cancel{10}}\ \underset{1}{\cancel{(2k+5)}}(k-3)}$$
$$=\frac{k+3}{2(k-3)},\quad k\neq 3,-\frac{5}{2}$$

e) $$\frac{x-4}{4-x}=\frac{-1\ \overset{1}{\cancel{(-x+4)}}}{\underset{1}{\cancel{4-x}}}$$
$$=-1,\quad x\neq 4$$

f) $$\frac{5(x^2-y^2)}{x^2-2xy+y^2}=\frac{5\ \overset{1}{\cancel{(x-y)}}(x+y)}{\underset{1}{\cancel{(x-y)}}(x-y)}$$
$$=\frac{5(x+y)}{x-y},\quad x\neq y$$

Section 6.1 Page 318 Question 9

$$\frac{x^2+2x-15}{x-3}=\frac{(x-3)(x+5)}{x-3}$$
$$=x+5,\quad x\neq 3$$

The statement is sometimes true. It is true for all values of x except $x = 3$.

Section 6.1 Page 318 Question 10

The original expression may have had other factors that have been divided out.
Example: $\frac{y(y+1)}{(y-6)(y+1)}$, $y\neq 6,-1$.

Section 6.1 Page 318 Question 11

Yes, Mike is correct, because $-1(5-x) = -5 + x$, or $x - 5$. The only thing to remember is that the factor in the denominator leads to a non-permissible value that should be stated in the reduced expression.

Section 6.1 Page 318 Question 12

Examples:
Start with a factored expression and expand to create the question for your friend.

$$\frac{(x+3)(x-1)}{(x+3)(x+1)}=\frac{x^2+2x-3}{x^2+4x+4}$$

Other similar examples are $\frac{x^2+x-6}{x^2+5x+6}$, $\frac{2x^2+6x}{x^2+3x}$.

Section 6.1 Page 318 Question 13

In the third step, Shali cancelled +2 from both the numerator and the denominator. This is not correct because 2 is not a factor of the numerator. The correct solution is as follows.

$$\begin{aligned}\frac{g^2-4}{2g-4}&=\frac{\cancel{(g-2)}(g+2)}{2\cancel{(g-2)}}\\&=\frac{g+2}{2},\ \ g\neq 2\end{aligned}$$

Section 6.1 Page 318 Question 14

Example: $\frac{4p}{(p-2)(p+1)}=\frac{4p}{p^2-p-2}$

Section 6.1 Page 319 Question 15

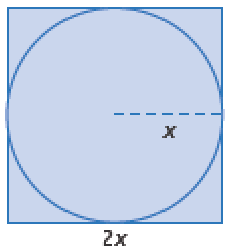
a) $t=\frac{d}{r}$, so an expression for the time is:

$$\frac{2n^2+11n+12}{2n^2-32}$$

b) $$\begin{aligned}\frac{2n^2+11n+12}{2n^2-32}&=\frac{(2n+3)(n+4)}{2(n^2-16)}\\&=\frac{(2n+3)(n+4)}{2(n-4)(n+4)}\\&=\frac{2n+3}{2(n-4)},\ \ n\neq\pm4\end{aligned}$$

Section 6.1 Page 319 Question 16

a)



b) $\dfrac{\text{Area of circle}}{\text{Area of square}} = \dfrac{\pi x^2}{4x^2}$

c) The denominator, $4x^2$, cannot be zero, so $x \neq 0$.

d) $\dfrac{\pi}{4}$

e) Percent $= \dfrac{\pi}{4} \times 100\%$

The percent of paper in the circle is 79%, to the nearest percent.

Section 6.1 Page 319 Question 17

a) The non-permissible value is $p = -2$, but this negative value has no meaning since p is a mass.

b) The least extra yield occurs when the expression $\dfrac{900p}{2+p}$ is least. If $p = 0$, there is no extra yield.

c) Try $p = 450$, $\dfrac{900p}{2+p} = \dfrac{900(450)}{2+450} \approx 896$

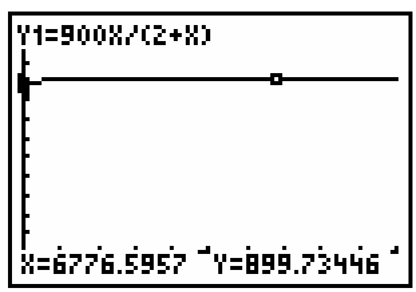
Try $p = 900$, $\dfrac{900p}{2+p} = \dfrac{900(900)}{2+900} \approx 898$

Try $p = 1000$, $\dfrac{900p}{2+p} = \dfrac{900(1000)}{2+1000} \approx 898$

Try $p = 6000$, $\dfrac{900p}{2+p} = \dfrac{900(6000)}{2+6000} \approx 900$

The greatest extra yield seems to be 900 kg.

A graph of the related function, $y = \dfrac{900p}{2+p}$, confirms this.



Section 6.1 Page 319 Question 18

a) Substitute $d = 100$ and $r = 2q$ into $t = \frac{d}{r}$.

$t = \frac{100}{2q}$

$t = \frac{50}{q}, \quad q \neq 0$

b) Substitute $d = 100$ and $r = p - 4$ into $t = \frac{d}{r}$.

$t = \frac{100}{p-4}$

$t = \frac{100}{p-4}, \quad p \neq 4$

Section 6.1 Page 319 Question 19

a) Cost for 30 students = 350 + 30(9)
= 620
The total cost for 30 students is $620.

b) Cost for n students = 350 + 9n
So, cost per student = $\frac{350+9n}{n}, \quad n \neq 0$

c) Substitute $n = 30$ into $\frac{350+9n}{n}$.

Cost per student $= \frac{350+9(30)}{30}$
≈ 20.67
The cost per student, if 30 students go, is $20.67.

Section 6.1 Page 320 Question 20

a) No. Terri divided out the term 5 but it is not a factor of the denominator.

b) Example: when $m = 3$

$\frac{5}{m+5} = \frac{5}{3+5}$ or $\frac{5}{8}$ but $\frac{1}{m+1} = \frac{1}{3+1}$ or $\frac{1}{4}$

Section 6.1 Page 320 Question 21

a) To change $\frac{3x}{4}$ into $\frac{15x}{20}$, multiply numerator and denominator by 5.

$$\frac{5(3x)}{5(4)}=\frac{15x}{20}$$

b) To change $\frac{3x}{4}$ into $\frac{3x^2-6x}{4x-8}$, multiply numerator and denominator by $(x-2)$.

$$\frac{3x(x-2)}{4(x-2)}=\frac{3x^2-6x}{4x-8}$$

Section 6.1 Page 320 Question 22

a)
$$\begin{aligned}\frac{x-2}{3}&=\frac{4(x-2)}{4(3)}\\&=\frac{4x-8}{12}\end{aligned}$$

b)
$$\begin{aligned}\frac{x-2}{3}&=\frac{3(x-2)}{3(3)}\\&=\frac{3x-6}{9}\end{aligned}$$

c)
$$\begin{aligned}\frac{x-2}{3}&=\frac{(x-2)(2x+5)}{3(2x+5)}\\&=\frac{2x^2+x-10}{6x+15},\ x\neq-\frac{5}{2}\end{aligned}$$

Section 6.1 Page 320 Question 23

a)
$$\begin{aligned}5&=\frac{5(5b)}{5b}\\&=\frac{25b}{5b},\quad b\neq 0\end{aligned}$$

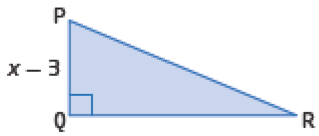
b)
$$\begin{aligned}\frac{x+1}{3}&=\frac{(x+1)(4a^2b)}{3(4a^2b)}\\&=\frac{4a^2bx+4a^2b}{12a^2b},\ a\neq 0, b\neq 0\end{aligned}$$

c) $\dfrac{a-b}{7x} = \dfrac{-2(a-b)}{-2(7x)}$

$= \dfrac{2b-2a}{-14x}, \quad x \neq 0$

Section 6.1 Page 320 Question 24

a)



b) Use the formula for area of a triangle, $A = \dfrac{bh}{2}$, where b is the base and h is the height.

QR is the base. So, QR = $\dfrac{2(\text{Area})}{\text{height}}$.

$\text{QR} = \dfrac{2(x^2-x-6)}{x-3}$

$\text{QR} = \dfrac{2(x-3)(x+2)}{x-3}$

$\text{QR} = 2(x+2)$

c) The non-permissible value is $x \neq 3$.

Section 6.1 Page 320 Question 25

a) $\dfrac{6x^2-x-1}{9x^2-1} = \dfrac{(2x+1)(3x-1)}{(3x-1)(3x+1)}$

$= \dfrac{2x+1}{3x+1}, \quad x \neq \pm\dfrac{1}{3}$

b) $\dfrac{2n^2+n-15}{5n-2n^2} = \dfrac{(n+3)(2n-5)}{n(5-2n)}$

$= \dfrac{(n+3)(2n-5)}{-n(2n-5)}$

$= \dfrac{n+3}{-n}$

$= \dfrac{-n-3}{n}, \; n \neq 0, \dfrac{5}{2}$

Section 6.1 Page 320 Question 26

a) $$\frac{(x+2)^2-(x+2)-20}{x^2-9}=\frac{x^2+4x+4-x-2-20}{x^2-9}$$
$$=\frac{x^2+3x-18}{x^2-9}$$
$$=\frac{(x+6)(x-3)}{(x+3)(x-3)}$$
$$=\frac{x+6}{x+3},\quad x\neq\pm3$$

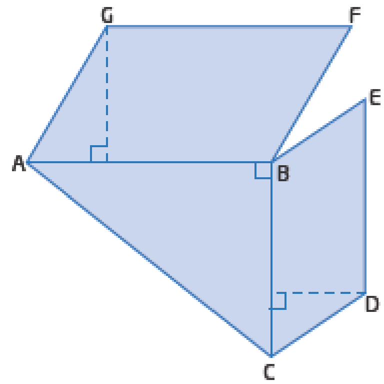
b)
$$\frac{4(x^2-9)^2-(x+3)^2}{x^2+6x+9}=\frac{[2(x^2-9)+(x+3)][2(x^2-9)-(x+3)]}{(x+3)(x+3)}$$
$$=\frac{[2x^2-18+x+3][2x^2-18-x-3]}{(x+3)(x+3)}$$
$$=\frac{[2x^2+x-15][2x^2-x-21]}{(x+3)(x+3)}$$
$$=\frac{(2x-5)\cancel{(x+3)}(2x-7)\cancel{(x+3)}}{\cancel{(x+3)}\cancel{(x+3)}}$$
$$=(2x-5)(2x-7),\quad x\neq-3$$

c) $$\frac{(x^2-x)^2-8(x^2-x)+12}{(x^2-4)^2-(x-2)^2}=\frac{[(x^2-x)-6][(x^2-x)-2]}{[(x^2-4)+(x-2)][(x^2-4)-(x-2)]}$$
$$=\frac{(x^2-x-6)\cancel{(x^2-x-2)}}{(x^2+x-6)\cancel{(x^2-x-2)}}$$
$$=\frac{(x-3)(x+2)}{(x+3)(x-2)},\quad x\neq2,-3,-1$$

d) $$\frac{(x^2+4x+4)^2-10(x^2+4x+4)+9}{(2x+1)^2-(x+2)^2}=\frac{[(x^2+4x+4)-1][(x^2+4x+4)-9]}{[(2x+1)+(x+2)][(2x+1)-(x+2)]}$$
$$=\frac{(x^2+4x+3)(x^2+4x-5)}{(3x+3)(x-1)}$$
$$=\frac{\cancel{(x+1)}(x+3)(x+5)\cancel{(x-1)}}{3\cancel{(x+1)}\cancel{(x-1)}}$$
$$=\frac{(x+3)(x+5)}{3},\quad x\neq\pm1$$

Section 6.1 Page 320 Question 27

Use the formula for area of a parallelogram, $A = bh$, where b is the base and h is the height, to find the base, AB, and height, BC, of $\triangle$ABC.
For parallelogram ABFG,



$$\begin{aligned}\text{AB} &= \frac{16x^2-1}{4x-1}\\ &= \frac{(4x-1)(4x+1)}{(4x-1)}\\ &= 4x+1,\quad x \neq \frac{1}{4}\end{aligned}$$

For parallelogram BCDE,

$$\begin{aligned}\text{BC} &= \frac{6x^2-x-12}{2x-3}\\ &= \frac{(3x+4)(2x-3)}{(2x-3)}\\ &= 3x+4,\quad x \neq \frac{3}{2}\end{aligned}$$

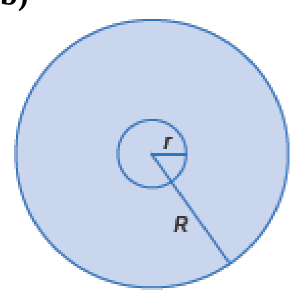
$$\begin{aligned}\text{Then, the area of } \triangle\text{ABC} &= \frac{(4x+1)(3x+4)}{2}\\ &= \frac{12x^2+19x+4}{2}\\ &= 6x^2+\frac{19}{2}x+2\end{aligned}$$

The area of $\triangle$ABC is ($6x^2+\frac{19}{2}x+2$) square units, $x \neq \frac{1}{4}, \frac{3}{2}$.

Section 6.1 Page 321 Question 28

a) Visualize the carpet laid flat: the exposed edge is a rectangle with length is L and thickness is t so its area is Lt.

b)



$$A = \pi R^2 - \pi r^2$$
$$A = \pi(R^2 - r^2)$$
$$A = \pi(R - r)(R + r)$$

c) $L = \frac{A}{t}$

$L = \frac{\pi(R-r)(R+r)}{t},\ t > 0,\ R > r$

Also, *r*, *R*, and *t* should be expressed in the same units.

Section 6.1 Page 321 Question 29

Examples:

a) $\frac{3x}{(x+2)(x-5)},\ x \neq -2,\ 5$

b) $\frac{x(x+3)}{(x-1)(x+3)} = \frac{x^2+3x}{x^2+2x-3},\ x \neq 1, -3$

The given expression has non-permissible value 1, so I multiplied numerator and denominator by $(x + 3)$ to introduce the other non-permissible value, –3.

Section 6.1 Page 321 Question 30

a) Example: Use $y = 1$.

$$\begin{aligned}\frac{y-3}{4} &= \frac{1-3}{4}\\ &= \frac{-2}{4}\\ &= -\frac{1}{2}\end{aligned}$$

$$\begin{aligned}\frac{2y^2-5y-3}{8y+4} &= \frac{2(1)^2-5(1)-3}{8(1)+4}\\ &= \frac{-6}{12}\\ &= -\frac{1}{2}\end{aligned}$$

b)
$$\begin{aligned}\frac{2y^2-5y-3}{8y+4} &= \frac{(2y+1)(y-3)}{4(2y+1)}\\ &= \frac{y-3}{4},\ y \neq -\frac{1}{2}\end{aligned}$$

c) The algebraic approach, in part b), proves that the two expressions are equal. Substituting a particular value does not prove the result for all permissible values.

