Chapter 2 Trigonometry

Section 2.1 Angles in Standard Position

Section 2.1 Page 83 Question 1

a) No; angle θ is not in standard position because its vertex is not at the origin.

b) Yes; angle θ is in standard position because its initial arm is on the positive *x*-axis and the vertex is at the origin.

c) No; angle θ is not in standard position because its initial arm is not on the positive *x*-axis.

d) Yes; angle θ is in standard position because its initial arm is on the positive *x*-axis and the vertex is at the origin.

Section 2.1 Page 83 Question 2

a) Diagram **F** shows 150°.

b) Diagram **C** shows 180°.

c) Diagram **A** shows 45°.

d) Diagram **D** shows 320°.

e) Diagram **B** shows 215°.

f) Diagram **E** shows 270°.

Section 2.1 Page 83 Question 3

a) 48° is is quadrant I.

b) 300° is in quadrant IV.

c) 185° is in quadrant III.

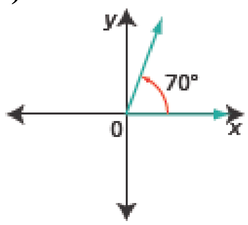
d) 75° is in quadrant I.

e) 220° is in quadrant III.

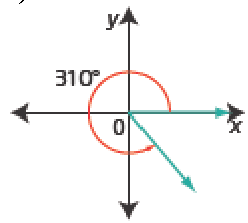
f) 160° is in quadrant II.

Section 2.1 Page 83 Question 4

a)



b)



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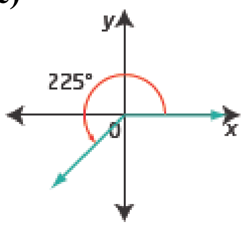

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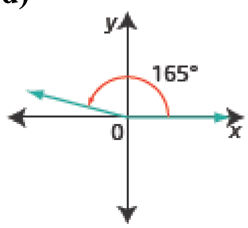


Section 2.1 Page 83 Question 5

a) 180° − 170° = 10°. The reference angle for 170° is 10°.

b) 360° − 345° = 15°. The reference angle for 345° is 15°.

c) The reference angle for 72° is 72°.

d) 215° – 180° = 35°. The reference angle for 215° is 35°.

Section 2.1 Page 83 Question 6

a) 180° − 45° = 135°, 180° + 45° = 225°, 360° − 45° = 315°
The three other angles in standard position, 0° < θ < 360°, that have a reference angle of 45° are 135°, 225°, and 315°.

b) 180° − 60° = 120°, 180° + 60° = 240°, 360° − 60° = 300°
The three other angles in standard position, 0° < θ < 360°, that have a reference angle of 60° are 120°, 240°, and 300°.

c) 180° − 30° = 150°, 180° + 30° = 210°, 360° − 30° = 330°
The three other angles in standard position, 0° < θ < 360°, that have a reference angle of 30° are 150°, 210°, and 330°.

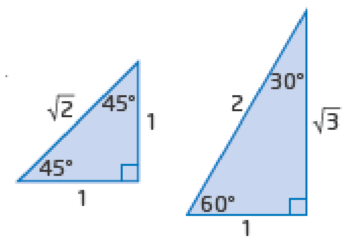
d) 180° − 75° = 105°, 180° + 75° = 255°, 360° − 75° = 285°
The three other angles in standard position, 0° < θ < 360°, that have a reference angle of 75° are 105°, 255°, and 285°.

Section 2.1 Page 83 Question 7

	Reference Angle	Quadrant	Angle in Standard Position
a)	72°	IV	360° − 72° = 288°
b)	56°	II	180° − 56° = 124°
c)	18°	III	180° + 18° = 198°
d)	35°	IV	360° − 35° = 325°

Section 2.1 Page 83 Question 8

To complete the table, refer to the special triangles shown.



θ	sin θ	cos θ	tan θ
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$
45°	$\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$	$\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

Section 2.1 Page 84 Question 9

$180° - 20.4° = 159.6°$
The angle measured in standard position is 159.6°.

Section 2.1 Page 84 Question 10

a) The coordinates of the other three trees are found using symmetries of the diagram: flowering dogwood (−3.5, 2), river birch (−3.5, −2), white pine (3.5, −2).

b) For the red maple,

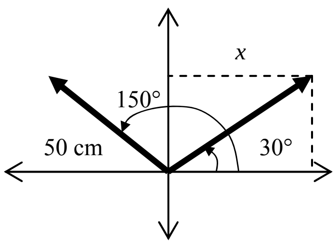
$$\tan\theta = \frac{2}{3.5}$$
$$\theta = \tan^{-1}\left(\frac{2}{3.5}\right)$$
$$\theta = 29.744...$$

The angle in standard position for the red maple is 30°, to the nearest degree.

Then, the angle in standard position for the flowering dogwood is 180° − 30° or 150°, to the nearest degree. The angle in standard position for the river birch is 180° + 30° or 210°, to the nearest degree. The angle in standard position for the white pine is 360° − 30° or 330°, to the nearest degree.

c) On the grid, there are 4 vertical units of distance between the red maple and the white pine. Since each grid mark represents 10 m, the distance between these two trees is 40 m.

Section 2.1 Page 84 Question 11



$$\cos 30° = \frac{x}{50}$$
$$\frac{\sqrt{3}}{2} = \frac{x}{50}$$
$$x = 25\sqrt{3}$$

By symmetry, the horizontal distance that the tip of the wiper travels in one swipe will be $2x$, or $50\sqrt{3}$ cm.

Section 2.1 Page 84 Question 12

a) Using the symmetries of the diagram, the coordinates are A′(x, −y), A″(−x, y) and A‴(−x, −y).

b) A′ is in quadrant IV, so ∠A′OC = 360° − θ.
A″ is in quadrant II, so ∠A″OC = 180° − θ.
A‴ is in quadrant III, so ∠A‴OC = 180° + θ.

Section 2.1 Page 84 Question 13

$$\sin 60° = \frac{v_1}{10}$$
$$\frac{\sqrt{3}}{2} = \frac{v_1}{10}$$
$$v_1 = 5\sqrt{3}$$

$$\sin 30° = \frac{v_2}{10}$$
$$\frac{1}{2} = \frac{v_2}{10}$$
$$v_2 = 5$$

Then, $v_1 - v_2 = 5\sqrt{3} - 5$.
The exact vertical displacement of the boom is $\left(5\sqrt{3} - 5\right)$ m.

Section 2.1 Page 85 Question 14

The 72° angle is in quadrant III, so in standard position the angle is 180° + 72 ° or 252°.

Section 2.1 Page 85 Question 15

An angle of 110° will be in quadrant II. Using a protractor to determine where this angle falls on the diagram, it is found that the terminal arm passes through Cu, Ag, and Au. These elements are copper, silver, and gold, respectively.

Section 2.1 Page 85 Question 16

a) The terminal arm of the blue angle falls at the end of the 12th day of 20 on the second ring. Since there are 360° in the circle, the blue angle measures $\left(\frac{12}{20}\right)360°$ or 216°.

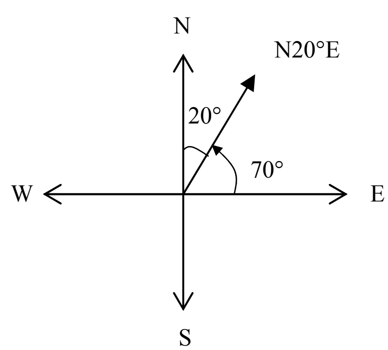
b) If the angle was in quadrant II, it would be the 8th day of 20. The angle would measure $\left(\frac{8}{20}\right)360°$ or 144°.

You can check this using reference angles. For 216°, the reference angle is 216° − 180° or 36°. So, the same reference angle in quadrant II is 180° – 36° or 144°.

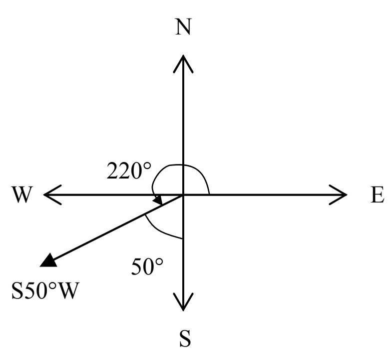
c) In quadrant IV, this reference angle would give and angle of 360° − 36° or 324°. This represents 2 short of 20 days, or 18 days.