Section 6.1 Page 321 Question 31

a) Use slope $= \dfrac{\text{vertical change}}{\text{horizontal change}}$.

$$\begin{aligned}\text{slope } &= \frac{p-5-3}{2p+1-p} \\ &= \frac{p-8}{p+1},\ \ p \neq -1\end{aligned}$$

b) Example: If $p = 1$, the slope is $\dfrac{1-8}{1+1} = -\dfrac{7}{2}$.

c) When $p = -1$ the slope is undefined and the line is vertical.

Section 6.1 Page 321 Question 32

Example: To express a fraction in lowest terms you divide the numerator and the denominator by any common factors. The same principle applies when simplifying a rational expression.

$$\begin{aligned}\frac{30}{80} &= \frac{3(10)}{8(10)} \\ &= \frac{3}{8}\end{aligned} \qquad\qquad \begin{aligned}\frac{30x^2+10}{50-80x} &= \frac{10(3x^2+1)}{10(5-8x)} \\ &= \frac{3x^2+1}{5-8x},\ \ x \neq \frac{5}{8}\end{aligned}$$

Section 6.2 Multiplying and Dividing Rational Expressions

Section 6.2 Page 327 Question 1

a) $$\frac{\overset{3}{\cancel{12}}\ \overset{m}{\cancel{m^2}}\ \cancel{f}}{\underset{1}{\cancel{5}}\ \cancel{c}\ \cancel{f}} \times \frac{\overset{3}{\cancel{15}}\ \cancel{c}}{\underset{1}{\cancel{4}}\ \underset{1}{\cancel{m}}} = 9m,\ \ c \neq 0,\ f \neq 0,\ m \neq 0$$

b) $$\frac{\overset{1}{\cancel{3}}\ \overset{1}{\cancel{(a-b)}}}{(a-1)\ \underset{1}{\cancel{(a+5)}}} \times \frac{(a-5)\ \overset{1}{\cancel{(a+5)}}}{\underset{5}{\cancel{15}}\ \underset{1}{\cancel{(a-b)}}} = \frac{a-5}{5(a-1)},\ \ a \neq 1, -5,\ a \neq b$$

c) $$\frac{(y-7)\ \overset{1}{\cancel{(y+3)}}}{(2y-3)\ \underset{1}{\cancel{(2y+3)}}} \times \frac{4\ \overset{1}{\cancel{(2y+3)}}}{\underset{1}{\cancel{(y+3)}}\,(y-1)} = \frac{4(y-7)}{(2y-3)(y-1)},\ \ y \neq \pm\frac{3}{2}, 1, -3$$

Section 6.2 Page 327 Question 2

a) $$\frac{d^2-100}{144}\times\frac{36}{d+10}=\frac{(d-10)\,\overset{1}{\cancel{(d+10)}}}{\underset{4}{\cancel{144}}}\times\frac{\overset{1}{\cancel{36}}}{\underset{1}{\cancel{d+10}}}$$
$$=\frac{d-10}{4},\quad d\neq-10$$

b) $$\frac{a+3}{a+1}\times\frac{a^2-1}{a^2-9}=\frac{\overset{1}{\cancel{a+3}}}{\underset{1}{\cancel{a+1}}}\times\frac{\overset{1}{\cancel{(a+1)}}(a-1)}{(a-3)\,\underset{1}{\cancel{(a+3)}}}$$
$$=\frac{a-1}{a-3},\quad a\neq-1,\pm3$$

c) $$\frac{4z^2-25}{2z^2-13z+20}\times\frac{z-4}{4z+10}=\frac{\overset{1}{\cancel{(2z-5)}}\,\overset{1}{\cancel{(2z+5)}}}{\underset{1}{\cancel{(2z-5)}}\,\underset{1}{\cancel{(z-4)}}}\times\frac{\overset{1}{\cancel{z-4}}}{2\,\underset{1}{\cancel{(2z+5)}}}$$
$$=\frac{1}{2},\quad z\neq\pm\frac{5}{2},4$$

d)
$$\frac{2p^2+5p-3}{2p-3}\times\frac{p^2-1}{6p-3}\times\frac{2p-3}{p^2+2p-3}=\frac{\overset{1}{\cancel{(2p-1)}}\,\overset{1}{\cancel{(p+3)}}}{\underset{1}{\cancel{2p-3}}}\times\frac{\overset{1}{\cancel{(p-1)}}(p+1)}{3\,\underset{1}{\cancel{(2p-1)}}}\times\frac{\overset{1}{\cancel{2p-3}}}{\underset{1}{\cancel{(p+3)}}\,\underset{1}{\cancel{(p-1)}}}$$
$$=\frac{p+1}{3},\quad p\neq-3,\ 1,\ \frac{3}{2},\ \frac{1}{2}$$

Section 6.2 Page 327 Question 3

a) The reciprocal of $\frac{2}{t}$ is $\frac{t}{2}$.

b) The reciprocal of $\frac{2x-1}{3}$ is $\frac{3}{2x-1}$.

c) The reciprocal of $\frac{-8}{3-y}$ is $\frac{3-y}{-8}$ or $\frac{y-3}{8}$.

d) The reciprocal of $\frac{2p-3}{p-3}$ is $\frac{p-3}{2p-3}$.

Section 6.2 Page 327 Question 4

a) $\dfrac{4t^2}{3s} \div \dfrac{2t}{s^2} = \dfrac{4t^2}{3s} \times \dfrac{s^2}{2t}$

The non-permissible values are $s \neq 0$, $t \neq 0$.

b) $\dfrac{r^2 - 7r}{r^2 - 49} \div \dfrac{3r^2}{r+7} = \dfrac{r^2 - 7r}{r^2 - 49} \times \dfrac{r+7}{3r^2}$

$$= \frac{r^2 - 7r}{(r-7)(r+7)} \times \frac{r+7}{3r^2}$$

The non-permissible values are $r \neq 0, \pm 7$.

c) $\dfrac{5}{n+1} \div \dfrac{10}{n^2 - 1} \div (n-1) = \dfrac{5}{n+1} \times \dfrac{n^2 - 1}{10} \times \dfrac{1}{n-1}$

The non-permissible values are $n \neq \pm 1$.

Section 6.2 Page 327 Question 5

$$\frac{2x-6}{x+3} \times \frac{x+3}{2} = \frac{\overset{1}{\cancel{2}}(x-3)}{\underset{1}{\cancel{x+3}}} \times \frac{\overset{1}{\cancel{x+3}}}{\underset{1}{\cancel{2}}}$$

$$= x - 3, \quad x \neq -3$$

Section 6.2 Page 327 Question 6

$$\frac{y^2}{y^2 - 9} \div \frac{y}{y-3} = \frac{\overset{y}{\cancel{y^2}}}{\underset{1}{\cancel{(y-3)}}(y+3)} \times \frac{\overset{1}{\cancel{y-3}}}{\underset{1}{\cancel{y}}}$$

$$= \frac{y}{y+3}, \quad y \neq 0, \pm 3$$

Section 6.2 Page 327 Question 7

a) $\dfrac{3-p}{p-3} = \dfrac{-1\,\overset{1}{\cancel{(-3+p)}}}{\underset{1}{\cancel{p-3}}}$

$$= -1, \quad p \neq 3$$

b) $$\frac{7k-1}{3k}\times\frac{1}{1-7k}=\frac{-1\,\overset{1}{\cancel{(-7k+1)}}}{3k}\times\frac{1}{\underset{1}{\cancel{1-7k}}}$$

$$=\frac{-1}{3k}\text{ or }-\frac{1}{3k},\quad k\neq 0,\frac{1}{7}$$

Section 6.2 Page 327 Question 8

a) $$\frac{2w^2-w-6}{3w+6}\div\frac{2w+3}{w+2}=\frac{\overset{1}{\cancel{(2w+3)}}\,(w-2)}{3\,\underset{1}{\cancel{(w+2)}}}\times\frac{\overset{1}{\cancel{w+2}}}{\underset{1}{\cancel{2w+3}}}$$

$$=\frac{w-2}{3},\quad w\neq -2,-\frac{3}{2}$$

b) $$\frac{v-5}{v}\div\frac{v^2-2v-15}{v^3}=\frac{\overset{1}{\cancel{v-5}}}{\cancel{v}}\times\frac{\overset{v^2}{\cancel{v^3}}}{\underset{1}{\cancel{(v-5)}}\,(v+3)}$$

$$=\frac{v^2}{v+3},\quad v\neq 0,5,-3$$

c) $$\frac{9x^2-1}{x+5}\div\frac{3x^2-5x-2}{2-x}=\frac{(3x-1)\,\overset{1}{\cancel{(3x+1)}}}{x+5}\times\frac{\overset{-1}{\cancel{2-x}}}{\underset{1}{\cancel{(3x+1)}}\,\underset{1}{\cancel{(x-2)}}}$$

$$=\frac{-1(3x-1)}{x+5}\text{ or }\frac{1-3x}{x+5},\quad x\neq 2,-5,-\frac{1}{3}$$

d) $$\frac{8y^2-2y-3}{y^2-1}\div\frac{2y^2-3y-2}{2y-2}\div\frac{3-4y}{y+1}$$

$$=\frac{\overset{-1}{\cancel{(4y-3)}}\,\overset{1}{\cancel{(2y+1)}}}{\underset{1}{\cancel{(y-1)}}\,\underset{1}{\cancel{(y+1)}}}\times\frac{2\,\overset{1}{\cancel{(y-1)}}}{\underset{1}{\cancel{(2y+1)}}\,(y-2)}\times\frac{\overset{1}{\cancel{y+1}}}{\underset{1}{\cancel{3-4y}}}$$

$$=\frac{-2}{y-2}\text{ or }\frac{2}{2-y},\quad y\neq \pm 1,2,-\frac{1}{2},\frac{3}{4}$$

Section 6.2 Page 327 Question 9

$$\frac{x-5}{x+3}\div\frac{x+1}{x-2}=\frac{x-5}{x+3}\times\frac{x-2}{x+1}$$

The non-permissible values in the original expression are –3 and 2. Then, when the division is converted to multiply by the reciprocal, –1 is also non-permissible.

Section 6.2 Page 327 Question 10

Height of stack ÷ number of sheets = thickness of each sheet

$$\frac{n^2-4}{n+1}\div(n-2)=\frac{\overset{1}{\cancel{(n-2)}}(n+2)}{n+1}\times\frac{1}{\underset{1}{\cancel{n-2}}}$$
$$=\frac{n+2}{n+1},\quad n\neq -1, 2$$

An expression for the thickness of one sheet of plywood is $\frac{n+2}{n+1},\ n\neq -1, 2$.

Section 6.2 Page 328 Question 11

a) $\frac{x-3}{5}$ metres per minute; $\frac{x-3}{\cancel{5}}\times\overset{12}{\cancel{60}}=12x-36$ metres in 60 min

The outer edge of the windmill blade travel ($12x$ – 36) metres in 1 h.

b) $\text{speed}=\frac{\text{distance}}{\text{time}}$

$$\text{speed}=900\div\frac{600}{n+1}$$
$$=\overset{3}{\cancel{900}}\times\frac{n+1}{\underset{2}{\cancel{600}}}$$
$$=\frac{3n+3}{2},\quad n\neq -1$$

The average speed of the plane is $\frac{3n+3}{2}$ kilometres per hour, $n\neq -1$.

c) $\text{height}=\frac{\text{volume}}{\text{area of base}}$

$$\text{height}=\frac{x^2+2x+1}{(2x-3)(x+1)}$$
$$=\frac{(x+1)\overset{1}{\cancel{(x+1)}}}{(2x-3)\underset{1}{\cancel{(x+1)}}}$$
$$=\frac{x+1}{2x-3},\quad x\neq -1, \frac{3}{2}$$

An expression for the height of the box, in metres, is $\frac{x+1}{2x-3},\ x\neq -1, \frac{3}{2}$.

Section 6.2 Page 328 Question 12

$$\frac{3m+1}{m-1}\div\frac{3m+1}{m^2-1}=\frac{3m+1}{m-1}\times\frac{(m-1)(m+1)}{3m+1}$$
$$=m+1$$

$$\frac{3m+1}{m^2-1}\div\frac{3m+1}{m-1}=\frac{3m+1}{(m-1)(m+1)}\times\frac{m-1}{3m+1}$$
$$=\frac{1}{m+1}$$

The answers are reciprocals of each other. This is always true. Compare the following general rational expressions:

$$\frac{a}{b}\div\frac{x}{y}=\frac{a}{b}\times\frac{y}{x}=\frac{ay}{bx}$$
$$\frac{x}{y}\div\frac{a}{b}=\frac{x}{y}\times\frac{b}{a}=\frac{bx}{ay}$$

Section 6.2 Page 328 Question 13

Example: $1\ \cancel{\text{yd}}\left(\frac{3\ \cancel{\text{ft}}}{1\ \cancel{\text{yd}}}\right)\left(\frac{12\ \cancel{\text{in.}}}{1\ \cancel{\text{ft}}}\right)\left(\frac{2.54\ \text{cm}}{1\ \cancel{\text{in.}}}\right)=91.44\ \text{cm}$

Section 6.2 Page 328 Question 14

a) Tessa's mistake is in the first step: she took the reciprocal of the dividend, not the divisor.

b) $$\frac{c^2-36}{2c}\div\frac{c+6}{8c^2}=\frac{(c-6)\overset{1}{\cancel{(c+6)}}}{\underset{1}{\cancel{2c}}}\times\frac{\overset{4c}{\cancel{8c^2}}}{\underset{1}{\cancel{c+6}}}$$
$$=4c(c-6)\text{ or }4c^2-24c,\quad c\neq 0,-6$$

c) The correct answer is the reciprocal of Tessa's answer. Taking reciprocals of either factor produces reciprocal answers. See Question 12 above.

Section 6.2 Page 328 Question 15

$$\text{length} = \frac{\text{Area}}{\text{width}}$$

$$\text{length} = \frac{x^2 - 9}{\frac{x^2 - 2x - 3}{x + 1}}$$

$$= \cancel{(x-3)}^{1}(x+3) \times \frac{\cancel{(x+1)}^{1}}{\cancel{(x-3)}_{1}\ \cancel{(x+1)}_{1}}$$

$$= x + 3, \quad x \neq 3, -1$$

An expression for the length is $x + 3$, $x \neq 3, -1$.

Section 6.2 Page 328 Question 16

$$\text{Area } \triangle = \frac{\text{base} \times \text{height}}{2}$$

$$\text{Area } \triangle\text{PQR} = \frac{1}{2} \times \frac{x^2 - 7x - 8}{x^2 - 4} \times \frac{x + 2}{x - 8}$$

$$= \frac{1}{2} \times \frac{\cancel{(x-8)}^{1}(x+1)}{(x-2)\ \cancel{(x+2)}_{1}} \times \frac{\cancel{x+2}^{1}}{\cancel{x-8}_{1}}$$

$$= \frac{x+1}{2(x-2)}, \quad x \neq \pm 2, 8$$

An expression for the area of the triangle is $\frac{x+1}{2(x-2)}$, $x \neq \pm 2, 8$.

Section 6.2 Page 329 Question 17

a) In $K = \frac{P}{2m}$, substitute $m = \frac{h}{w}$.

$$K = \frac{P}{2} \div \frac{h}{w}$$

$$K = \frac{Pw}{2h}, \quad m \neq 0, w \neq 0, h \neq 0$$

b) In $y = \frac{2\pi}{d}$, substitute $d = \frac{x}{r}$.

$$y = \frac{2\pi}{\frac{x}{r}}$$

$$y = 2\pi\left(\frac{r}{x}\right)$$

$$y = \frac{2\pi r}{x}, \quad x \neq 0, r \neq 0, d \neq 0$$

c) Substitute $r = \dfrac{v}{w}$ into $a = w^2 r$.

$$a = w^2\left(\frac{v}{w}\right)$$
$$a = wv, \quad w \neq 0$$

Section 6.2 Page 329 Question 18

If $\dfrac{V_1}{V_2} = \dfrac{T_1}{T_2}$, then $V_1 = \dfrac{V_2 T_1}{T_2}$. Substitute $V_2 = \dfrac{n^2 - 16}{n-1}$, $T_1 = \dfrac{n-1}{3}$ and $T_2 = \dfrac{n+4}{6}$.

$$V_1 = \frac{n^2-16}{n-1} \times \frac{n-1}{3} \div \frac{n+4}{6}$$
$$V_1 = \frac{(n-4)\,\cancelto{1}{(n+4)}}{\cancelto{1}{n-1}} \times \frac{\cancelto{1}{n-1}}{\cancelto{1}{3}} \times \frac{\cancelto{2}{6}}{\cancelto{1}{n+4}}$$
$$V_1 = 2(n-4), \quad n \neq 1, \pm 4$$

Section 6.2 Page 329 Question 19

a) Yes. The product has the difference of squares pattern. When you expand $(x - \sqrt{5})(x + \sqrt{5}) = x^2 - 5$.

b) $$\left(\frac{x+\sqrt{3}}{x^2-3}\right)\left(\frac{x^2-7}{x-\sqrt{7}}\right) = \frac{\cancelto{1}{(x+\sqrt{3})}}{\cancelto{1}{(x+\sqrt{3})}\,(x-\sqrt{3})} \times \frac{\cancelto{1}{(x-\sqrt{7})}\,(x+\sqrt{7})}{\cancelto{1}{x-\sqrt{7}}}$$
$$= \frac{x-\sqrt{7}}{x-\sqrt{3}}$$

c) $$\frac{x^2-7}{x-\sqrt{7}} = \frac{x^2-7}{x-\sqrt{7}} \times \frac{x+\sqrt{7}}{x+\sqrt{7}}$$
$$= \frac{(x^2-7)(x+\sqrt{7})}{x^2-7}$$
$$= x + \sqrt{7}$$

This is the same expression as obtained by factoring $\dfrac{x^2-7}{x-\sqrt{7}}$ in part b).

Section 6.2 Page 329 Question 20

a) In $\dfrac{V^2 \sin x}{2g}$, substitute $V = 85$, $x = 52°$, and $g = 9.8$.

$$\begin{aligned}\frac{V^2 \sin x}{2g} &= \frac{(85)^2 \sin 52°}{2(9.8)} \\ &\approx 290\end{aligned}$$

The canister reaches a height of approximately 290 m.

b) In $\dfrac{V^2 \sin x}{2g}$, substitute $V = \dfrac{x+3}{x-5}$ and $x = 30°$.

$$\begin{aligned}\frac{V^2 \sin x}{2g} &= \frac{\left(\frac{x+3}{x-5}\right)^2 \sin 30°}{2g} \\ &= \frac{(x+3)^2}{(x-5)^2} \times \frac{1}{2} \times \frac{1}{2g} \\ &= \frac{(x+3)^2}{4g(x-5)^2}\end{aligned}$$

In this case, the canister reaches a height of $\dfrac{(x+3)^2}{4g(x-5)^2}$ metres.

Section 6.2 Page 330 Question 21

I agree. In both, you first convert any division to multiplication by multiplying by the reciprocal. Then, you divide numerators and denominators by any common factors.

Example: $\left(\dfrac{2}{3}\right)\left(\dfrac{1}{5}\right) = \dfrac{(2)(1)}{(3)(5)} = \dfrac{2}{15}$ and $\dfrac{2}{3} \div \dfrac{1}{5} = \left(\dfrac{2}{3}\right)\left(\dfrac{5}{1}\right) = \dfrac{10}{3}$

$$\begin{aligned}\frac{(x+2)}{(x+3)} \times \frac{(x+1)}{(x+3)} &= \frac{(x+2)(x+1)}{(x+3)(x+3)} \\ &= \frac{x^2+3x+2}{x^2+6x+9},\ x \neq -3\end{aligned}$$

$$\begin{aligned}\frac{(x+2)}{(x+3)} \div \frac{(x+1)}{(x+3)} &= \frac{(x+2)}{(x+3)} \times \frac{(x+3)}{(x+1)} \\ &= \frac{(x+2)}{(x+1)},\ x \neq -3, -1\end{aligned}$$

Section 6.2 Page 330 Question 22

a) Use slope = $\dfrac{\text{vertical change}}{\text{horizontal change}}$.

$$\begin{aligned}\text{slope } &= \frac{2p+3-(p+1)}{p-1-(2p-5)}\\ &= \frac{2p+3-p-1}{p-1-2p+5}\\ &= \frac{p+2}{4-p}\end{aligned}$$

b) The slopes of perpendicular lines are negative reciprocals. So, slope of line perpendicular to MN is $\dfrac{-1(4-p)}{p+2} = \dfrac{p-4}{p+2}$.

Section 6.2 Page 330 Question 23

a) $\tan \text{B} = \dfrac{b}{a}$

b) $\dfrac{\sin \text{B}}{\cos \text{B}} = \dfrac{\frac{b}{c}}{\frac{a}{c}} = \dfrac{b}{a}$

c) They are the same, $\tan \text{B} = \dfrac{\sin \text{B}}{\cos \text{B}}$.

6.3 Adding and Subtracting Rational Expressions

Section 6.3 Page 336 Question 1

a) $\dfrac{11x}{6} - \dfrac{4x}{6} = \dfrac{7x}{6}$

b) $\dfrac{7}{x} + \dfrac{3}{x} = \dfrac{10}{x}, x \neq 0$

c) $\dfrac{5t+3}{10}+\dfrac{3t+5}{10}=\dfrac{5t+3+3t+5}{10}$

$$=\frac{8t+8}{10}$$

$$=\frac{\overset{1}{\cancel{2}}(4t+4)}{\underset{5}{\cancel{10}}}$$

$$=\frac{4t+4}{5}$$

d) $\dfrac{m^2}{m+1}+\dfrac{m}{m+1}=\dfrac{m^2+m}{m+1}$

$$=\frac{m\,\overset{1}{\cancel{(m+1)}}}{\underset{1}{\cancel{m+1}}}$$

$$=m,\quad m\neq -1$$

e) $\dfrac{a^2}{a-4}-\dfrac{a}{a-4}-\dfrac{12}{a-4}=\dfrac{a^2-a-12}{a-4}$

$$=\frac{(a+3)\,\overset{1}{\cancel{(a-4)}}}{\underset{1}{\cancel{a-4}}}$$

$$=a+3,\quad a\neq 4$$

Section 6.3 Page 336 Question 2

$$\frac{3x-7}{9}+\frac{6x+7}{9}=\frac{3x-7+6x+7}{9}$$

$$=\frac{9x}{9}$$

$$=x$$

Section 6.3 Page 336 Question 3

a) $\dfrac{1}{(x-3)(x+1)}-\dfrac{4}{(x+1)}=\dfrac{1-4(x-3)}{(x-3)(x+1)}$

$$=\frac{1-4x+12}{(x-3)(x+1)}$$

$$=\frac{13-4x}{(x-3)(x+1)},\quad x\neq 3,-1$$

b)
$$\begin{aligned}\frac{x-5}{x^2+8x-20}+\frac{2x+1}{x^2-4}&=\frac{x-5}{(x+10)(x-2)}+\frac{2x+1}{(x-2)(x+2)}\\&=\frac{(x-5)(x+2)+(2x+1)(x+10)}{(x+10)(x-2)(x+2)}\\&=\frac{x^2-3x-10+2x^2+21x+10}{(x+10)(x-2)(x+2)}\\&=\frac{3x^2+18x}{(x+10)(x-2)(x+2)}\\&=\frac{3x(x+6)}{(x+10)(x-2)(x+2)},\quad x\neq-10,\pm2\end{aligned}$$

Section 6.3 Page 336 Question 4

a) Examples for common denominators: 24, 36
LCD = 12

b) Examples for common denominators: $10a^2y^2$, $50a^3y^3$
LCD = $10a^2y^2$

c) Examples for common denominators: $(9 - x^2)(3 + x)$, $9 - x^2$
LCD = $9 - x^2$

Section 6.3 Page 336 Question 5

a)
$$\begin{aligned}\frac{1}{3a}+\frac{2}{5a}&=\frac{5}{5(3a)}+\frac{2(3)}{3(5a)}\\&=\frac{5}{15a}+\frac{6}{15a}\\&=\frac{11}{15a},\quad a\neq0\end{aligned}$$

b)
$$\begin{aligned}\frac{3}{2x}+\frac{1}{6}&=\frac{3(3)}{3(2x)}+\frac{1(x)}{6(x)}\\&=\frac{9}{6x}+\frac{x}{6x}\\&=\frac{9+x}{6x},\quad x\neq0\end{aligned}$$

c) $4-\frac{6}{5x}=\frac{4(5x)}{5x}-\frac{6}{5x}$

$$=\frac{20x}{5x}-\frac{6}{5x}$$

$$=\frac{20x-6}{5x}$$

$$=\frac{2(10x-3)}{5x},\quad x\neq 0$$

d) $\frac{4z}{xy}-\frac{9x}{yz}=\frac{4z(z)}{xy(z)}-\frac{9x(x)}{yz(x)}$

$$=\frac{4z^2}{xyz}-\frac{9x^2}{xyz}$$

$$=\frac{4z^2-9x^2}{xyz}$$

$$=\frac{(2z-3x)(2z+3x)}{xyz},\ \ x\neq 0,\ y\neq 0,\ z\neq 0$$

e)

$$\frac{2s}{5t^2}+\frac{1}{10t}-\frac{6}{15t^3}=\frac{2s(6t)}{5t^2(6t)}+\frac{1(3t^2)}{10t(3t^2)}-\frac{6(2)}{15t^3(2)}$$

$$=\frac{12st}{30t^3}+\frac{3t^2}{30t^3}-\frac{12}{30t^3}$$

$$=\frac{12st+3t^2-12}{30t^3}$$

$$=\frac{\overset{1}{\cancel{3}}(4st+t^2-4)}{\underset{10}{\cancel{30}}\,t^3}$$

$$=\frac{4st+t^2-4}{10t^3},\ \ t\neq 0$$

f) $\frac{6xy}{a^2b}-\frac{2x}{ab^2y}+1=\frac{6xy(by)}{a^2b(by)}-\frac{2x(a)}{(ab^2y)a}+\frac{a^2b^2y}{a^2b^2y}$

$$=\frac{6bxy^2}{a^2b^2y}-\frac{2ax}{a^2b^2y}+\frac{a^2b^2y}{a^2b^2y}$$

$$=\frac{6bxy^2-2ax+a^2b^2y}{a^2b^2y},\quad a\neq 0,\ b\neq 0,\ y\neq 0$$

Section 6.3 Page 336 Question 6

a) $\dfrac{8}{x^2-4}-\dfrac{5}{x+2}=\dfrac{8}{(x-2)(x+2)}-\dfrac{5(x-2)}{(x+2)(x-2)}$

$$=\frac{8-5x+10}{(x-2)(x+2)}$$

$$=\frac{18-5x}{(x-2)(x+2)},\quad x\neq\pm2$$

b) $\dfrac{1}{x^2-x-12}+\dfrac{3}{x+3}=\dfrac{1}{(x-4)(x+3)}+\dfrac{3(x-4)}{(x+3)(x-4)}$

$$=\frac{1+3x-12}{(x-4)(x+3)}$$

$$=\frac{3x-11}{(x-4)(x+3)},\quad x\neq4,-3$$

c) $\dfrac{3x}{x+2}-\dfrac{x}{x-2}=\dfrac{3x(x-2)}{(x+2)(x-2)}-\dfrac{x(x+2)}{(x+2)(x-2)}$

$$=\frac{3x^2-6x-x^2-2x}{(x+2)(x-2)}$$

$$=\frac{2x^2-8x}{(x+2)(x-2)}$$

$$=\frac{2x(x-4)}{(x+2)(x-2)},x\neq\pm2$$

d)