Section 2.1 Page 85 Question 17

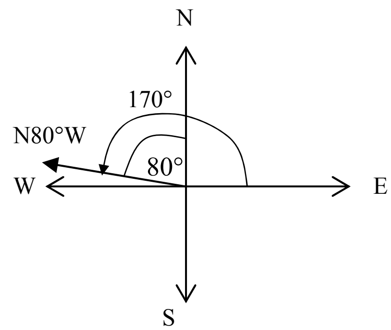
a) N20°E is equivalent to 70° in standard position.



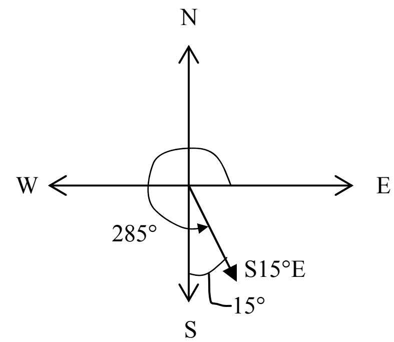
b) S50°W is equivalent to 180° + 40° or 220° in standard position.



c) N80°W is equivalent to 90° + 80° or 170° in standard position.



d) S15°E is equivalent to 270° + 15° or 285° in standard position.



Section 2.1 Page 86 Question 18

a) $\sin\theta = \frac{h_{arm}}{45}$, where h_{arm} is the height of the fingertips above the centre of rotation.

$h_{arm} = 45\sin\theta$

Then, the height of the arm above the table is given by $h = 12 + 45\sin\theta$.

θ	0°	15°	30°	45°	60°	75°	90°
$h = 12 + 45\sin\theta$ (in centimetres)	12.0	23.6	34.5	43.8	51.0	55.5	57.0

b) From the table, it is clear that the increase in h is not constant. For example, from 0° to 15° the height increases 11.6 cm, while from 30° to 45° the height increases 9.3 cm. the rate of increase lessens as the measure of θ approaches 90°.

c) If θ were extended 90°, my conjecture is that the height would decrease, reaching 12 cm when θ = 180°, and that the height would decrease slowly for angles just past 90° but the rate of decrease would be greater towards the horizontal position.

Section 2.1 Page 86 Question 19

Let x and y represent the two angles. Supplementary angles have a sum of 180°, so $x + y = 180°$.

If the terminal arms of the two angles are perpendicular, then one angle is 90° more than the other, or $y = x + 90°$.

Substituting, $x + x + 90° = 180°$
Then, $x = 45°$.
The two angles must be 45° and 135°.

Section 2.1 Page 86 Question 20

a) Let y_s represent the height of the seat above the centre of rotation.

$$\sin 72° = \frac{y_s}{9}$$
$$y_s = 9 \sin 72°$$
$$y_s = 8.559...$$

So, the height of Carl's seat above the ground is 11 + 8.559… or approximately 19.56 m.

b) i) If the speed is 4 rev/min, then in 5 s Carl has moved $\left(\frac{4}{60}\right)5$ or $\frac{1}{3}$ of a revolution.

So , the second stop is 120° past the 72° position. The angle of the seat that Carl is on, in standard position, is 72° + 120° or 192°.

ii) The second stop is 12° below the horizontal. So, in this position

$$\sin 12° = \frac{y_s}{9}$$
$$y_s = 9 \sin 12°$$
$$y_s = 1.871...$$

In this case, Carls seat is 11 − 1.871… or approximately 9.13 m above the ground.

Section 2.1 Page 86 Question 21

a) $\sin \theta = \frac{\text{CD}}{\text{OC}}$

$\sin \theta = \frac{\text{CD}}{1}$, since the radius is 1.

So, option B is correct.

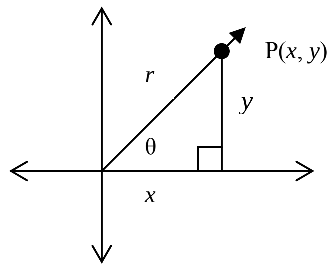
b) $\tan \theta = \frac{\text{BA}}{\text{OA}}$

$\sin \theta = \frac{\text{BA}}{1}$, since the radius is 1.

So, option D is correct.

Section 2.1 Page 86 Question 22

Using the Pythagorean Theorem,
$x^2 + y^2 = r^2$



Section 2.1 Page 86 Question 23

a)

θ	20°	40°	60°	80°
sin θ	0.3420	0.6428	0.8660	0.9848
sin (180° − θ)	0.3420	0.6428	0.8660	0.9848
sin (180° + θ)	−0.3420	−0.6428	−0.8660	−0.9848
sin (360° − θ)	−0.3420	−0.6428	−0.8660	−0.9848

b) My conjecture is that sin θ and sin (180° − θ) have the same value. Also, sin (180° + θ) and sin (360° − θ) have the opposite value to sin θ (i.e., same numeric value but negative).

c) Similar results will hold true for values of cosine and tangent, but they will be negative in different quadrants. Refer to the diagram in the previous question and think of a point on the terminal arm in each quadrant. Cosine will be negative in quadrants II and III, because cosine involves the adjacent side which is negative in those quadrants. Tangent will be negative in quadrants II and IV, because either the adjacent side or the opposite side is negative in those quadrants.

Section 2.1 Page 86 Question 24

a) Substitute $V = 110$ and $\theta = 30°$ into the formula $d = \dfrac{V^2 \cos\theta \sin\theta}{16}$.

$$d = \frac{110^2 \cos 30° \sin 30°}{16}$$

$$d = \frac{3025}{4}\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)$$

$$d = \frac{3025\sqrt{3}}{16}$$

The exact distance that Daria hit the ball with this driver was $\dfrac{3025\sqrt{3}}{16}$ ft.

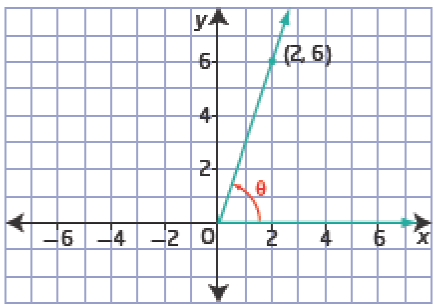
b) To get a longer hit Daria should increase the angle of hit. An angle of 45° gives the greatest distance in the formula for *d*.

c) An angle of elevation of 45° will probably produce the hit that travels the greatest distance. For this angle sine and cosine are both the same, so their effect on the formula will be balanced.

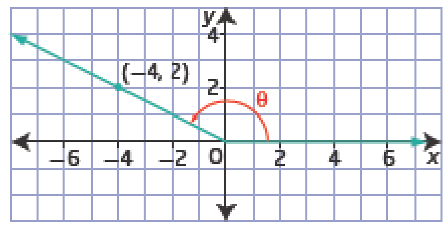
2.2 Trigonometric Ratios of Any Angle

Section 2.2 Page 96 Question 1

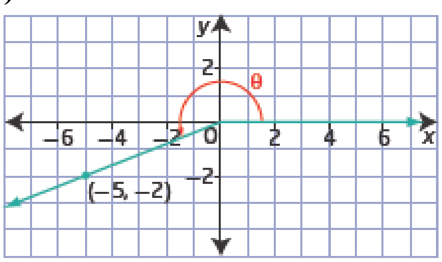
a)



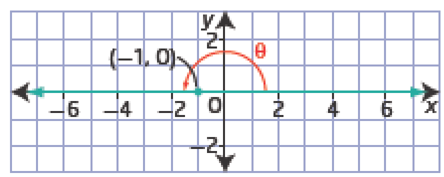
b)



c)



d)



Section 2.2 Page 96 Question 2

a) $\sin 60° = \frac{\sqrt{3}}{2}, \ \cos 60° = \frac{1}{2}, \ \tan 60° = \sqrt{3}$

b) $\sin 225° = -\frac{1}{\sqrt{2}} \text{ or } -\frac{\sqrt{2}}{2}, \ \cos 225° = -\frac{1}{\sqrt{2}} \text{ or } -\frac{\sqrt{2}}{2}, \ \tan 60° = 1$

c) $\sin 150° = \frac{1}{2}, \ \cos 150° = -\frac{\sqrt{3}}{2}, \ \tan 150° = -\frac{1}{\sqrt{3}} \text{ or } -\frac{\sqrt{3}}{3}$

d) $\sin 90° = 1, \ \cos 90° = 0, \ \tan 90° \text{ is undefined}$

Section 2.2 Page 96 Question 3

a) Use the Pythagorean Theorem to determine the hypotenuse: $r = 5$.

$\sin\theta = \frac{y}{r}$, $\cos\theta = \frac{x}{r}$, $\tan\theta = \frac{y}{x}$

$\sin\theta = \frac{4}{5}$, $\cos\theta = \frac{3}{5}$, $\tan\theta = \frac{4}{3}$

b) $r^2 = x^2 + y^2$

$r^2 = (-12)^2 + (-5)^2$

$r^2 = 144 + 25$

$r^2 = 169$

$r = 13$

$\sin\theta = \frac{y}{r}$, $\cos\theta = \frac{x}{r}$, $\tan\theta = \frac{y}{x}$

$\sin\theta = \frac{-5}{13}$, $\cos\theta = \frac{-12}{13}$, $\tan\theta = \frac{-5}{-12} = \frac{5}{12}$

c) $r^2 = x^2 + y^2$

$r^2 = (8)^2 + (-15)^2$

$r^2 = 64 + 225$

$r^2 = 289$

$r = 17$

$\sin\theta = \frac{y}{r}$, $\cos\theta = \frac{x}{r}$, $\tan\theta = \frac{y}{x}$

$\sin\theta = \frac{-15}{17}$, $\cos\theta = \frac{8}{17}$, $\tan\theta = \frac{-15}{8}$

d) $r^2 = x^2 + y^2$

$r^2 = (1)^2 + (-1)^2$

$r^2 = 2$

$r = \sqrt{2}$

$\sin\theta = \frac{y}{r}$, $\cos\theta = \frac{x}{r}$, $\tan\theta = \frac{y}{x}$

$\sin\theta = \frac{-1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$, $\cos\theta = \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$, $\tan\theta = \frac{-1}{1} = -1$

Section 2.2 Page 96 Question 4

a) The cosine ratio is negative and the sine ratio is positive in quadrant II.

b) The cosine ratio and the tangent ratio are both positive in quadrant I.

c) The sine ratio and the cosine ratio are both negative in quadrant III.

d) The tangent ratio is negative and the cosine ratio is positive in quadrant IV.

Section 2.2 Page 96 Question 5

a) First calculate r.

$r^2 = x^2 + y^2$
$r^2 = (-5)^2 + (12)^2$
$r^2 = 25 + 144$
$r^2 = 169$
$r = 13$

$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$

$\sin\theta = \frac{12}{13}$ $\cos\theta = \frac{-5}{13}$ or $-\frac{5}{13}$ $\tan\theta = \frac{12}{-5}$ or $-\frac{12}{5}$

b) $r^2 = x^2 + y^2$
$r^2 = (5)^2 + (-3)^2$
$r^2 = 25 + 9$
$r^2 = 34$
$r = \sqrt{34}$

$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$

$\sin\theta = \frac{-3}{\sqrt{34}}$ or $-\frac{3\sqrt{34}}{34}$ $\cos\theta = \frac{5}{\sqrt{34}}$ or $\frac{5\sqrt{34}}{34}$ $\tan\theta = \frac{-3}{5}$ or $-\frac{3}{5}$

c) $r^2 = x^2 + y^2$
$r^2 = (6)^2 + (3)^2$
$r^2 = 36 + 9$
$r^2 = 45$
$r = \sqrt{45}$

$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$

$\sin\theta = \frac{3}{\sqrt{45}} = \frac{1}{\sqrt{5}}$ or $\frac{\sqrt{5}}{5}$ $\cos\theta = \frac{6}{\sqrt{45}} = \frac{2}{\sqrt{5}}$ or $\frac{2\sqrt{5}}{5}$ $\tan\theta = \frac{3}{6} = \frac{1}{2}$

d) $r^2 = x^2 + y^2$
$r^2 = (-24)^2 + (-10)^2$
$r^2 = 576 + 100$
$r^2 = 676$
$r = 26$

$\sin\theta = \dfrac{y}{r}$ $\qquad$ $\cos\theta = \dfrac{x}{r}$ $\qquad$ $\tan\theta = \dfrac{y}{x}$

$\sin\theta = \dfrac{-10}{26} = -\dfrac{5}{13}$ $\qquad$ $\cos\theta = \dfrac{-24}{26} = -\dfrac{12}{13}$ $\qquad$ $\tan\theta = \dfrac{10}{24} = \dfrac{5}{12}$

Section 2.2 Page 96 Question 6

a) The angle is in quadrant II, so sin 155° is positive.

b) The angle is in quadrant IV, so cos 320° is positive.

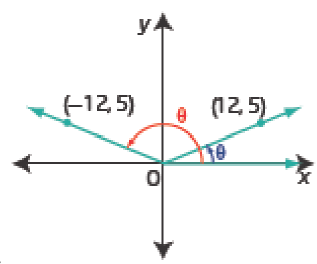
c) The angle is in quadrant II, so tan 120° is negative.

d) The angle is in quadrant III, so cos 220° is negative.

Section 2.2 Page 96 Question 7

a) Given $\sin\theta = \dfrac{5}{13}$, $y = 5$ and $r = 13$. Use the Pythagorean Theorem to determine x.

$$r^2 = x^2 + y^2$$
$$13^2 = x^2 + 5^2$$
$$169 = x^2 + 25$$
$$x^2 = 144$$



$$x = \pm 12$$

b) Determine the reference angle.

$$\sin\theta = \frac{5}{13}$$
$$\theta = \sin^{-1}\left(\frac{5}{13}\right)$$
$$\theta = 22.619...$$

Then, to the nearest degree, in quadrant I, θ = 23° and in quadrant II, θ = 180° − 23° or 157°.

Section 2.2 Page 96 Question 8

a) $\cos\theta = \dfrac{x}{r} = -\dfrac{2}{3}$, so $x = -2$ and $r = 3$ for an angle in quadrant II.

Use the Pythagorean Theorem to determine y.

$$r^2 = x^2 + y^2$$
$$3^2 = (-2)^2 + y^2$$
$$9 = 4 + y^2$$
$$y = \sqrt{5}$$

Then, $\sin\theta = \frac{y}{r}$ and $\tan\theta = \frac{y}{x}$

$\sin\theta = \frac{\sqrt{5}}{3}$ $\tan\theta = \frac{\sqrt{5}}{-2} = -\frac{\sqrt{5}}{2}$

b) $\sin\theta = \frac{y}{r} = \frac{3}{5}$, so $y = 3$ and $r = 5$ for an angle in quadrant I.

Use the Pythagorean Theorem to determine x.

$r^2 = x^2 + y^2$
$5^2 = x^2 + 3^2$
$25 = x^2 + 9$
$x = 4$

Then, $\cos\theta = \frac{x}{r}$ and $\tan\theta = \frac{y}{x}$

$\cos\theta = \frac{4}{5}$ $\tan\theta = \frac{3}{4}$

c) $\tan\theta = \frac{y}{x} = -\frac{4}{5}$, so $x = 5$ and $y = -4$ for an angle in quadrant IV.

Use the Pythagorean Theorem to determine r.

$r^2 = x^2 + y^2$
$r^2 = (5)^2 + (-4)^2$
$r^2 = 25 + 16$
$r = \sqrt{41}$

Then, $\sin\theta = \frac{y}{r}$ and $\cos\theta = \frac{x}{r}$

$\sin\theta = \frac{-4}{\sqrt{41}}$ or $-\frac{4\sqrt{41}}{41}$ $\cos\theta = \frac{5}{\sqrt{41}}$ or $\frac{5\sqrt{41}}{41}$

d) $\sin\theta = \frac{y}{r} = -\frac{1}{3}$, so $y = -1$ and $r = 3$ for an angle in quadrant III.