$\dfrac{5}{y+1}-\dfrac{1}{y}-\dfrac{y-4}{y^2+y}=\dfrac{5(y)}{(y+1)y}-\dfrac{1(y+1)}{y(y+1)}-\dfrac{(y-4)}{y(y+1)}$

$$=\frac{5y-y-1-y+4}{y(y+1)}$$

$$=\frac{3y+3}{y(y+1)}$$

$$=\frac{3\,\overset{1}{\cancel{(y+1)}}}{y\,\underset{1}{\cancel{(y+1)}}}$$

$$=\frac{3}{y},\quad y\neq0,-1$$

e)

$$\frac{2h}{h^2-9}+\frac{h}{h^2+6h+9}-\frac{3}{h-3}=\frac{2h}{(h-3)(h+3)}+\frac{h}{(h+3)(h+3)}-\frac{3}{h-3}$$

$$=\frac{2h(h+3)}{(h-3)(h+3)(h+3)}+\frac{h(h-3)}{(h-3)(h+3)(h+3)}-\frac{3(h+3)(h+3)}{(h-3)(h+3)(h+3)}$$

$$=\frac{2h^2+6h+h^2-3h-3h^2-18h-27}{(h-3)(h+3)(h+3)}$$

$$=\frac{-15h-27}{(h-3)(h+3)(h+3)}$$

$$=\frac{-3(5h+9)}{(h-3)(h+3)(h+3)},\quad h\neq\pm3$$

f)

$$\frac{2}{x^2+x-6}+\frac{3}{x^3+2x^2-3x}=\frac{2}{(x+3)(x-2)}+\frac{3}{x(x+3)(x-1)}$$

$$=\frac{2(x)(x-1)}{x(x+3)(x-2)(x-1)}+\frac{3(x-2)}{x(x+3)(x-2)(x-1)}$$

$$=\frac{2x^2-2x+3x-6}{x(x+3)(x-2)(x-1)}$$

$$=\frac{2x^2+x-6}{x(x+3)(x-2)(x-1)}$$

$$=\frac{(2x-3)(x+2)}{x(x+3)(x-2)(x-1)},\quad x\neq0,1,2,-3$$

Section 6.3 Page 336 Question 7

a)

$$\frac{3x+15}{x^2-25}+\frac{4x^2-1}{2x^2+9x-5}=\frac{3\cancel{(x+5)}}{(x-5)\cancel{(x+5)}}+\frac{\cancel{(2x-1)}(2x+1)}{\cancel{(2x-1)}(x+5)}$$

$$=\frac{3(x+5)}{(x-5)(x+5)}+\frac{(2x+1)(x-5)}{(x+5)(x-5)}$$

$$=\frac{3x+15+2x^2-9x-5}{(x-5)(x+5)}$$

$$=\frac{2x^2-6x+10}{(x-5)(x+5)}$$

$$=\frac{2(x^2-3x+5)}{(x-5)(x+5)},\quad x\neq\pm5,\frac{1}{2}$$

b)

$$\begin{aligned}
\frac{2x}{x^3+x^2-6x}-\frac{x-8}{x^2-5x-24} &= \frac{2\cancel{x}}{\cancel{x}(x+3)(x-2)}-\frac{\overset{1}{\cancel{x-8}}}{\cancel{(x-8)}(x+3)} \\
&= \frac{2}{(x+3)(x-2)}-\frac{1(x-2)}{(x+3)(x-2)} \\
&= \frac{2-x+2}{(x+3)(x-2)} \\
&= \frac{4-x}{(x+3)(x-2)}, \quad x \neq 0, 2, -3, 8
\end{aligned}$$

c)

$$\begin{aligned}
\frac{n+3}{n^2-5n+6}+\frac{6}{n^2-7n+12} &= \frac{n+3}{(n-3)(n-2)}+\frac{6}{(n-3)(n-4)} \\
&= \frac{(n+3)(n-4)}{(n-3)(n-2)(n-4)}+\frac{6(n-2)}{(n-3)(n-2)(n-4)} \\
&= \frac{n^2-n-12+6n-12}{(n-3)(n-2)(n-4)} \\
&= \frac{n^2+5n-24}{(n-3)(n-2)(n-4)} \\
&= \frac{(n+8)\cancel{(x-3)}}{\cancel{(n-3)}(n-2)(n-4)} \\
&= \frac{n+8}{(n-2)(n-4)}, \quad n \neq 2, 3, 4
\end{aligned}$$

d)

$$\begin{aligned}
\frac{2w}{w^2+5w+6}-\frac{w-6}{w^2+6w+8} &= \frac{2w}{(w+3)(w+2)}-\frac{w-6}{(w+4)(w+2)} \\
&= \frac{2w(w+4)}{(w+3)(w+2)(w+4)}-\frac{(w-6)(w+3)}{(w+3)(w+2)(w+4)} \\
&= \frac{2w^2+8w-w^2+3w+18}{(w+3)(w+2)(w+4)} \\
&= \frac{w^2+11w+18}{(w+3)(w+2)(w+4)} \\
&= \frac{(w+9)\cancel{(w+2)}}{(w+3)\cancel{(w+2)}(w+4)} \\
&= \frac{w+9}{(w+3)(w+4)}, \quad w \neq -2, -3, -4
\end{aligned}$$

Section 6.3 Page 336 Question 8

In the third line, multiplying by −7 should give $-7x + 14$. Also, Linda has forgotten to list the non-permissible values.

$$\begin{aligned}\frac{6}{x-2}+\frac{4}{x^2-4}-\frac{7}{x+2}&=\frac{6(x+2)+4-7(x-2)}{(x-2)(x+2)}\\&=\frac{6x+12+4-7x+14}{(x-2)(x+2)}\\&=\frac{-x+30}{(x-2)(x+2)},\ x\neq\pm2\end{aligned}$$

Section 6.3 Page 336 Question 9

Yes, the expression can be simplified further. Factor −1 from the numerator then simplify.

$$\begin{aligned}\frac{-x+5}{(x-5)(x+5)}&=\frac{-1\cancel{(x-5)}}{\cancel{(x-5)}(x+5)}\\&=\frac{-1}{x+5},\quad x\neq\pm5\end{aligned}$$

Section 6.3 Page 337 Question 10

a)
$$\begin{aligned}\frac{2-\frac{6}{x}}{1-\frac{9}{x^2}}&=\frac{2x-6}{x}\div\frac{x^2-9}{x^2}\\&=\frac{2\cancel{(x-3)}}{\cancel{x}}\times\frac{\overset{x}{\cancel{x^2}}}{\cancel{(x-3)}(x+3)}\\&=\frac{2x}{x+3},\quad x\neq0,\pm3\end{aligned}$$

b)
$$\begin{aligned}\frac{\frac{3}{2}+\frac{3}{t}}{\frac{t}{t+6}-\frac{1}{t}}&=\frac{3t+6}{2t}\div\frac{t^2-t-6}{t(t+6)}\\&=\frac{3\cancel{(t+2)}}{2\cancel{t}}\times\frac{\cancel{t}(t+6)}{(t-3)\cancel{(t+2)}}\\&=\frac{3(t+6)}{2(t-3)},\quad t\neq0,3,-2,-6\end{aligned}$$

c)

$$\frac{\frac{3}{m}-\frac{3}{2m+3}}{\frac{3}{m^2}+\frac{1}{2m+3}}=\frac{3(2m+3)-3(m)}{m(2m+3)}\div\frac{3(2m+3)+m^2}{m^2(2m+3)}$$

$$=\frac{6m+9-3m}{\cancel{m}\,\cancel{(2m+3)}}\times\frac{\overset{m}{\cancel{m^2}}\,\cancel{(2m+3)}}{6m+9+m^2}$$

$$=\frac{3\,\cancel{(m+3)}}{1}\times\frac{m}{\cancel{(m+3)}\,(m+3)}$$

$$=\frac{3m}{m+3},\quad m\neq 0,-3,-\frac{3}{2}$$

d)

$$\frac{\frac{1}{x+4}+\frac{1}{x-4}}{\frac{x}{x^2-16}+\frac{1}{x+4}}=\frac{(x-4)+(x+4)}{(x-4)(x+4)}\div\frac{x+x-4}{(x-4)(x+4)}$$

$$=\frac{\cancel{2}x}{\cancel{(x-4)}\,\cancel{(x+4)}}\times\frac{\cancel{(x-4)}\,\cancel{(x+4)}}{\cancel{2}(x-2)}$$

$$=\frac{x}{x-2},\quad x\neq\pm4,2$$

Section 6.3 Page 337 Question 11

a)

$$\frac{\frac{AD}{B}+C}{D}=\left(\frac{AD+CB}{B}\right)\div D$$

$$=\left(\frac{AD+CB}{B}\right)\left(\frac{1}{D}\right)$$

$$=\frac{AD+CB}{BD}$$

$$=\frac{AD}{BD}+\frac{CB}{BD}$$

$$=\frac{A}{B}+\frac{C}{D}$$

b) $\left[\dfrac{\left(\dfrac{AB}{D}+C\right)D}{F}+E\right]F=\left(\dfrac{AB}{D}+C\right)D+EF$

$$=AB+CD+EF$$

Section 6.3 Page 337 Question 12

Let h represent the length of the hypotenuse.

$$h^2=\left(\frac{x}{2}\right)^2+\left(\frac{x-1}{4}\right)^2$$
$$h^2=\frac{x^2}{4}+\frac{x^2-2x+1}{16}$$
$$h^2=\frac{4x^2+x^2-2x+1}{16}$$
$$h=\sqrt{\frac{5x^2-2x+1}{16}}$$
$$h=\frac{\sqrt{5x^2-2x+1}}{4}$$

Section 6.3 Page 337 Question 13

a) $\dfrac{200}{m}$ tells the expected number of weeks to gain 200 kg; $\dfrac{200}{m+4}$ tells the number of weeks to gain 200 kg when the calf is on the healthy growth program.

b) $\dfrac{200}{m}-\dfrac{200}{m+4}$

c) $\dfrac{200}{m}-\dfrac{200}{m+4}=\dfrac{200(m+4)-200m}{m(m+4)}$

$$=\frac{800}{m(m+4)},\quad m\neq 0,-4$$

Yes, the simplified expression still represents the difference between the expected and the actual times the calf took to gain 200 kg because the expressions are equivalent.

Section 6.3 Page 337 Question 14

a) At a typing speed of *n* words per minute, the time to type 200 words is $\frac{200}{n}$ minutes.

b) The time to type the three assignments is $\left(\frac{200}{n}+\frac{500}{n}+\frac{1000}{n}\right)$ minutes

c) $\frac{200}{n}+\frac{500}{n}+\frac{1000}{n}=\frac{1700}{n}$. The expression tells the number of minutes to type all three assignments at a typing speed of *n* words per minute.

d) If the speed decreases by 5 words per minute with each new assignment, then the extra time to type them is given by

$$\begin{aligned}\left(\frac{200}{n}+\frac{500}{n-5}+\frac{1000}{n-10}\right)-\frac{1700}{n} &= -\frac{1500}{n}+\frac{500}{n-5}+\frac{1000}{n-10}\\ &= \frac{-1500(n-5)(n-10)+500n(n-10)+1000n(n-5)}{n(n-5)(n-10)}\\ &= \frac{-1500n^2+22\ 500n-75\ 000+500n^2-5000n+1000n^2-5000n}{n(n-5)(n-10)}\\ &= \frac{12\ 500n-75\ 000}{n(n-5)(n-10)}\end{aligned}$$

If the typing speed deceases by 5 words per minute for each new assignment then it will take $\frac{12\ 500n-75\ 000}{n(n-5)(n-10)}$ minutes longer to type the three assignments.

Section 6.3 Page 338 Question 15

a)
$$\begin{aligned}\frac{x-2}{x+5}+\frac{x^2-2x-3}{x^2-x-6}\times\frac{x^2+2x}{x^2-4x} &= \frac{x-2}{x+5}+\frac{\cancel{(x-3)}(x+1)}{\cancel{(x-3)}\cancel{(x+2)}}\times\frac{\cancel{x}\cancel{(x+2)}}{\cancel{x}(x-4)}\\ &= \frac{(x-2)(x-4)}{(x+5)(x-4)}+\frac{(x+1)(x+5)}{(x+5)(x-4)}\\ &= \frac{x^2-6x+8+x^2+6x+5}{(x+5)(x-4)}\\ &= \frac{2x^2+13}{(x+5)(x-4)},\quad x\neq 0,3,4,-2,-5\end{aligned}$$

b) $$\frac{2x^2-x}{x^2+3x}\times\frac{x^2-x-12}{2x^2-3x+1}-\frac{x-1}{x+2}=\frac{\cancel{x}\,\cancel{(2x-1)}}{\cancel{x}\,\cancel{(x+3)}}\times\frac{(x-4)\,\cancel{(x+3)}}{\cancel{(2x-1)}\,(x-1)}-\frac{x-1}{x+2}$$
$$=\frac{(x-4)(x+2)}{(x-1)(x+2)}-\frac{(x-1)(x-1)}{(x-1)(x+2)}$$
$$=\frac{x^2-2x-8-(x^2-2x+1)}{(x-1)(x+2)}$$
$$=\frac{-9}{(x-1)(x+2)},\quad x\neq 0,1,-2,-3,\frac{1}{2}$$

c)

$$\frac{x-2}{x+5}-\frac{x^2-2x-3}{x^2-x-6}\times\frac{x^2+2x}{x^2-4x}=\frac{x-2}{x+5}-\frac{\cancel{(x-3)}\,(x+1)}{\cancel{(x-3)}\,\cancel{(x+2)}}\times\frac{\cancel{x}\,\cancel{(x+2)}}{\cancel{x}\,(x-4)}$$
$$=\frac{(x-2)(x-4)}{(x+5)(x-4)}-\frac{(x+1)(x+5)}{(x+5)(x-4)}$$
$$=\frac{x^2-6x+8-x^2-6x-5}{(x+5)(x-4)}$$
$$=\frac{3-12x}{(x+5)(x-4)}$$
$$=\frac{3(1-4x)}{(x+5)(x-4)},\quad x\neq 0,3,4,-5,-2$$

d) $$\frac{x+1}{x+6}-\frac{x^2-4}{x^2+2x}\div\frac{2x^2+7x+3}{2x^2+x}=\frac{x+1}{x+6}-\frac{(x-2)\,\cancel{(x+2)}}{\cancel{x}\,\cancel{(x+2)}}\times\frac{\cancel{x}\,\cancel{(2x+1)}}{\cancel{(2x+1)}\,(x+3)}$$
$$=\frac{(x+1)(x+3)}{(x+6)(x+3)}-\frac{(x-2)(x+6)}{(x+6)(x+3)}$$
$$=\frac{x^2+4x+3-x^2-4x+12}{(x+6)(x+3)}$$
$$=\frac{15}{(x+6)(x+3)},\quad x\neq 0,-2-3,-6,-\frac{1}{2}$$

Section 6.3 Page 338 Question 16

Let x represent her speed, in kilometres per hour, during the first 20 km.
Then, $x - 2$ represents her speed for the remaining 16 km.
Use the formula $\text{time} = \dfrac{\text{distance}}{\text{speed}}$ to write an expression for the total time.

$$\text{Total time} = \frac{20}{x} + \frac{16}{x-2}$$

An expression for the total time of her bike ride is $\left(\dfrac{20}{x} + \dfrac{16}{x-2}\right)$ hours.

Section 6.3 Page 338 Question 17

Example: Jo runs 1 km/h faster than Sam. Write an expression for how much longer it takes Sam to run 10 km.
Let x represent Sam's running speed. Then, $x + 1$ represents Jo's running speed.
An expression for how much longer it takes Sam to run 10 km is $\left(\dfrac{10}{x} - \dfrac{10}{x+1}\right)$hours.

Section 6.3 Page 338 Question 18

a) Incorrect: $\dfrac{a}{b} - \dfrac{b}{a} = \dfrac{a^2 - b^2}{ab}$. Find the LCD first, do not just combine pieces.

b) Incorrect: $\dfrac{ca - cb}{c + cd} = \dfrac{a+b}{1+d}$. Factor c from the numerator and from the denominator, remembering that $c(1) = c$.

c) Incorrect: $\dfrac{a}{4} - \dfrac{6-b}{4} = \dfrac{a - 6 + b}{4}$. Distribute the subtraction to both terms in the numerator of the second rational expression by first putting the numerator in a bracket.

d) Incorrect: $\dfrac{1}{1 - \frac{a}{b}} = \dfrac{b}{b-a}$. Simplify the denominator first, then divide.

e) Incorrect: $\dfrac{1}{a-b} = \dfrac{-1}{b-a}$. Multiplying both numerator and denominator by −1, which is the same as multiplying the whole expression by 1, changes every term to its opposite.