Use the Pythagorean Theorem to determine x.

$r^2 = x^2 + y^2$
$3^2 = x^2 + (-1)^2$
$9 = x^2 + 1$
$x = -\sqrt{8}$

Then, $\cos\theta = \frac{x}{r}$ and $\tan\theta = \frac{y}{x}$

$\cos\theta = \frac{-\sqrt{8}}{3}$ or $-\frac{2\sqrt{2}}{3}$ $\tan\theta = \frac{-1}{-\sqrt{8}}$ or $\frac{1}{2\sqrt{2}}$ or $\frac{\sqrt{2}}{4}$

e) $\tan\theta = \frac{y}{x} = \frac{-1}{-1}$, so $x = -1$ and $y = -1$ for an angle in quadrant III.

Use the Pythagorean Theorem to determine r.

$r^2 = x^2 + y^2$

$r^2 = (-1)^2 + (-1)^2$

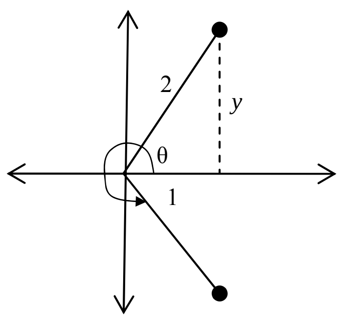
$r = \sqrt{2}$

Then, $\sin\theta = \frac{y}{r}$ and $\cos\theta = \frac{x}{r}$

$\sin\theta = \frac{-1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$ $\cos\theta = \frac{-1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$

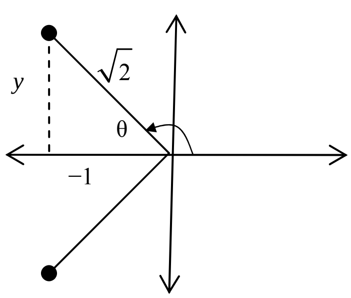
Section 2.2 Page 97 Question 9

a) The diagram shows the two possible positions of θ, $0° \le \theta < 360°$, for which $\cos\theta = \frac{1}{2}$.



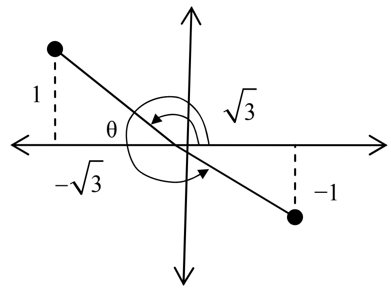
$\cos\theta = \frac{1}{2}$ is part of the 30°-60°-90° right triangle with sides 1, 2, and $\sqrt{3}$.
The reference angle for θ is 60°.
In quadrant I, θ = 60°.
In quadrant IV, θ = 360° − 60° or 300°.

b) The diagram shows the two possible positions of θ, $0° \le \theta < 360°$, for which $\cos\theta = -\frac{1}{\sqrt{2}}$.



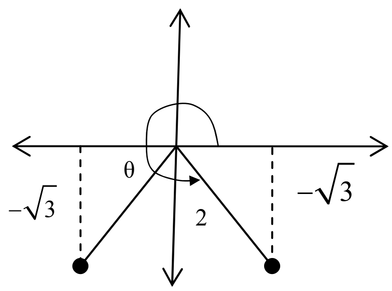
For $\cos\theta = -\frac{1}{\sqrt{2}}$, refer to the 45°-45°-90° right triangle with sides 1, 1, and $\sqrt{2}$.
The reference angle for θ is 45°.
In quadrant II, θ = 180° − 45° or 135°.
In quadrant III, θ = 180° + 45° or 225°.

c) The diagram shows the two possible positions of θ, 0° ≤ θ < 360°, for which $\tan \theta = -\frac{1}{\sqrt{3}}$.



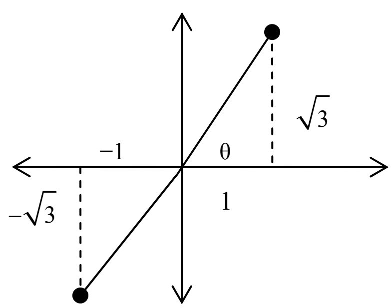
For $\tan \theta = -\frac{1}{\sqrt{3}}$, refer to the 30°-60°-90° right triangle with sides 1, 2, and $\sqrt{3}$.
The reference angle for θ is 30°.
In quadrant II, θ = 180° − 30° or 150°.
In quadrant IV, θ = 360° − 30° or 330°.

d) The diagram shows the two possible positions of θ, 0° ≤ θ < 360°, for which $\sin \theta = -\frac{\sqrt{3}}{2}$.



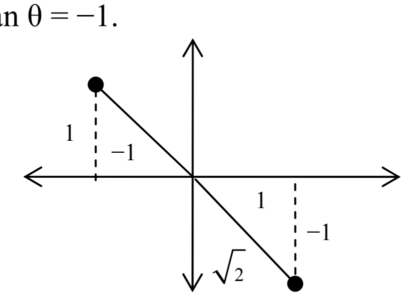
For $\sin \theta = -\frac{\sqrt{3}}{2}$, refer to the 30°-60°-90° right triangle with sides 1, 2, and $\sqrt{3}$.
The reference angle for θ is 60°.
In quadrant III, θ = 180° + 60° or 240°. In quadrant IV, θ = 360° − 60° or 300°.

e) The diagram shows the two possible positions of θ, 0° ≤ θ < 360°, for which $\tan \theta = \sqrt{3}$.



For $\tan \theta = \sqrt{3}$, refer to the 30°-60°-90° right triangle with sides 1, 2, and $\sqrt{3}$.
The reference angle for θ is 60°.
In quadrant I, θ = 60°.
In quadrant IV, θ = 180° + 60° or 240°.

f) The diagram shows the two possible positions of θ, 0° ≤ θ < 360°, for which tan θ = −1.



For tan θ = −1, refer to the 45°-45°-90° right triangle with sides 1, 1, and $\sqrt{2}$.
The reference angle for θ is 45°.
In quadrant II, θ = 180° − 45° or 135°.
In quadrant IV, θ = 360° − 45° or 315°.

Section 2.2 Page 97 Question 10

θ	sin θ	cos θ	tan θ
0°	0	1	0
90°	1	0	undefined
180°	0	−1	0
270°	−1	0	undefined
360°	0	1	0

Section 2.2 Page 97 Question 11

a) Given P(−8, 6), use $x = -8$, $y = 6$, and the Pythagorean Theorem to determine r.

$r^2 = x^2 + y^2$
$r^2 = (-8)^2 + 6^2$
$r^2 = 64 + 36$
$r^2 = 100$
$r = 10$

$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$

$\sin\theta = \frac{6}{10}$ or $\frac{3}{5}$ $\cos\theta = \frac{-8}{10}$ or $-\frac{4}{5}$ $\tan\theta = \frac{6}{-8}$ or $-\frac{3}{4}$

b) Given P(5, −12), use $x = 5$, $y = -12$, and the Pythagorean Theorem to determine r.

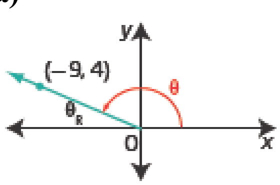
$r^2 = x^2 + y^2$
$r^2 = 5^2 + (-12)^2$
$r^2 = 25 + 144$
$r^2 = 169$
$r = 13$

$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$

$\sin\theta = \frac{-12}{13}$ $\cos\theta = \frac{5}{13}$ $\tan\theta = \frac{-12}{5}$

Section 2.2 Page 97 Question 12

a)



b) $\tan \theta_R = \frac{4}{9}$

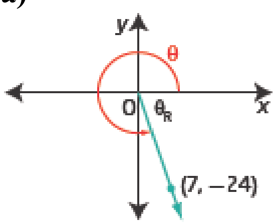
$\theta_R = \tan^{-1}\left(\frac{4}{9}\right)$

$\theta_R \approx 24°$

c) $\theta = 180° - 24°$
$\theta = 156°$, to the nearest degree

Section 2.2 Page 97 Question 13

a)



b) $\tan \theta_R = \frac{24}{7}$

$\theta_R = \tan^{-1}\left(\frac{24}{7}\right)$

$\theta_R \approx 74°$

c) $\theta = 360° - 74°$
$\theta = 286°$, to the nearest degree

Section 2.2 Page 97 Question 14

a) Given P(2, 4), then $x = 2$ and $y = 4$. Use the Pythagorean Theorem to determine r.
$r^2 = x^2 + y^2$
$r^2 = 2^2 + 4^2$
$r^2 = 20$
$r = \sqrt{20}$

Then, $\sin \theta = \frac{y}{r}$

$\sin \theta = \frac{4}{\sqrt{20}} = \frac{4}{2\sqrt{5}}$ or $\frac{2\sqrt{5}}{5}$

b) Given Q(4, 8), then $x = 4$ and $y = 8$.
Then, $r^2 = x^2 + y^2$
$r^2 = 4^2 + 8^2$
$r^2 = 80$
$r = \sqrt{80}$

$\sin \theta = \frac{y}{r}$

$\sin \theta = \frac{8}{\sqrt{80}} = \frac{8}{4\sqrt{5}}$ or $\frac{2\sqrt{5}}{5}$

c) Given R(8, 16), then $x = 8$ and $y = 16$.
Then, $r^2 = x^2 + y^2$
$r^2 = 8^2 + 16^2$

$$r^2 = 320$$
$$r = \sqrt{320}$$
$$\sin\theta = \frac{y}{r}$$
$$\sin\theta = \frac{16}{\sqrt{320}} = \frac{16}{8\sqrt{5}} \text{ or } \frac{2\sqrt{5}}{5}$$

d) The sine ratio is the same because the points P, Q, and R are all on the terminal arm of angle θ. The right triangles formed by *x*, *y*, and *r* in each case are all similar triangles so the ratio of corresponding sides are equal.

Section 2.2 Page 97 Question 15

a) Given P(k, 24) and $r = 25$,

$$\sin\theta = \frac{y}{r}$$
$$\sin\theta = \frac{24}{25}$$
$$\theta = \sin^{-1}\left(\frac{24}{25}\right)$$
$$\theta = 73.739...$$

The sine ratio is positive in quadrants I and II, so θ is approximately 74° or 180° − 74° = 106°.

b) Use the Pythagorean Theorem to determine *x*.

$$r^2 = x^2 + y^2$$
$$25^2 = x^2 + 24^2$$
$$x^2 = 625 - 576$$
$$x^2 = 49$$
$$x = \pm 7$$

Then, in quadrant I, $x = 7$, $y = 24$, and $r = 25$:

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x}$$
$$\sin\theta = \frac{24}{25} \qquad \cos\theta = \frac{7}{25} \qquad \tan\theta = \frac{24}{7}$$

Then, in quadrant II, $x = -7$, $y = 24$, and $r = 25$:

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x}$$
$$\sin\theta = \frac{24}{25} \qquad \cos\theta = \frac{-7}{25} = -\frac{7}{25} \qquad \tan\theta = \frac{24}{-7} = -\frac{24}{7}$$

Section 2.2 Page 97 Question 16

$\cos\theta = \frac{x}{r}$, so given $\cos\theta = \frac{1}{5}$, $x = 1$, and $r = 5$.

$\tan\theta = \frac{y}{x}$, so given $\tan\theta = 2\sqrt{6}$, $y = 2\sqrt{6}$.

Then, $\sin\theta = \frac{y}{r}$

$$\sin\theta = \frac{2\sqrt{6}}{5}$$

Section 2.2 Page 97 Question 17

At the equator, where the angle of dip is 0°, a point on the terminal arm is (1, 0).
So, $x = 1$, $y = 0$, and $r = 1$.

Then, $\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$

$\sin 0° = \frac{0}{1}$ or 0 $\cos 0° = \frac{1}{1}$ or 1 $\tan 0° = \frac{0}{1} = 0$

At the North and South Poles, where the angle of dip is 90°, a point on the terminal arm is (0, 1).
So, $x = 1$, $y = 0$, and $r = 1$.

Then, $\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$

$\sin 90° = \frac{1}{1}$ or 1 $\cos 90° = \frac{0}{1}$ or 0 $\tan 90° = \frac{1}{0}$ or undefined

Section 2.2 Page 98 Question 18

a) sin 151° = sin 29° is a true statement, because in quadrant I and II the sine ratio is positive and 29° is the reference angle for 151°.

b) cos 135° = sin 225° is a true statement, because in quadrant II the cosine ratio is negative, in quadrant III the sine ratio is negative, and 45° is the reference angle for both angles.

c) tan 135° = tan 225° is a false statement, because in quadrant II the tangent ratio is negative and in quadrant III the tangent ratio is positive.

d) sin 60° = cos 330° is a true statement, because using the special 30°-60°-90° reference triangle both have a value of $\frac{\sqrt{3}}{2}$.

e) sin 270° = cos 180° is a true statement, because both have a value of −1.

Section 2.2 Page 98 Question 19

θ	sin θ	cos θ	tan θ
0°	0	1	0
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$
45°	$\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$	$\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	1	0	undefined
120°	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$
135°	$\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$	$-\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$	−1
150°	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{3}}$ or $-\frac{\sqrt{3}}{3}$
180°	0	−1	0
210°	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$
225°	$-\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$	$-\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$	1
240°	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\sqrt{3}$
270°	−1	0	undefined
300°	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\sqrt{3}$
315°	$-\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$	$\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$	−1
330°	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{3}}$ or $-\frac{\sqrt{3}}{3}$
360°	0	1	0

Section 2.2 Page 98 Question 20

a) The measure for ∠A is 45°, for ∠B is 135°, for ∠C is 225°, and for ∠D is 315°.

b) Use the special 45°-45°-90° reference triangle with sides $\frac{1}{\sqrt{2}}$, $\frac{1}{\sqrt{2}}$, and 1.

Then, A is on the terminal arm of 45°: $A\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$.

B is on the terminal arm of 135°: $B\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$.

C is on the terminal arm of 225°: $C\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$.

D is on the terminal arm of 315°: $D\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$.

Section 2.2 Page 98 Question 21

a)

Angle	Sine	Cosine	Tangent
0°	0	1	0
15°	0.2588	0.9659	0.2679
30°	0.5	0.8660	0.5774
45°	0.7071	0.7071	1
60°	0.8660	0.5	1.7321
75°	0.9659	0.2588	3.7321
90°	1	0	undefined
105°	0.9659	−0.2588	−3.7321
120°	0.8660	−0.5	−1.7321
135°	0.7071	−0.7071	−1
150°	0.5	−0.8660	−0.5774
165°	0.2588	−0.9659	−0.2679
180°	0	−1	0

b) As θ increases from 0° to 90° the sine ratio increases from 0 to 1, while the cosine ratio decreases from 1 to 0. As θ increases from 90° to 180° the sine ratio decreases from 1 to 0, while the cosine ratio decreases from 0 to −1. As θ increases from 0° to 90° the tangent ratio increases from 0 to undefined, then from 90° to 180° the tangent ratio starts with large negative values but increases to 0.

c) The values of sine and cosine seem to be related. For example, sin 30° = cos 60°. However in quadrant II, the sign is different. For example, −sin 120° = cos 150°. For $0° \le \theta \le 90°$, cos θ = sin (90° − θ). For $90° \le \theta \le 180°$, cos θ = −sin (θ – 90°).

d) In quadrant I, sine, cosine, and tangent values are all positive. In quadrant II, sine values are positive but cosine and tangent values are negative.

e) In quadrant III, the sine and cosine values will be negative, but the tangent values will be positive. In quadrant IV, the cosine values will be positive, but the sine and tangent values will be negative.

Section 2.2 Page 98 Question 22

a) Choose a point on the line $y = 6x$. When $x = 1$, $y = 6$.
Use the Pythagorean Theorem to determine r.