Section 6.3 Page 338 Question 19

a) Agree. Each term in the numerator is divided by the denominator, and then can be simplified.

b) Disagree. Examples: $\frac{3x+2y}{xy}=\frac{3x}{xy}+\frac{2y}{xy}=\frac{3}{y}+\frac{2}{x}$, but this may not be the original. $\frac{3-y}{y}+\frac{2+x}{x}$ simplifies to the same expression.

Section 6.3 Page 339 Question 20

a) In $R=\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}$ substitute $R_1 = 2$, $R_2 = 3$, and $R_3 = 4$.

$$R=\frac{1}{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}$$

$$R=\frac{1}{\frac{6+4+3}{12}}$$

$$R=\frac{12}{13}$$

The total resistance is $\frac{12}{13}$ ohms.

b) $R=\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}$

$$R=1\div\frac{R_2R_3+R_1R_3+R_1R_2}{R_1R_2R_3}$$

$$R=\frac{R_1R_2R_3}{R_2R_3+R_1R_3+R_1R_2}$$

c) In $R=\frac{R_1R_2R_3}{R_2R_3+R_1R_3+R_1R_2}$ substitute $R_1 = 2$, $R_2 = 3$, and $R_3 = 4$.

$$R=\frac{2(3)(4)}{3(4)+2(4)+2(3)}$$

$$R=\frac{24}{26}$$

$$R=\frac{12}{13}$$

The total resistance is $\frac{12}{13}$ ohms.

d) Example: I prefer the simplified version from part b) because it is not a double fraction.

Section 6.3 Page 339 Question 21

Example:

Arithmetic:	Algebra:
If $\frac{2}{3}=\frac{6}{9}$, then $\frac{2}{3}=\frac{2-6}{3-9}$ $=\frac{-4}{-6}$ $=\frac{2}{3}$	If $\frac{x}{2}=\frac{3x}{6}$, then $\frac{x}{2}=\frac{x-3x}{2-6}$ $=\frac{-2x}{-4}$ $=\frac{x}{2}$

Section 6.3 Page 339 Question 22

a) Use slope = $\frac{\text{vertical change}}{\text{horizontal change}}$.

$$
\begin{aligned}
\text{slope}_{AB} &= \frac{\frac{p}{3}-\frac{2p-3}{4}}{\frac{p-1}{2}-\frac{p}{3}} \\
&= \frac{\frac{4p-3(2p-3)}{12}}{\frac{3(p-1)-2p}{6}} \\
&= \frac{9-2p}{\cancel{12}_{2}} \times \frac{\cancel{6}}{p-3} \\
&= \frac{9-2p}{2(p-3)}, \quad p \neq 3
\end{aligned}
$$

b) When $p = 3$ the slope is undefined, so the line is vertical.

c) When $p < 3$ and p is an integer, the slope is negative. Example: When $p = 2$, $\text{slope}_{AB} = -2.5$.

d) When $p = 4$, the slope is positive, from $p = 5$ to $p = 10$ the slope is always negative.

Section 6.3 Page 339 Question 23

$$\left(\frac{p}{p-x}+\frac{q}{q-x}+\frac{r}{r-x}\right)-\left(\frac{x}{p-x}+\frac{x}{q-x}+\frac{x}{r-x}\right)$$
$$=\left(\frac{p}{p-x}-\frac{x}{p-x}\right)+\left(\frac{q}{q-x}-\frac{x}{q-x}\right)+\left(\frac{r}{r-x}-\frac{x}{r-x}\right)$$
$$=\frac{p-x}{p-x}+\frac{q-x}{q-x}+\frac{r-x}{r-x}$$
$$=1+1+1$$
$$=3$$

Section 6.3 Page 339 Question 24

Examples: $\frac{2}{5}+\frac{1}{5}=\frac{2+1}{5}=\frac{3}{5}$ and $\frac{2}{x}+\frac{1}{x}=\frac{2+1}{x}=\frac{3}{x}$ and

$\frac{2}{5}+\frac{1}{3}=\frac{2(3)+1(5)}{15}=\frac{11}{15}$ $\frac{2}{x}+\frac{1}{y}=\frac{2(y)+1(x)}{xy}=\frac{2y+x}{xy}$

Section 6.3 Page 340 Question 25

a) The student's suggestion is correct. Example: Find the average of $\frac{1}{2}$ and $\frac{3}{4}$.

$$\left(\frac{1}{2}+\frac{3}{4}\right)\div 2=\left(\frac{2+3}{4}\right)\times\left(\frac{1}{2}\right)$$
$$=\frac{5}{8}$$

Halfway between $\frac{1}{2}$ and $\frac{3}{4}$, or $\frac{4}{8}$ and $\frac{6}{8}$, is $\frac{5}{8}$.

b) $\left(\frac{3}{a}+\frac{7}{2a}\right)\div 2=\left(\frac{6+7}{2a}\right)\left(\frac{1}{2}\right)$
$$=\frac{13}{4a},\quad a\neq 0$$

Section 6.3 Page 340 Question 26

Yes. Example: $\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$ and $\frac{1}{2}+\frac{1}{3}=\frac{1}{\frac{6}{5}}=\frac{5}{6}$

$\frac{1}{x}+\frac{1}{y}=\frac{x+y}{xy}$ and $\frac{1}{x}+\frac{1}{y}=\frac{1}{\frac{xy}{x+y}}=\frac{x+y}{xy}$

Section 6.3 Page 340 Question 27

a) $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$

$=\frac{u+v}{uv}$

b) Substitute $u = 80$ and $v = 6.4$.

$\frac{1}{f}=\frac{80+6.4}{80(6.4)}$

$\frac{1}{f}=\frac{86.4}{512}$

$f=\frac{512}{86.4}\approx 5.93$

When $u = 80$ and $v = 6.4$, $f \approx 5.93$ cm.

c) Given that $\frac{1}{f}=\frac{u+v}{uv}$, then $f=\frac{uv}{u+v}$.

Section 6.3 Page 340 Question 28

Step 3 Yes

Step 4a) $A = 2$, $B = 1$

Step 4b) $A = 3$, $B = 3$

Step 5 Always: $\frac{3}{x-4}+\frac{-2}{x-1}=\frac{3(x-1)+-2(x-4)}{(x-4)(x-1)}$

$$=\frac{x+5}{(x-4)(x-1)}$$

Section 6.4 Rational Equations

Section 6.4 Page 348 Question 1

a)

$$\frac{x-1}{3}-\frac{2x-5}{4}=\frac{5}{12}+\frac{x}{6}$$
$$12\left[\frac{x-1}{3}-\frac{2x-5}{4}\right]=12\left[\frac{5}{12}+\frac{x}{6}\right]$$
$$\frac{\overset{4}{\cancel{12}}(x-1)}{\cancel{3}}-\frac{\overset{3}{\cancel{12}}(2x-5)}{\cancel{4}}=\frac{\cancel{12}(5)}{\cancel{12}}+\frac{\overset{2}{\cancel{12}}(x)}{\cancel{6}}$$
$$4(x-1)-3(2x-5)=5+2x$$

b)

$$\frac{2x+3}{x+5}+\frac{1}{2}=\frac{7}{2x+10}$$
$$\frac{2x+3}{x+5}+\frac{1}{2}=\frac{7}{2(x+5)}$$
$$2(x+5)\left[\frac{2x+3}{x+5}+\frac{1}{2}\right]=2(x+5)\left[\frac{7}{2(x+5)}\right]$$
$$2\overset{1}{\cancel{(x+5)}}\left(\frac{2x+3}{\underset{1}{\cancel{x+5}}}\right)+\cancel{2}(x+5)\left(\frac{1}{\cancel{2}}\right)=\cancel{2}\overset{1}{\cancel{(x+5)}}\left(\frac{7}{\cancel{2}\underset{1}{\cancel{(x+5)}}}\right)$$
$$2(2x+3)+(x+5)=7$$

c)

$$\frac{4x}{x^2-9}-\frac{5}{x+3}=2$$
$$\frac{4x}{(x-3)(x+3)}-\frac{5}{x+3}=2$$
$$(x-3)(x+3)\left[\frac{4x}{(x-3)(x+3)}-\frac{5}{x+3}\right]=(x+3)(x-3)2$$
$$\cancel{(x-3)}\cancel{(x+3)}\frac{4x}{\cancel{(x-3)}\cancel{(x+3)}}-(x-3)\cancel{(x+3)}\frac{5}{\cancel{x+3}}=2(x-3)(x+3)$$
$$4x-5(x-3)=2(x-3)(x+3)$$

Section 6.4 Page 348 Question 2

a)
$$\frac{f+3}{2}-\frac{f-2}{3}=2$$
$$\frac{6(f+3)}{2}-\frac{6(f-2)}{3}=6(2)$$
$$3(f+3)-2(f-2)=12$$
$$3f+9-2f+4=12$$
$$f=-1$$

b)
$$\frac{3-y}{3y}+\frac{1}{4}=\frac{1}{2y}$$
$$12y\left(\frac{3-y}{3y}\right)+12y\left(\frac{1}{4}\right)=12y\left(\frac{1}{2y}\right)$$
$$4(3-y)+3y=6$$
$$12-4y+3y=6$$
$$6=y,\quad y\neq 0$$

c)
$$\frac{9}{w-3}-\frac{4}{w-6}=\frac{18}{w^2-9w+18}$$
$$\frac{9}{w-3}-\frac{4}{w-6}=\frac{18}{(w-3)(w-6)}$$
$$(w-3)(w-6)\left(\frac{9}{w-3}-\frac{4}{w-6}\right)=(w-3)(w-6)\left[\frac{18}{(w-3)(w-6)}\right]$$
$$9(w-6)-4(w-3)=18$$
$$9w-54-4w+12=18$$
$$5w=60$$
$$w=12,\quad w\neq 3,6$$

Section 6.4 Page 348 Question 3

a)
$$\frac{6}{t}+\frac{t}{2}=4$$
$$2t\left(\frac{6}{t}+\frac{t}{2}\right)=2t(4)$$
$$12+t^2=8t$$
$$t^2-8t+12=0$$
$$(t-6)(t-2)=0$$
$$t=6 \text{ or } t=2,\quad t\neq 0$$

b)
$$\frac{6}{c-3}=\frac{c+3}{c^2-9}-5$$
$$\frac{6}{c-3}=\frac{\overset{1}{\cancel{c+3}}}{(c-3)\cancel{(c+3)}}-5$$
$$(c-3)\left(\frac{6}{c-3}\right)=(c-3)\left(\frac{1}{c-3}-5\right)$$
$$6=1-5(c-3)$$
$$6=1-5c+15$$
$$5c=10$$
$$c=2,\quad c\neq \pm 3$$

c)

$$\frac{d}{d+4}=\frac{2-d}{d^2+3d-4}+\frac{1}{d-1}$$
$$\frac{d}{d+4}=\frac{2-d}{(d-1)(d+4)}+\frac{1}{d-1}$$
$$(d-1)(d+4)\left(\frac{d}{d+4}\right)=(d-1)(d+4)\left[\frac{2-d}{(d-1)(d+4)}+\frac{1}{d-1}\right]$$
$$(d-1)d=2-d+d+4$$
$$d^2-d-6=0$$
$$(d-3)(d+2)=0$$
$$d=3 \text{ or } d=-2,\ d\neq 1,-4$$

d)

$$\frac{x^2+x+2}{x+1}-x=\frac{x^2-5}{x^2-1}$$
$$\frac{x^2+x+2}{x+1}-x=\frac{x^2-5}{(x-1)(x+1)}$$
$$(x-1)(x+1)\left[\frac{x^2+x+2}{x+1}-x\right]=(x-1)(x+1)\left[\frac{x^2-5}{(x-1)(x+1)}\right]$$
$$(x-1)(x^2+x+2)-(x-1)(x+1)x=x^2-5$$
$$x^3+x^2+2x-x^2-x-2-x^3+x=x^2-5$$
$$0=x^2-2x-3$$
$$0=(x-3)(x+1)$$

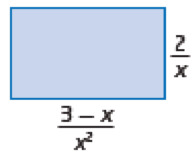
Then, $x=3$, $x\neq\pm1$.

Section 6.4 Page 348 Question 4

The solution for $\frac{-3y}{y-1}+6=\frac{6y-9}{y-1}$ cannot be $y = 1$ because this is a non-permissible value for the equation.

Section 6.4 Page 348 Question 5

a) $$\text{Length} - \text{Width} = \frac{3-x}{x^2} - \frac{2}{x}$$
$$= \frac{3-x-2x}{x^2}$$
$$= \frac{3-3x}{x^2}$$
$$= \frac{3(1-x)}{x^2}, \quad x \neq 0$$



For the width to exist, $x > 0$.

b) Area = length × width
$$= \frac{3-x}{x^2} \times \frac{2}{x}$$
$$= \frac{2(3-x)}{x^3}, \quad x \neq 0, x > 0$$

c) Perimeter = 2(length + width)
$$28 = 2\left[\frac{3-x}{x^2} + \frac{2}{x}\right]$$
$$14 = \frac{3-x+2x}{x^2}$$
$$14x^2 = 3 + x$$
$$14x^2 - x - 3 = 0$$
$$(7x+3)(2x-1) = 0$$
Since x cannot be negative, $x = 0.5$ cm.

Section 6.4 Page 348 Question 6

a)
$$\frac{26}{b+5} = 1 + \frac{3}{b-2}$$
$$(b+5)(b-2)\left(\frac{26}{b+5}\right) = (b+5)(b-2)\left[1 + \frac{3}{b-2}\right]$$
$$26(b-2) = (b+5)(b-2) + 3(b+5)$$
$$26b - 52 = b^2 + 5b - 2b - 10 + 3b + 15$$
$$0 = b^2 - 20b + 57$$
Substitute $a = 1$, $b = -20$, and $c = 57$ into the quadratic formula.
$$b = \frac{-(-20) \pm \sqrt{(-20)^2 - 4(1)(57)}}{2(1)}$$
$$b = \frac{20 \pm \sqrt{172}}{2}$$
$$b \approx 16.56 \text{ or } b \approx 3.44$$

b)

$$\frac{c}{c+2}-3=\frac{-6}{c^2-4}$$

$$\frac{c}{c+2}-3=\frac{-6}{(c-2)(c+2)}$$

$$(c-2)(c+2)\left[\frac{c}{c+2}-3\right]=(c-2)(c+2)\left[\frac{-6}{(c-2)(c+2)}\right]$$

$$c(c-2)-3(c-2)(c+2)=-6$$

$$c^2-2c-3c^2+12=-6$$

$$0=2c^2+2c-18$$

$$c^2+c-9=0$$

Substitute $a = 1$, $b = 1$, and $c = –9$ into the quadratic formula.

$$c=\frac{-1\pm\sqrt{1^2-4(1)(-9)}}{2(1)}$$

$$c=\frac{-1\pm\sqrt{37}}{2}$$

$$c\approx 2.54 \text{ or } c\approx -3.54$$

Section 6.4 Page 348 Question 7

Substitute $w = 30$ into $\frac{l}{w}=\frac{l+w}{l}$.

$$\frac{l}{30}=\frac{l+30}{l}$$

$$l^2=30l+900$$

$$l^2-30l-900=0$$

Substitute $a = 1$, $b = –30$, and $c = –900$ into the quadratic formula.

$$l=\frac{-(-30)\pm\sqrt{(-30)^2-4(1)(-900)}}{2(1)}$$

$$l=\frac{30\pm\sqrt{4500}}{2}$$

$$l=\frac{30\pm30\sqrt{5}}{2}$$

$$l=15\pm15\sqrt{5}$$

$$l=15(1\pm\sqrt{5})$$

The length of the frame is exactly $15(1+\sqrt{5})$ cm or 48.5 cm, to the nearest tenth of a centimetre.