$r^2 = x^2 + y^2$
$r^2 = 1^2 + 6^2$
$r = \sqrt{37}$

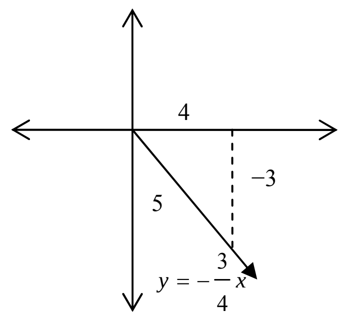
Then, $\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$

$\sin\theta = \frac{6}{\sqrt{37}}$ or $\frac{6\sqrt{37}}{37}$ $\cos\theta = \frac{1}{\sqrt{37}}$ or $\frac{\sqrt{37}}{37}$ $\tan\theta = \frac{6}{1} = 6$

b) $4y + 3x = 0$, for $x \geq 0$

So, $y = -\frac{3}{4}x$

Since the slope is negative, the angle is in quadrant IV. The point (4, −3) lies on the line and $r = 5$.



$\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$

$\cos\theta = \frac{4}{5}$ $\tan\theta = \frac{-3}{4} = -\frac{3}{4}$

Then, $\tan\theta + \cos\theta = -\frac{3}{4} + \frac{4}{5}$

$= \frac{-15+16}{20}$

$= \frac{1}{20}$

Section 2.2 Page 98 Question 23

As θ increases from 0° to 90°:

- the value of x will decrease from 12 to 0
- the value of y will increase from 0 to 12
- the value of sin θ will increase from 0 to 1
- the value of cos θ will decrease from 1 to 0
- the value of tan θ will increase from 0 to undefined

Section 2.2 Page 99 Question 24

Since $\cos\theta = \frac{x}{r}$ and cos θ = a, then $x = a$, $r = 1$, and $y = \sqrt{1-a^2}$.

$\tan \theta = \frac{y}{x}$

$\tan \theta = \frac{\sqrt{1-a^2}}{a}$

Section 2.2 Page 99 Question 25

OA = OB = 1 They are both radii.
Therefore, ΔOAB is isosceles with ∠OAB = ∠ABO = 60°. So, in actual fact ΔOAB is equilateral and AB = 1.
CB = 2

Since OA is the terminal arm of 60°, the coordinates of A are $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$. The coordinates of C are (−1, 0). Then, use the Pythagorean Theorem in the right triangle with hypotenuse AC.

$$AC^2 = \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{3}{2}\right)^2$$

$$AC^2 = \frac{3}{4} + \frac{9}{4}$$

$$AC^2 = 3$$

$$AC = \sqrt{3}$$

Then, in ∠ABC:

$AC^2 + AB^2 = 3 + 1$ $BC^2 = 2^2$

$AC^2 + AB^2 = 4$ $BC^2 = 4$

So the sides of ΔABC satisfy the Pythagorean Theorem, with ∠CAB = 90° because BC is the hypotenuse.

Section 2.2 Page 99 Question 26

For any angle that is not in quadrant I, the reference angle is the acute angle made between the terminal arm and the *x*-axis. Use the reference angle measure to determine the numeric value of the trigonometric ratios but then adjust the sign to negative if necessary for the quadrant that the terminal arm is in.

Section 2.2 Page 99 Question 27

Point P(−5, −9) is in quadrant III. The reference triangle, formed with the *x*-axis, has sides −5 and −9, and hypotenuse $\sqrt{106}$. Use the reference triangle to determine the measure of the acute angle made by the *x*-axis and the terminal arm. Then, add 180° to determine the actual angle measure.

Section 2.2 Page 99 Question 28

$\sin\theta = \frac{y}{r}$

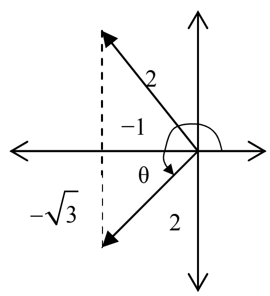
The *y*-coordinate for any point in quadrant I will be the same as the *y*-coordinate of the reflection of that point in quadrant II. Similarly, the *y*-coordinates of points in quadrant III match one other point in quadrant IV. So for any non-quadrantal angle between 0° and 360° there is always one other matching angle that has the same sine ratio.

Section 2.2 Page 99 Question 29

$\cos\theta = \frac{x}{r}$ So, given $\cos\theta = -\frac{1}{2}$, $x = -1$ and $r = 2$. This means that the angle is either in quadrant II or III.

But since $\sin\theta = \frac{y}{r} = -\frac{\sqrt{3}}{2}$, $y = -\sqrt{3}$. So the angle must be in quadrant III.

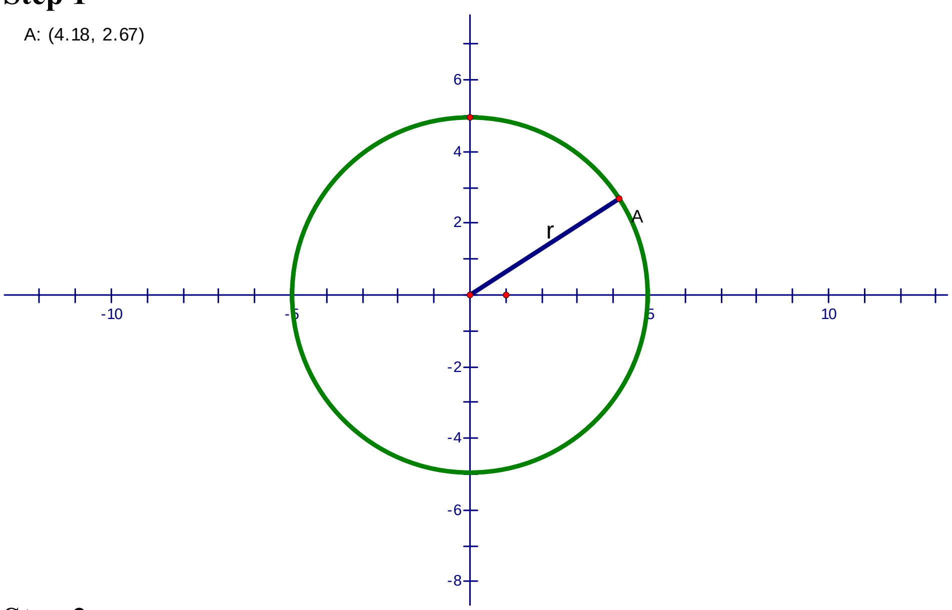
The reference angle is 60°, so the angle must be 180° + 60° or 240°.



Section 2.2 Page 99 Question 30

Answers will vary. Example:

Step 1



Step 2

a) $x = 4.18, y = 2.67$

b) $\sin A = \frac{y}{5}$; sin A = 0.534

c) $\cos A = \frac{x}{5}$, cos A = 0.836; $\tan A = \frac{y}{x}$, tan A = 0.6388…

Step 3 Animation will vary.

Step 4
a) The sine and cosine increase and decrease between 1 and −1. The tangent can take any value.
b) The sine and cosine ratios are both 0.707 at 45° and at 225°.
c) The signs of the ratios change. In quadrant II, the sine ratios are positive, but the cosine and tangent are negative. In quadrant III, the tangent ratios are positive but the sine and cosine are negative. In quadrant IV, the cosine ratios are positive but the sine and tangent are negative.
d) The sine ratio divided by the cosine ratio is equal to the tangent ratio. Yes, this is true for all angles.

2.3 The Sine Law

Section 2.3 Page 108 Question 1

a)
$$\begin{aligned}\frac{a}{\sin 35°} &= \frac{10}{\sin 40°}\\ a &= \frac{10\sin 35°}{\sin 40°}\\ a &= 8.923...\end{aligned}$$
The unknown side *a* = 8.9, to the nearest tenth.

b)
$$\begin{aligned}\frac{b}{\sin 48°} &= \frac{65}{\sin 75°}\\ b &= \frac{65\sin 48°}{\sin 75°}\\ b &= 50.008...\end{aligned}$$
The unknown side *b* = 50.0, to the nearest tenth.

c) $\dfrac{\sin\theta}{12} = \dfrac{\sin 50°}{65}$

$\sin\theta = \dfrac{12\sin 50°}{65}$

$\theta = \sin^{-1}\left(\dfrac{12\sin 50°}{65}\right)$

$\theta = 8.130...$

The measure of angle θ is 8°, to the nearest degree.

d) $\dfrac{\sin A}{25} = \dfrac{\sin 62°}{32}$

$\sin A = \dfrac{25\sin 62°}{32}$

$\angle A = \sin^{-1}\left(\dfrac{25\sin 62°}{32}\right)$

$\angle A = 43.614...$

The measure of ∠A is 44°, to the nearest degree.