Section 6.4 Page 348 Question 8

Let x represent the first number. Then, the second number will be $25 - x$.

$$\frac{1}{x}+\frac{1}{25-x}=\frac{1}{4}$$
$$4(25-x)+4x=x(25-x)$$
$$100-4x+4x=25x-x^2$$
$$x^2-25x+100=0$$
$$(x-20)(x-5)=0$$

So, $x = 20$ or $x = 5$.
The two numbers are 5 and 20.

Section 6.4 Page 349 Question 9

$$\frac{x+6}{x+1-2}=\frac{9}{2}$$
$$2(x+6)=9(x-1)$$
$$2x+12=9x-9$$
$$21=7x$$
$$x=3$$

The numbers are 3 and 4.

Section 6.4 Page 349 Question 10

Let x represent the number of students expected to go on the trip.

$$\frac{540}{x}+3=\frac{540}{x-6}$$
$$540(x-6)+3x(x-6)=540x$$
$$540x-3240+3x^2-18x=540x$$
$$3x^2-18x-3240=0$$
$$x^2-6x-1080=0$$
$$(x-36)(x+30)=0$$

So, $x = 36$.
Thirty students actually went on the trip.

Section 6.4 Page 349 Question 11

Let n represent the first integer. Then, the next consecutive integer is $n + 1$.

$$\frac{1}{n}+\frac{1}{n+1}=\frac{11}{30}$$
$$30(n+1)+30n=11n(n+1)$$
$$60n+30=11n^2+11n$$
$$11n^2-49n-30=0$$
$$(11n+6)(n-5)=0$$
$$n=-\frac{6}{11} \text{ or } n=5$$

Then, since n represents an integer, the two numbers are 5 and 6.

Section 6.4 Page 349 Question 12

a) With both taps running it should take less than 2 min because more water is going in at the same time.

b)

	Time to Fill Tub (min)	**Fraction Filled in 1 min**	**Fraction Filled in *x* minutes**
Cold Tap	2	$\frac{1}{2}$	$\frac{x}{2}$
Hot Tap	3	$\frac{1}{3}$	$\frac{x}{3}$
Both Taps	x	$\frac{1}{x}$	1

c) $\frac{x}{2}+\frac{x}{3}=1$

d)
$$\frac{x}{2}+\frac{x}{3}=1$$
$$6\left(\frac{x}{2}+\frac{x}{3}\right)=6(1)$$
$$3x+2x=6$$
$$5x=6$$
$$x=1.2$$

The time to fill the tub with both taps running is 1.2 min.

Section 6.4 Page 349 Question 13

	Time to Fill Pool (h)	**Fraction Filled in 1 h**	**Fraction Filled in *x* hours**
Hose A	3	$\frac{1}{3}$	$\frac{x}{3}$
Hose B	x	$\frac{1}{x}$	1
Both Hoses	2	$\frac{1}{2}$	$\frac{x}{2}$

$$\begin{aligned} \frac{x}{2}-\frac{x}{3}&=1 \\ 6\left(\frac{x}{2}-\frac{x}{3}\right)&=6(1) \\ 3x-2x&=6 \\ x&=6 \end{aligned}$$

It would take 6 h to fill the pool using only hose B.

Section 6.4 Page 349 Question 14

a)

	Distance (km)	**Rate (km/h)**	**Time (h)**
Downstream	18	$x+3$	$\frac{18}{x+3}$
Upstream	8	$x-3$	$\frac{8}{x-3}$

b) $\frac{18}{x+3}=\frac{8}{x-3}$

c)
$$\begin{aligned} \frac{18}{x+3}&=\frac{8}{x-3} \\ 18(x-3)&=8(x+3) \\ 18x-54&=8x+24 \\ 10x&=78 \\ x&=7.8 \end{aligned}$$

The rate of the kayakers in still water is 7.8 km/h.

d) $x\neq\pm3$

Section 6.4 Page 350 Question 15

	Time to harvest all (h)	Fraction of harvest done per hour	Fraction of harvest done in *t* hours
Nikita alone	72	$\frac{1}{72}$	$\frac{t}{72}$
Neighbour alone	48	$\frac{1}{48}$	$\frac{t}{48}$
Together	t	$\frac{1}{t}$	1

$$\frac{t}{72}+\frac{t}{48}=1$$
$$48t+72t=(72)(48)$$
$$120t=3456$$
$$t=28.8$$

If they work together, the harvest will take 28.8 h.

Section 6.4 Page 350 Question 16

	Distance (km)	Speed (km/h)	Time (h)
To Forde Lake	70	$x-5$	$\frac{70}{x-5}$
Beyond Forde Lake	60	x	$\frac{60}{x}$

$$\frac{70}{x-5}-4(24)=\frac{60}{x}$$
$$70x-96x(x-5)=60(x-5)$$
$$70x-96x^2+480x=60x-300$$
$$0=96x^2-490x-300$$
$$0=48x^2-245x-150$$
$$x=\frac{-(-245)\pm\sqrt{(-245)^2-4(48)(-150)}}{2(48)}$$
$$x=\frac{245\pm\sqrt{88\,825}}{96}$$
$$x=5.6566...$$

The average speed of the herd beyond Forde Lake was 5.7 km/h, to the nearest tenth of a kilometre per hour.

Section 6.4 Page 350 Question 17

Let x kilometres per hour represent Ted's speed east of Swift Current.

	Distance (km)	Speed (km/h)	Time (h)
West of Swift Current	275	$x - 10$	$\frac{275}{x-10}$
East of Swift Current	300	x	$\frac{300}{x}$

$$\frac{275}{x-10} = \frac{300}{x} + \frac{1}{2}$$
$$2(x)275 = 300(2)(x-10) + x(x-10)$$
$$550x = 600x - 6000 + x^2 - 10x$$
$$0 = x^2 + 40x - 6000$$
$$0 = (x+100)(x-60)$$
$$x = -100 \text{ or } x = 60$$

Ted's average speed east of Swift Current was 60 km/h and west of Swift Current it was 50 km/h.

Section 6.4 Page 350 Question 18

Let x kilometres per hour represent the speed of the current.

	Distance (km)	Speed (km/h)	Time (h)
Up river	2	$6 - x$	$\frac{2}{6-x}$
Down river	2	$6 + x$	$\frac{2}{6+x}$

Use the fact that the total time to paddle up river and back is 1 h to write an equation.

$$\frac{2}{6-x} + \frac{2}{6+x} = 1$$
$$2(6+x) + 2(6-x) = (6+x)(6-x)$$
$$12 + 2x + 12 - 2x = 36 - x^2$$
$$x^2 = 12$$
$$x = \sqrt{12}$$
$$x \approx 3.5$$

The speed of the current is approximately 3.5 km/h.

Section 6.4 Page 350 Question 19

Let x pages per day represent the reading rate for the first half.

	Reading Rate in Pages per Day	**Number of Pages Read**	**Number of Days**
First Half	x	259	$\frac{259}{x}$
Second Half	$x + 12$	259	$\frac{259}{x+12}$

$$\frac{259}{x}+\frac{259}{x+12}=21$$
$$259(x+12)+259x=21x(x+12)$$
$$259x+3108+259x=21x^2+252x$$
$$0=21x^2-266x-3108$$
$$0=3x^2-38x-444$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-(-38)\pm\sqrt{(-38)^2-4(3)(-444)}}{2(3)}$$
$$x=\frac{38\pm\sqrt{6772}}{6}$$
$$x\approx 20$$

The reading rate for the first half of the book is about 20 pages per day.

Section 6.4 Page 350 Question 20

a) For the 30% solution $A = 0.3$. Substitute $A = 0.3$, $s = 1$, and $C = 0.1$.

$$C=\frac{A}{s+w}$$
$$0.1=\frac{0.3}{1+w}$$
$$0.1(1+w)=0.3$$
$$1+w=\frac{0.3}{0.1}$$
$$w=3-1$$
$$w=2$$

To get a 10% solution, 2 L must be added to the 1-L bottle of 30% solution.

b) For a 10% solution, $A = 0.1$. Substitute $A = 0.1$, $s = 0.5$, and $C = 0.02$.

$$C = \frac{A}{s+w}$$
$$0.02 = \frac{0.1}{0.5+w}$$
$$0.02(0.5+w) = 0.1$$
$$0.5+w = \frac{0.1}{0.02}$$
$$w = 5 - 0.5$$
$$w = 4.5$$

To get a 2% solution, 4.5 L must be added to the half-litre bottle of 10% solution.

Section 6.4 Page 350 Question 21

Since $b = \frac{1}{a}$, $a = \frac{1}{b}$. Substitute to eliminate b.

$$\frac{\frac{1}{a} - \frac{1}{b}}{\frac{1}{a} + \frac{1}{b}} = \frac{4}{5}$$
$$\frac{\frac{1}{a} - a}{\frac{1}{a} + a} = \frac{4}{5}$$
$$\frac{1-a^2}{1+a^2} = \frac{4}{5}$$
$$5(1-a^2) = 4(1+a^2)$$
$$5 - 5a^2 = 4 + 4a^2$$
$$1 = 9a^2$$
$$a = \sqrt{\frac{1}{9}}$$
$$a = \pm\frac{1}{3}$$

Section 6.4 Page 351 Question 22

a)
$$\frac{1}{x} = \frac{1}{2}\left(\frac{1}{a} + \frac{1}{b}\right)$$
$$\frac{1}{x} = \frac{1}{2}\left(\frac{b+a}{ab}\right)$$
$$\frac{1}{x} = \frac{b+a}{2ab}$$
$$x = \frac{2ab}{a+b}$$

b)
$$6 = \frac{2a(a+8)}{a+a+8}$$
$$6(2a+8) = 2a^2 + 16a$$
$$12a + 48 = 2a^2 + 16a$$
$$0 = 2a^2 + 4a - 48$$
$$0 = a^2 + 2a - 24$$
$$0 = (a+6)(a-4)$$
So, $a = -6$ or $a = 4$.
The two numbers are –6 and 2, or 4 and 12.

Section 6.4 Page 351 Question 23

a)

$$\frac{1}{x} - \frac{1}{y} = a$$
$$y - x = axy$$
$$y = axy + x$$
$$y = x(ay+1)$$
$$\frac{y}{ay+1} = x$$

or

$$\frac{1}{x} - \frac{1}{y} = a$$
$$\frac{y-x}{xy} = a$$
$$y - x = axy$$
$$y = axy + x$$
$$y = x(ay+1)$$
$$\frac{y}{ay+1} = x$$

In both, $x \neq 0$, $y \neq 0$, $ay \neq -1$.

b)
$$d = v_0 t + \frac{1}{2}gt^2$$
$$d - \frac{1}{2}gt^2 = v_0 t$$
$$\frac{2d - gt^2}{t} = v_0, \quad t \neq 0$$

c)

$$I=\frac{E}{R+\frac{r}{n}}$$

$$R+\frac{r}{n}=\frac{E}{I}$$

$$\frac{r}{n}=\frac{E}{I}-R$$

$$\frac{r}{n}=\frac{E-IR}{I}$$

$$n=\frac{rI}{E-IR},\quad R\neq-\frac{r}{n},n\neq0,I\neq0,E\neq IR$$

Section 6.4 Page 351 Question 24

a) Rational expressions combine operations and variables in one or more terms. Rational equations involve rational expressions and an equal sign.

Example: $\frac{1}{x}+\frac{1}{y}$ is a rational expression, which can be simplified but not solved.

$\frac{1}{x}+\frac{1}{2x}=5$ is a rational equation that can be solved.

b)

$$\frac{5}{x}-\frac{1}{x-1}=\frac{1}{x-1}$$

$$x(x-1)\left(\frac{5}{x}\right)-x(x-1)\left(\frac{1}{x-1}\right)=x(x-1)\left(\frac{1}{x-1}\right)$$ Multiply each term by the LCD.

$$(x-1)(5)-x(1)=x(1)$$ Divide common factors.

$$5x-5-x=x$$ Multiply.

$$3x=5$$ Collect like terms.

$$x=\frac{5}{3}$$ Divide.

c) Example: Add the second term on the left to both sides, to give

$$\frac{5}{x}=\frac{2}{x-1}$$

Section 6.4 Page 351 Question 25

a) Let x represent the number of pages that the ink-jet printer prints per minute. Then, the laser printer prints $x + 24$ pages per minute. Write an equation for the printing done by both in 14 min.

$$\begin{aligned} 14x + 14(x + 24) &= 490 \\ 28x + 336 &= 490 \\ 28x &= 154 \\ x &= 5.5 \end{aligned}$$

The ink-jet printer prints 5.5 pages per minute.

b) Example: I try to use new paper rarely. However, if I consider all the paper entering my life, such as the new phone book each year, flyers, newspapers, and so on, the total is more than I initially thought. I doubt if it is 20 000 pages per year, though.

c) Examples: Re-use paper so both sides are used. Read newspapers and magazines on line rather than getting a hard copy.

Section 6.4 Page 351 Question 26

a) Let n represent your average score for the next 4 quizzes.

Total score on first 6 quizzes = 6(36)

Total score on next 4 quizzes = $4n$

Total score if average is to be 40 on 10 quizzes = 40(10)

$6(36) + 4n = 40(10)$

$216 + 4n = 400$

$4n = 184$

$n = 46$

Your average mark on the next 4 quizzes needs to be 46.

b) $\frac{45}{50}$ is 90%, so $\frac{10(40) + 5(x)}{15} = 45$. For this equation to be true, you would need 55 on each of the remaining quizzes, which is not possible.

Section 6.4 Page 351 Question 27

a) Tyler made an error in the last term on the left of the third step.

$$\begin{aligned}\frac{2}{x-1}-3&=\frac{5x}{x+1}\\ 2(x+1)-3(x-1)(x+1)&=5x(x-1)\\ 2x+2-3x^2+3&=5x^2-5x\\ 0&=8x^2-7x-5\end{aligned}$$

b) Use the quadratic formula with $a = 8$, $b = –7$, $c = –5$.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-(-7)\pm\sqrt{(-7)^2-4(8)(-5)}}{2(8)}$$
$$x=\frac{7\pm\sqrt{209}}{16}$$

c) To the nearest hundredth, $x = 1.34$ or $x = –0.47$.

Chapter 6 Review

Chapter 6 Review Page 352 Question 1

a) $b \neq 0$, because division by zero is not defined.

b) Example: Some rational expressions have non-permissible values.

For $\frac{2}{x-3}$, x may not take on the value 3.

Chapter 6 Review Page 352 Question 2

Agree. Example: There are an unlimited number of ways of expressing 1 and unlimited equivalent expressions can be created by multiplying, or dividing, by 1.

Chapter 6 Review Page 352 Question 3

a) $2y \neq 0$, so $y \neq 0$

b) $x + 1 \neq 0$, so $x \neq -1$

c) The denominator is always 3, so no non-permissible values.

d) $(a-3)(a+2) \neq 0$, so $a \neq -2, 3$

e) $2m^2 - m - 3 \neq 0$
$(m+1)(2m-3) \neq 0$
So $m \neq -1, \dfrac{3}{2}$.

f) $2t^2 - 8 \neq 0$
$2(t-2)(t+2) \neq 0$
So $t \neq \pm 2$.

Chapter 6 Review Page 352 Question 4

a) $$\frac{2s-8s}{s} = \frac{-6s}{s} = -6, \quad s \neq 0$$

b) $$\frac{5x-3}{3-5x} = \frac{-1(3-5x)}{3-5x} = -1, \quad x \neq \frac{3}{5}$$

c) $$\frac{2-b}{4b-8} = \frac{\overset{-1}{\cancel{2-b}}}{4\cancel{(b-2)}} = -\frac{1}{4}, \quad b \neq 2$$

Chapter 6 Review Page 352 Question 5

a) $$\frac{2x(x-3)}{2x(5)} = \frac{2x^2-6x}{10x}, \quad x \neq 0$$

b) $$\frac{x-3}{x^2-9} = \frac{\overset{1}{\cancel{x-3}}}{\underset{1}{\cancel{(x-3)}}(x+3)} = \frac{1}{x+3}, \quad x \neq \pm 3$$

c) $$\frac{c-2d}{3f} = \frac{3(c-2d)}{3(3f)} = \frac{3c-6d}{9f}, \quad f \neq 0$$

d) $$\frac{m+1}{m+4} = \frac{m+1}{m+4} \times \frac{m-4}{m-4} = \frac{m^2-3m-4}{m^2-16}, \quad m \neq \pm 4$$

Chapter 6 Review Page 352 Question 6

a) Factor the denominator(s), set each factor equal to zero and solve.

Example: since $\frac{m-4}{m^2-9}=\frac{m-4}{(m+3)(m-3)}$, the non-permissible values are ±3.

b) i) $$\frac{3x^2-13x-10}{3x+2}=\frac{(3x+2)(x-5)}{3x+2}$$
$$=x-5,\quad x\neq-\frac{2}{3}$$

ii) $$\frac{a^2-3a}{a^2-9}=\frac{a(a-3)}{(a+3)(a-3)}$$
$$=\frac{a}{a+3},\quad a\neq\pm3$$

iii) $$\frac{3y-3x}{4x-4y}=\frac{3\,\overset{-1}{\cancel{(y-x)}}}{4\,\cancel{(x-y)}}$$
$$=-\frac{3}{4},\quad x\neq y$$

iv) $$\frac{81x^2-36x+4}{18x-4}=\frac{(9x-2)(9x-2)}{2(9x-2)}$$
$$=\frac{9x-2}{2},\quad x\neq\frac{2}{9}$$

Chapter 6 Review Page 352 Question 7

a) $$\text{Length}=\frac{\text{Area}}{\text{Width}}$$
$$\text{Length}=\frac{x^2-1}{x-1}$$
$$=\frac{(x+1)(x-1)}{x-1}$$
$$=x+1,\quad x\neq1$$

A simplified expression for the length is $x + 1$.

b) The non-permissible values are $x \neq 1$, as this value would make the width zero, and $x \neq -1$, as this value would make the length zero.

Chapter 6 Review Page 352 Question 8

Example: The same processes are used for rational expressions as for fractions. Multiplying involves finding the product of the numerators and then the product of the denominators. To divide, you multiply by the reciprocal of the divisor. The differences are that rational expressions involve variables and may have non-permissible values.

$\left(\frac{1}{2}\right)\left(\frac{3}{5}\right)=\frac{(1)(3)}{(2)(5)}$ $=\frac{3}{5}$	$\frac{x+2}{2}\times\frac{x+3}{5}=\frac{(x+2)(x+3)}{(2)(5)}$ $=\frac{x^2+5x+6}{10}$
$\frac{3}{4}\div\frac{1}{2}=\left(\frac{3}{4}\right)\left(\frac{2}{1}\right)$ $=\frac{3}{2}$	$\frac{x+2}{4}\div\frac{x+1}{2}=\frac{x+2}{4}\times\frac{2}{x+1}$ $=\frac{x+2}{2(x+1)},\ x\neq -1$

Chapter 6 Review Page 352 Question 9

a) $$\frac{2p}{r}\times\frac{10q}{8p}=\frac{\overset{1}{\cancel{2}}\,\overset{1}{\cancel{p}}}{r}\times\frac{\overset{5}{\cancel{10}}\,q}{\underset{\underset{2}{\cancel{4}}}{\cancel{8}}\,\cancel{p}}$$

$$=\frac{5q}{2r},\quad p\neq 0,\ r\neq 0$$

b) $$4m^3t\times\frac{1}{16mt^4}=\cancel{4}\,\overset{m^2}{\cancel{m^3}}\,\cancel{t}\times\frac{1}{\underset{4}{\cancel{16}}\,\cancel{m}\,\underset{t^3}{\cancel{t^4}}}$$

$$=\frac{m^2}{4t^3},\quad m\neq 0,\ t\neq 0$$

c) $$\frac{3a+3b}{8}\times\frac{4}{a+b}=\frac{3(a+b)}{\underset{2}{\cancel{8}}}\times\frac{\overset{1}{\cancel{4}}}{a+b}$$

$$=\frac{3}{2},\quad a\neq -b$$

d) $\dfrac{x^2-4}{x^2+25}\times\dfrac{2x^2+10x}{x^2+2x}=\dfrac{(x-2)\overset{1}{\cancel{(x+2)}}}{x^2+25}\times\dfrac{2\overset{1}{\cancel{x}}(x+5)}{\underset{1}{\cancel{x}}\underset{1}{\cancel{(x+2)}}}$

$$=\frac{2(x-2)(x+5)}{x^2+25},\quad x\neq 0,-2$$

e) $\dfrac{d^2+3d+2}{2d+2}\times\dfrac{2d+6}{d^2+5d+6}=\dfrac{\overset{1}{\cancel{(d+2)}}\overset{1}{\cancel{(d+1)}}}{\underset{1}{\cancel{2}}\underset{1}{\cancel{(d+1)}}}\times\dfrac{\overset{1}{\cancel{2}}\overset{1}{\cancel{(d+3)}}}{\underset{1}{\cancel{(d+3)}}\underset{1}{\cancel{(d+2)}}}$

$$=1,\quad d\neq -1,-2,-3$$

f) $\dfrac{y^2-8y-9}{y^2-10y+9}\times\dfrac{y^2-9y+8}{y^2-1}\times\dfrac{y^2-25}{5-y}=\dfrac{\cancel{(y-9)}\cancel{(y+1)}}{\cancel{(y-9)}(y-1)}\times\dfrac{(y-8)\cancel{(y-1)}}{\cancel{(y+1)}\cancel{(y-1)}}\times\dfrac{\overset{-1}{\cancel{(y-5)}}(y+5)}{\cancel{5-y}}$

$$=\frac{-(y-8)(y+5)}{y-1},\quad y\neq \pm 1,5,9$$

Chapter 6 Review Page 353 Question 10

a) $2t\div\dfrac{1}{4}=2t\times\dfrac{4}{1}$

$$=8t$$

b) $\dfrac{a^3}{b^4}\div\dfrac{a^3}{b^3}=\dfrac{a^3}{b^4}\times\dfrac{b^3}{a^3}$

$$=\frac{1}{b},\quad a\neq 0,\ b\neq 0$$

c) $\dfrac{7}{x^2-y^2}\div\dfrac{-35}{x-y}=\dfrac{\overset{1}{\cancel{7}}}{\cancel{(x-y)}(x+y)}\times\dfrac{\overset{1}{\cancel{x-y}}}{\underset{-5}{\cancel{-35}}}$

$$=\frac{-1}{5(x+y)},\quad x\neq \pm y$$

d) $\dfrac{3a+9}{a-3}\div\dfrac{a^2+6a+9}{a-3}=\dfrac{3\cancel{(a+3)}}{\underset{1}{\cancel{a-3}}}\times\dfrac{\overset{1}{\cancel{a-3}}}{\cancel{(a+3)}(a+3)}$

$$=\frac{3}{a+3},\quad a\neq \pm 3$$

e) $\dfrac{3x-2}{x^3+3x^2+2x} \div \dfrac{9x^2-4}{3x^2+8x+4} \div \dfrac{1}{x} = \dfrac{\cancel{3x-2}}{\cancel{x}(x^2+3x+2)} \times \dfrac{\cancel{(3x+2)}(x+2)}{\cancel{(3x+2)}\cancel{(3x-2)}} \times \dfrac{\cancel{x}}{1}$

$$= \frac{1}{(x+2)(x+1)} \times \frac{x+2}{1}$$

$$= \frac{1}{x+1}, \quad x \neq 0, \pm\frac{2}{3}, -1, -2$$

f) $\dfrac{\dfrac{4-x^2}{6}}{\dfrac{x-2}{2}} = \dfrac{4-x^2}{6} \times \dfrac{2}{x-2}$

$$= \frac{(2+x)\overset{-1}{\cancel{(2-x)}}}{\underset{3}{\cancel{6}}} \times \frac{\overset{1}{\cancel{2}}}{\cancel{(x-2)}}$$

$$= \frac{-(2+x)}{3}, \quad x \neq 2$$

Chapter 6 Review Page 353 Question 11

a) $\dfrac{9}{2m} \div \dfrac{3}{m} \times \dfrac{m}{3} = \dfrac{9}{2\cancel{m}} \times \dfrac{\cancel{m}}{3} \times \dfrac{m}{3}$

$$= \frac{m}{2}, \quad m \neq 0$$

b) $\dfrac{x^2-3x+2}{x^2-4} \times \dfrac{x+3}{x^2+3x} \div \dfrac{1}{x+2} = \dfrac{\overset{1}{\cancel{(x-2)}}(x-1)}{\underset{1}{\cancel{(x-2)}}\underset{1}{\cancel{(x+2)}}} \times \dfrac{\overset{1}{\cancel{x+3}}}{x\cancel{(x+3)}} \times \dfrac{\overset{1}{\cancel{x+2}}}{1}$

$$= \frac{x-1}{x}, \quad x \neq 0, \pm 2, -3$$

c) $\dfrac{a-3}{a-4} \div \dfrac{30}{a+3} \times \dfrac{5a-20}{a^2-9} = \dfrac{\overset{1}{\cancel{a-3}}}{\underset{1}{\cancel{a-4}}} \times \dfrac{\overset{1}{\cancel{a+3}}}{\underset{6}{\cancel{30}}} \times \dfrac{\cancel{5}\overset{1}{\cancel{(a-4)}}}{\underset{1}{\cancel{(a-3)}}\underset{1}{\cancel{(a+3)}}}$

$$= \frac{1}{6}, \quad a \neq \pm 3, 4$$

d) $$\frac{3x+12}{3x^2-5x-12}\times\frac{x-3}{x+4}\div\frac{15}{3x+4}=\frac{\overset{1}{\cancel{3}}\,\overset{1}{\cancel{(x+4)}}}{\underset{1}{\cancel{(3x+4)}}\,\underset{1}{\cancel{(x-3)}}}\times\frac{\overset{1}{\cancel{x-3}}}{\underset{1}{\cancel{x+4}}}\times\frac{\overset{1}{\cancel{3x+4}}}{\underset{5}{\cancel{15}}}$$
$$=\frac{1}{5},\quad x\neq 3,-4,-\frac{4}{3}$$

Chapter 6 Review Page 353 Question 12

$$\text{Height}=\frac{\text{Volume}}{\text{length}\times\ \text{width}}$$
$$\begin{aligned}\text{Height}&=\frac{2x^3+5x^2-12x}{(2x-3)(x+4)}\\&=\frac{x(2x^2+5x-12)}{(2x-3)(x+4)}\\&=\frac{x(2x-3)(x+4)}{(2x-3)(x+4)}\\&=x\end{aligned}$$

An expression for the height of the prism is *x* centimetres.

Chapter 6 Review Page 353 Question 13

Example: The advantage of using the LCD is that less simplifying needs to be done.

a) LCD is $10x$

b) LCD is $(x-2)(x+1)$

Chapter 6 Review Page 353 Question 14

a) $\frac{m}{5}+\frac{3}{5}=\frac{m+3}{5}$

b) $$\begin{aligned}\frac{2m}{x}-\frac{m}{x}&=\frac{2m-m}{x}\\&=\frac{m}{x},\quad x\neq 0\end{aligned}$$

c) $$\begin{aligned}\frac{x}{x+y}+\frac{y}{x+y}&=\frac{x+y}{x+y}\\&=1,\quad x\neq -y\end{aligned}$$

d) $$\begin{aligned}\frac{x-2}{3}-\frac{x+1}{3}&=\frac{x-2-(x+1)}{3}\\&=\frac{-3}{3}\\&=-1\end{aligned}$$

e) $$\frac{x}{x^2-y^2}-\frac{y}{y^2-x^2}=\frac{x}{(x-y)(x+y)}-\frac{y}{(y-x)(y+x)}$$
$$=\frac{x-(-1)y}{(x-y)(x+y)}$$
$$=\frac{x+y}{(x-y)(x+y)}$$
$$=\frac{1}{x-y},\quad x\neq\pm y$$

Chapter 6 Review Page 353 Question 15

a) $$\frac{4x-3}{6}-\frac{x-2}{4}=\frac{2(4x-3)-3(x-2)}{12}$$
$$=\frac{8x-6-3x+6}{12}$$
$$=\frac{5x}{12}$$

b) $$\frac{2y-1}{3y}+\frac{y-2}{2y}-\frac{y-8}{6y}=\frac{2(2y-1)}{6y}+\frac{3(y-2)}{6y}-\frac{y-8}{6y}$$
$$=\frac{4y-2+3y-6-y+8}{6y}$$
$$=\frac{6y}{6y}$$
$$=1,\quad y\neq 0$$

c) $$\frac{9}{x-3}+\frac{7}{x^2-9}=\frac{9(x+3)}{(x-3)(x+3)}+\frac{7}{(x-3)(x+3)}$$
$$=\frac{9x+27+7}{(x-3)(x+3)}$$
$$=\frac{9x+34}{(x-3)(x+3)},\quad x\neq\pm 3$$

d) $\dfrac{a}{a+3}-\dfrac{a^2-3a}{a^2+a-6}=\dfrac{a}{a+3}-\dfrac{a(a-3)}{(a+3)(a-2)}$

$$\begin{aligned}&=\frac{a(a-2)-a(a-3)}{(a+3)(a-2)}\\&=\frac{a^2-2a-a^2+3a}{(a+3)(a-2)}\\&=\frac{a}{(a+3)(a-2)},\quad a\neq 2,-3\end{aligned}$$

e)