Section 2.3 Page 108 Question 2

a) $\angle C = 180° - (88° + 35°)$

$\angle C = 57°$

$\dfrac{c}{\sin C} = \dfrac{b}{\sin B}$

$\dfrac{c}{\sin 57°} = \dfrac{44}{\sin 88°}$

$c = \dfrac{44\sin 57°}{\sin 88°}$

$c = 36.923...$

The length of AB is 36.9 mm, to the nearest tenth of a millimetre.

b) $\dfrac{c}{\sin C} = \dfrac{a}{\sin A}$

$\dfrac{c}{\sin 118°} = \dfrac{45}{\sin 52°}$

$c = \dfrac{45\sin 118°}{\sin 52°}$

$c = 50.421...$

The length of AB is 50.4 m, to the nearest tenth of a metre.

Section 2.3 Page 108 Question 3

a)
$$\frac{\sin C}{c}=\frac{\sin B}{b}$$
$$\frac{\sin C}{28}=\frac{\sin 62°}{31}$$
$$\sin C=\frac{28\sin 62°}{31}$$
$$\angle C=\sin^{-1}\left(\frac{28\sin 62°}{31}\right)$$
$$\angle C=52.892...$$
The measure of $\angle C$ is 53°, to the nearest degree.

b)
$$\frac{\sin A}{a}=\frac{\sin B}{b}$$
$$\frac{\sin A}{15}=\frac{\sin 98°}{17.5}$$
$$\sin A=\frac{15\sin 98°}{17.5}$$
$$\angle A=\sin^{-1}\left(\frac{15\sin 98°}{17.5}\right)$$
$$\angle A=58.081...$$
The measure of $\angle A$ is 58°, to the nearest degree.

Section 2.3 Page 108 Question 4

a) This is an ambiguous case.
First find the acute measure of $\angle C$:
$$\frac{\sin C}{c}=\frac{\sin B}{b}$$
$$\frac{\sin C}{13}=\frac{\sin 67°}{12}$$
$$\sin C=\frac{13\sin 67°}{12}$$
$$\angle C=\sin^{-1}\left(\frac{13\sin 67°}{12}\right)$$
$$\angle C=85.721...$$
The acute measure of $\angle C$ is 86°, to the nearest degree.
Then, the obtuse measure of $\angle C$ is $180° - 85.721° = 94.279…°$.
The obtuse measure of $\angle C$ is 94°, to the nearest degree.

Then, find the measure of ∠A.

$\angle A = 180° - (85.721...° + 67°)$ or $\angle A = 180° - (94.279...° + 67°)$

$\angle A = 27.279...°$ $\quad\quad$ $\angle A = 18.721...°$

The measure of ∠A is 27° or 19°, to the nearest degree.

Now find the measure of side *a*.

$$\frac{a}{\sin A} = \frac{b}{\sin B} \quad \text{or} \quad \frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{a}{\sin 27.279...°} = \frac{12}{\sin 67°} \quad\quad \frac{a}{\sin 18.721...°} = \frac{12}{\sin 67°}$$

$$a = \frac{12 \sin 27.279...°}{\sin 67°} \quad\quad a = \frac{12 \sin 18.721...°}{\sin 67°}$$

$$a = 5.974... \quad\quad a = 4.184...$$

Summary:

Acute case: ∠C = 86°, ∠A = 27°, side *a* is 6.0 m, to the nearest tenth of a metre.

Obtuse case: ∠C = 94°, ∠A = 19°, side *a* is 4.2 m, to the nearest tenth of a metre.

<Note: If you use the rounded degree values, then *a* is 5.9 m for the acute case. For the obtuse case, using rounded degree values results in the same answer.>

b) $\angle C = 180° - (42° + 84°)$

$\angle C = 54°$

Find the length of *a*, side BC:

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{a}{\sin 42°} = \frac{50}{\sin 84°}$$

$$a = \frac{50 \sin 42°}{\sin 84°}$$

$$a = 33.640...$$

The length of side *a* is 33.6 m, to the nearest tenth of a metre.

Find the length of *c*, side AB:

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$

$$\frac{c}{\sin 54°} = \frac{50}{\sin 84°}$$

$$c = \frac{50 \sin 54°}{\sin 84°}$$

$$c = 40.673...$$

The length of side *c* is 40.7 m, to the nearest tenth of a metre.

c) $\angle B = 180° - (22° + 39°)$
$\angle B = 119°$
Find the length of *a*, side BC:
$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{a}{\sin 22°} = \frac{29}{\sin 119°}$$
$$a = \frac{29 \sin 22°}{\sin 119°}$$
$$a = 12.420...$$
The length of side *a* is 12.4 mm, to the nearest tenth of a millimetre.

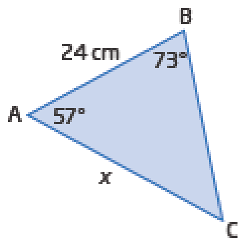
Find the length of *c*, side AB:
$$\frac{c}{\sin C} = \frac{b}{\sin B}$$
$$\frac{c}{\sin 39°} = \frac{29}{\sin 119°}$$
$$c = \frac{29 \sin 39°}{\sin 119°}$$
$$c = 20.866...$$
The length of side *c* is 20.9 mm, to the nearest millimetre.

d) $\angle B = 180° - (48° + 61°)$
$\angle B = 71°$
Find the length of *a*, side BC:
$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{a}{\sin 48°} = \frac{21}{\sin 71°}$$
$$a = \frac{21 \sin 48°}{\sin 71°}$$
$$a = 16.505...$$
The length of side *a* is 16.5 cm, to the nearest tenth of a centimetre.

Find the length of *c*, side AB:
$$\frac{c}{\sin C} = \frac{b}{\sin B}$$
$$\frac{c}{\sin 61°} = \frac{21}{\sin 71°}$$
$$c = \frac{21 \sin 61°}{\sin 71°}$$
$$c = 19.425...$$
The length of side *c* is 19.4 cm, to the nearest centimetre.

Section 2.3 Page 108 Question 5

a)

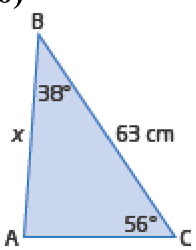


$\angle C = 180° - (57° + 73°)$
$\angle C = 50°$

$$\frac{x}{\sin 73°} = \frac{24}{\sin 50°}$$
$$x = \frac{24 \sin 73°}{\sin 50°}$$
$$x = 29.960...$$

The length of AC is 30.0 cm, to the nearest tenth of a centimetre.

b)

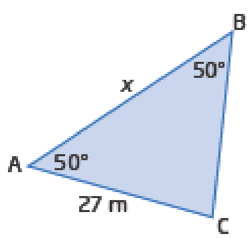


$\angle A = 180° - (38° + 56°)$
$\angle A = 86°$

$$\frac{x}{\sin 56°} = \frac{63}{\sin 86°}$$
$$x = \frac{63 \sin 56°}{\sin 86°}$$
$$x = 52.356...$$

The length of AB is 52.4 cm, to the nearest tenth of a centimetre.

c)

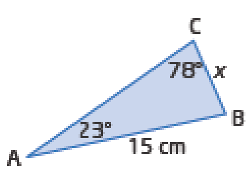


$\angle C = 180° - (50° + 50°)$
$\angle C = 80°$

$$\frac{x}{\sin 80°} = \frac{27}{\sin 50°}$$
$$x = \frac{27 \sin 80°}{\sin 50°}$$
$$x = 34.710...$$

The length of AB is 34.7 m, to the nearest tenth of a metre.

d)



$$\frac{x}{\sin 23°} = \frac{15}{\sin 78°}$$
$$x = \frac{15 \sin 23°}{\sin 78°}$$
$$x = 5.991...$$

The length of BC is 6.0 cm, to the nearest tenth of a centimetre.