$\dfrac{a}{a-b}-\dfrac{2ab}{a^2-b^2}+\dfrac{b}{a+b}=\dfrac{a}{a-b}-\dfrac{2ab}{(a-b)(a+b)}+\dfrac{b}{a+b}$

$$\begin{aligned}&=\frac{a(a+b)-2ab+b(a-b)}{(a-b)(a+b)}\\&=\frac{a^2+ab-2ab+ab-b^2}{(a-b)(a+b)}\\&=\frac{a^2-b^2}{a^2-b^2}\\&=1,\quad a\neq \pm b\end{aligned}$$

f) $\dfrac{2x}{4x^2-9}+\dfrac{x}{2x^2+5x+3}-\dfrac{1}{2x-3}=\dfrac{2x}{(2x-3)(2x+3)}+\dfrac{x}{(2x+3)(x+1)}-\dfrac{1}{2x-3}$

$$\begin{aligned}&=\frac{2x(x+1)+x(2x-3)-(2x+3)(x+1)}{(2x-3)(2x+3)(x+1)}\\&=\frac{2x^2+2x+2x^2-3x-2x^2-5x-3}{(2x-3)(2x+3)(x+1)}\\&=\frac{2x^2-6x-3}{(2x-3)(2x+3)(x+1)},\quad x\neq \pm\frac{3}{2},-1\end{aligned}$$

Chapter 6 Review Page 353 Question 16

a) $\dfrac{1}{a}+\dfrac{1}{b}=\dfrac{a+b}{ab}$

b) $$\begin{aligned}\text{Left Side}&=\frac{1}{a}+\frac{1}{b}\\&=\frac{b}{ab}+\frac{a}{ab}\\&=\frac{a+b}{ab}\\&=\text{Right Side}\end{aligned}$$

Chapter 6 Review Page 354 Question 17

Exam mark, $d = \dfrac{a+b+c}{3}$;

Final mark $= \left(\dfrac{1}{2}\right)\left(\dfrac{a+b+c}{3}\right) + \left(\dfrac{1}{2}\right)d$

$= \dfrac{a+b+c+3d}{6}$

Example:

$\dfrac{60+70+80}{3} = d$

$\dfrac{60+70+80+3(70)}{6} = 70$

Chapter 6 Review Page 354 Question 18

a) i) $c + 10$ represents the amount that Beth spends per chair, $10 more per chair than she planned

ii) $c - 10$ represents the amount that Helen spends per chair, $10 less per chair than planned

iii) $\dfrac{200}{c-10}$ represents the number of chairs Helen bought

iv) $\dfrac{250}{c+10}$ represents the number of chairs Beth bought

v) $\dfrac{200}{c-10} + \dfrac{250}{c+10}$ represents the total number of chairs purchased by the two sisters

b) $\dfrac{450c-500}{c^2-100}$ or $\dfrac{50(9c-10)}{(c-10)(c+10)}$, $c \neq \pm 10$

Chapter 6 Review Page 354 Question 19

Example: When solving a rational equation, you multiply all terms by the LCD to eliminate the denominators. In addition and subtraction of rational expressions, you use a LCD to simplify by grouping terms over one denominator.

Add or subtract.	**Solve.**
$\dfrac{x}{3} + \dfrac{x}{2}$ $= \dfrac{2x}{6} + \dfrac{3x}{6}$ $= \dfrac{5x}{6}$	$\dfrac{x}{3} + \dfrac{x}{2} = 5$ $2x + 3x = 30$ $5x = 30$ $x = 6$

Chapter 6 Review Page 354 Question 20

a) $\dfrac{s-3}{s+3}=2$

$$\begin{aligned} s-3&=2(s+3)\\ s-3&=2s+6\\ s&=-9,\quad s\neq -3 \end{aligned}$$

b)

$$\begin{aligned} \frac{x+2}{3x+2}&=\frac{x+3}{x-1}\\ (x+2)(x-1)&=(x+3)(3x+2)\\ x^2+x-2&=3x^2+11x+6\\ 0&=2x^2+10x+8\\ 0&=2(x^2+5x+4)\\ 0&=(x+4)(x+1)\\ x&=-4 \text{ or } x=-1,\quad x\neq 1,-\frac{2}{3} \end{aligned}$$

c)

$$\begin{aligned} \frac{z-2}{z}+\frac{1}{5}&=\frac{-4}{5z}\\ \frac{5(z-2)+z}{5z}&=\frac{-4}{5z}\\ 5z-10+z&=-4\\ 6z&=6\\ z&=1,\quad z\neq 0 \end{aligned}$$

d)

$$\begin{aligned} \frac{3m}{m-3}+2&=\frac{3m-1}{m+3}\\ 3m(m+3)+2(m-3)(m+3)&=(3m-1)(m-3)\\ 3m^2+9m+2m^2-18&=3m^2-10m+3\\ 2m^2+19m-21&=0\\ (2m+21)(m-1)&=0\\ m&=-\frac{21}{2} \text{ or } m=1,\quad m\neq \pm 3 \end{aligned}$$

e) $\dfrac{x}{x-3}=\dfrac{3}{x-3}-3$

$$x=3-3(x-3)$$
$$x=3-3x+9$$
$$4x=12$$
$$x=3,\quad x\neq 3$$

There is no solution, since $x = 3$ is non-permissible**.**

f) $\dfrac{x-2}{2x+1}=\dfrac{1}{2}+\dfrac{x-3}{2x}$

$$(2x)(x-2)=x(2x+1)+(x-3)(2x+1)$$
$$2x^2-4x=2x^2+x+2x^2-5x-3$$
$$0=2x^2-3$$
$$x=\pm\sqrt{\frac{3}{2}}\text{ or }\pm\frac{\sqrt{6}}{2},\quad x\neq 0,-\frac{1}{2}$$

g) $\dfrac{3}{x+2}+\dfrac{5}{x-3}=\dfrac{3x}{x^2-x-6}-1$

$$\frac{3}{x+2}+\frac{5}{x-3}=\frac{3x}{(x-3)(x+2)}-1$$
$$3(x-3)+5(x+2)=3x-(x-3)(x+2)$$
$$3x-9+5x+10=3x-x^2+x+6$$
$$x^2+4x-5=0$$
$$(x+5)(x-1)=0$$
$$x=-5\text{ or }x=1,\quad x\neq 3,-2$$

Chapter 6 Review Page 354 Question 21

Let n represent the first number. Then, $12 - n$ represents the other number.

$$\frac{1}{n}+\frac{1}{12-n}=\frac{3}{8}$$
$$8(12-n)+8n=3n(12-n)$$
$$96-8n+8n=36n-3n^2$$
$$3n^2-36n+96=0$$
$$n^2-12n+32=0$$
$$(n-8)(n-4)=0$$

So, $n = 8$ or $n = 4$.

The two numbers are 4 and 8.

Chapter 6 Review Page 354 Question 22

Let x hours represent the time it would take Elaine to paint the room by herself.

	Time to Paint Room (h)	**Fraction Painted in 1 h**	**Fraction Painted in *x* hours**
Matt alone	5	$\frac{1}{5}$	$\frac{x}{5}$
Elaine alone	x	$\frac{1}{x}$	1
Working together	3	$\frac{1}{3}$	$\frac{x}{3}$

$$\frac{x}{3}-\frac{x}{5}=1$$
$$5x-3x=15$$
$$2x=15$$
$$x=7.5$$

It would take Elaine 7.5 h to paint the room by herself.

Chapter 6 Review Page 354 Question 23

a) Let x metres per second represent the speed of the elevator on the way up. Then, the speed as the elevator descends is $x + 0.7$ metres per second.

	Distance (m)	**Speed (m/s)**	**Time (s)**
Going Up	160	x	$\frac{160}{x}$
Descending	160	$x + 0.7$	$\frac{160}{x+0.7}$

$$\frac{160}{x}+36+\frac{160}{x+0.7}=2.5(60)$$

b)
$$\frac{160}{x}+36+\frac{160}{x+0.7}=2.5(60)$$
$$\frac{160}{x}+\frac{160}{x+0.7}=150-36$$
$$160(x+0.7)+160x=x(x+0.7)(114)$$
$$160x+112+160x=114x^2+79.8x$$
$$0=114x^2-240.2x-112$$
$$0=570x^2-1201x-560$$

Use the quadratic formula with $a = 570$, $b = -1201$, and $c = -560$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-1201) \pm \sqrt{(-1201)^2 - 4(570)(-560)}}{2(570)}$$
$$x = \frac{1201 \pm \sqrt{2\ 719\ 201}}{1140}$$
$$x = 2.5$$

The rate of ascent is 2.5 m/s.

c) $2.5 \text{ m/s} = \dfrac{2.5(60)(60)}{1000} \text{ km/h}$. The rate of ascent is 9 km/h.

Chapter 6 Practice Test

Chapter 6 Practice Test Page 355 Question 1

For $\dfrac{x(x+2)}{(x-3)(x+1)}$, $x \neq 3, -1$.

Answer **D** is correct.

Chapter 6 Practice Test Page 355 Question 2

$$\begin{aligned}\frac{x^2 - 7x + 6}{x^2 - 2x - 24} &= \frac{(x-1)(x-6)}{(x-6)(x+4)} \\ &= \frac{x-1}{x+4}\end{aligned}$$

Answer **B** is correct.

Chapter 6 Practice Test Page 355 Question 3

$$\begin{aligned}\frac{8}{3y} + \frac{5y}{4} - \frac{5}{8} &= \frac{8(8) + 5y(6y) - 5(3y)}{24y} \\ &= \frac{64 + 30y^2 - 15y}{24y}\end{aligned}$$

Answer **A** is correct.

Chapter 6 Practice Test Page 355 Question 4

$$\frac{3x-12}{9x^2} \div \frac{x-4}{3x} = \frac{\cancel{3}^{1}\,\cancel{(x-4)}^{1}}{\cancel{9}_{3}\,\cancel{x^2}_{x}} \times \frac{\cancel{3}^{1}\,\cancel{x}}{\cancel{x-4}}$$
$$= \frac{1}{x}$$

Answer **A** is correct.

Chapter 6 Practice Test Page 355 Question 5

$$\begin{aligned}\frac{6}{t-3} &= \frac{4}{t+4}\\ 6(t+4) &= 4(t-3)\\ 6t+24 &= 4t-12\\ 2t &= -36\\ t &= -18\end{aligned}$$

Answer **D** is correct.

Chapter 6 Practice Test Page 355 Question 6

$$\frac{3x-5}{x^2-9} \times \frac{2x-6}{3x^2-2x-5} \div \frac{x-3}{x+3} = \frac{3x-5}{(x-3)(x+3)} \times \frac{2(x-3)}{(3x-5)(x+1)} \times \frac{x+3}{x-3}$$

The non-permissible values are $x \neq \pm 3, -1, \frac{5}{3}$.

Chapter 6 Practice Test Page 355 Question 7

$$\begin{aligned}\frac{2x^2+kx-10}{2x^2+7x+6} &= \frac{2x-5}{2x+3}\\ \frac{2x^2+kx-10}{(2x+3)(x+2)} &= \frac{2x-5}{2x+3}\\ 2x^2+kx-10 &= (2x-5)(x+2)\\ 2x^2+kx-10 &= 2x^2-x-10\end{aligned}$$

Therefore, $k = -1$.

Chapter 6 Practice Test Page 355 Question 8

$$\frac{5y}{6}+\frac{1}{y-2}-\frac{y+1}{3y-6}$$
$$=\frac{5y}{6}+\frac{1}{y-2}-\frac{y+1}{3(y-2)}$$
$$=\frac{5y(y-2)+6-2(y+1)}{6(y-2)}$$
$$=\frac{5y^2-10y+6-2y-2}{6(y-2)}$$
$$=\frac{5y^2-12y+4}{6(y-2)}$$
$$=\frac{(5y-2)(y-2)}{6(y-2)}$$
$$=\frac{5y-2}{6},\ \ y\neq 2$$

Chapter 6 Practice Test Page 355 Question 9

Let x represent the time for the smaller auger to fill the bin.

$$\frac{6}{x}+\frac{6}{x-5}=1$$

Chapter 6 Practice Test Page 355 Question 10

Example: For both you use a LCD. When solving, you multiply by the LCD to eliminate the denominators, while in addition and subtraction of rational expressions, you use the LCD to group terms over a single denominator.

Add or subtract.	**Solve.**
$\frac{x}{4}-\frac{x}{7}$ $=\frac{7x}{28}-\frac{4x}{28}$ $=\frac{3x}{28}$	$\frac{x}{5}+\frac{x}{3}=16$ $15\left(\frac{x}{5}\right)+15\left(\frac{x}{3}\right)=15(16)$ $3x+5x=240$ $8x=240$ $x=30$

Chapter 6 Practice Test Page 355 Question 11

$$2-\frac{5}{x^2-x-6}=\frac{x+3}{x+2}$$
$$2-\frac{5}{(x-3)(x+2)}=\frac{x+3}{x+2}$$
$$2(x-3)(x+2)-5=(x+3)(x-3)$$
$$2x^2-2x-12-5=x^2-9$$
$$x^2-2x-8=0$$
$$(x-4)(x+2)=0, \quad x\neq 3,-2$$

Therefore the solution is $x = 4$.

Chapter 6 Practice Test Page 355 Question 12

Use the fact that the difference between successive terms of an arithmetic sequence is constant.

$$\frac{5x+3}{5x}-\frac{2x-1}{2x}=\frac{2x-1}{2x}-\frac{3-x}{x}$$
$$2(5x+3)-5(2x-1)=5(2x-1)-10(3-x)$$
$$10x+6-10x+5=10x-5-30+10x$$
$$46=20x$$
$$x=2.3$$

Chapter 6 Practice Test Page 355 Question 13

Let x kilometres per hour represent the speed of the plane in calm air.

Winnipeg to Calgary	**Distance (km)**	**Speed (km/h)**	**Time (h)**
Against Strong Headwind	1200	$x - 50$	$\frac{1200}{x-50}$
In Calm Air	1200	x	$\frac{1200}{x}$

$$\frac{1200}{x-50} = \frac{1200}{x} + \frac{1}{2}$$

$$2x(1200) = 2(1200)(x-50) + x(x-50)$$

$$2400x = 2400x - 120\ 000 + x^2 - 50x$$

$$0 = x^2 - 50x - 120\ 000$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-50) \pm \sqrt{(-50)^2 - 4(1)(-120\ 000)}}{2(1)}$$

$$x = \frac{50 \pm \sqrt{482\ 500}}{2}$$

$$x \approx 372$$

The speed of the plane in calm air is 372 km/h, to the nearest kilometre per hour.