Section 2.3 Page 108 Question 6

a) Given ∠A = 39°, a = 10 cm, and b = 14 cm:
$b \sin A = 14 \sin 39°$
$b \sin A = 8.810\ldots$
Then, $b \sin A < a < b$ so there are two solutions.

b) Given ∠A = 123°, a = 23 cm, and b = 12 cm:
∠A is obtuse and $a > b$ so there is one solution.

c) Given ∠A = 145°, a = 18 cm, and b = 10 cm:
∠A is obtuse and $a > b$ so there is one solution.

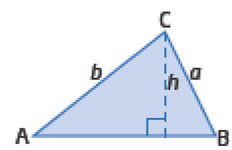
d) Given ∠A = 124°, a = 1 cm, and b = 2 cm:
∠A is obtuse and $a < b$ so there is no solution.

Section 2.3 Page 108 Question 7

a) $\sin A = \frac{h}{b}$

$h = b \sin A$

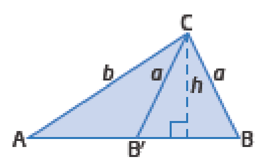
Then, from the diagram, $a > b \sin A$, or $a > h$ and $b > h$.



b) $\sin A = \frac{h}{b}$

$h = b \sin A$

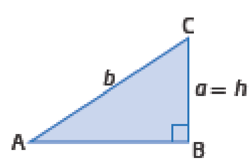
Then, from the diagram, $b \sin A < a < b$.



c) $\sin A = \frac{h}{b}$

$h = b \sin A$

Also, from the diagram, $a = b \sin A$.



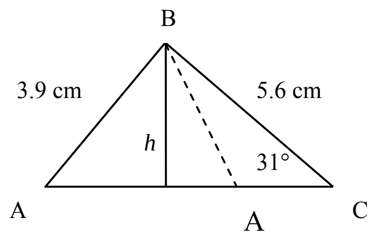
d) $\sin A = \frac{h}{b}$

$h = b \sin A$

From the diagram, $a \geq b > b \sin A$.

Section 2.3 Page 109 Question 8

a) The diagram shows the given information.
This is an ambiguous case, since h = 5.6 sin 31° = 2.884…
Because h is less than 5.6 and 3.9, two solutions are possible.



$$\frac{\sin A}{5.6} = \frac{\sin 31°}{3.9}$$
$$\sin A = \frac{5.6 \sin 31°}{3.9}$$
$$\angle A = \sin^{-1}\left(\frac{5.6 \sin 31°}{3.9}\right)$$
$$\angle A = 47.692...$$
Or, $\angle A = 180° - 47.692…°$
$\angle A = 132.308…°$

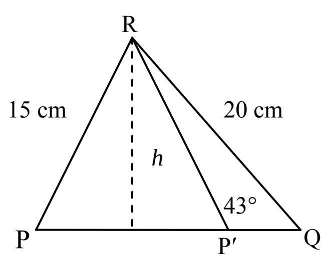
$\angle B = 180° - (31° + 48°)$ or $\angle B = 180° - (31° + 132°)$
$\angle B = 101°$ $\angle B = 17°$

Determine b for each measure of ∠B:

$$\frac{b}{\sin 101°} = \frac{3.9}{\sin 31°} \qquad \frac{b}{\sin 17°} = \frac{3.9}{\sin 31°}$$
$$b = \frac{3.9 \sin 101°}{\sin 31°} \qquad b = \frac{3.9 \sin 17°}{\sin 31°}$$
$$b = 7.433... \qquad b = 2.213...$$

In ΔABC, b = 7.4 cm, to the nearest tenth of a centimetre, and ∠A = 48° and ∠B = 101°, both to the nearest degree, or
b = 2.2 cm, to the nearest tenth of a centimetre, and ∠A = 132° and ∠B = 17°, both to the nearest degree.

b)



h = 20 sin 43° = 13.639…
Since this is less than 15 and less than 20, two solutions are possible, as shown in the diagram.

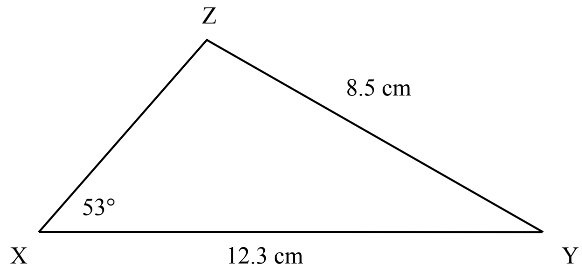
$$\frac{\sin P}{20} = \frac{\sin 43°}{15}$$
$$\sin P = \frac{20 \sin 43°}{15}$$
$$\angle P = \sin^{-1}\left(\frac{20 \sin 43°}{15}\right)$$
$$\angle P = 65.413...$$
So $\angle P = 65°$, or $\angle P = 180° - 65° = 115°$.
Then, $\angle R = 180° - (65° + 43°) = 72°$, or $\angle R = 180° - (115° + 43°) = 22°$.

Now determine r for each value of $\angle R$:

$$\frac{r}{\sin 72°} = \frac{15}{\sin 43°} \qquad \text{or} \qquad \frac{r}{\sin 22°} = \frac{15}{\sin 43°}$$
$$r = \frac{15 \sin 72°}{\sin 43°} \qquad\qquad r = \frac{15 \sin 22°}{\sin 43°}$$
$$r = 20.917... \qquad\qquad r = 8.239...$$

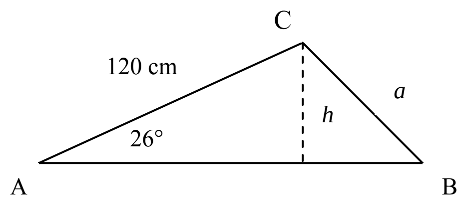
Therefore, in ΔPQR, $\angle P = 65°$, $\angle R = 72°$, and PQ = 20.9 cm, or
$\angle P = 115°$, $\angle R = 22°$, and PQ = 8.2 cm.

c)



$h = 12.3 \sin 53° = 9.823…$
Since $8.5 < h$, no solution is possible.

Section 2.3 Page 109 Question 9



$h = 120 \sin 26°$
$h = 52.604…$

a) There is one oblique triangle if $a \geq 120$ cm.

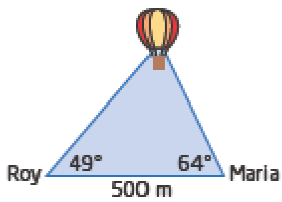
b) There is one right triangle if *a* = 52.6 cm.

c) The are two oblique triangles if 52.6 cm < *a* < 120 cm.

d) There is no such triangle if *a* < 52.6 cm.

Section 2.3 Page 109 Question 10

a)



b) Determine the measure of the third angle, ∠B.

$\angle B = 180° - (49° + 64°)$

$\angle B = 67°$

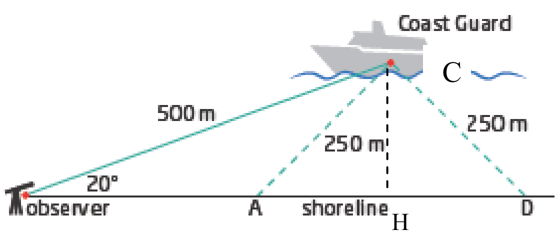
Then, $\dfrac{BM}{\sin 49°} = \dfrac{500}{\sin 67°}$

$BM = \dfrac{500 \sin 49°}{\sin 67°}$

$BM = 409.943...$

The hot air balloon is 409.9 m from Maria, to the nearest tenth of a metre.

Section 2.3 Page 109 Question 11



Let C be the position of the coast guard ship and H the foot of the perpendicular from C to the shoreline.

$\dfrac{\sin D}{500} = \dfrac{\sin 20°}{250}$

$\sin D = \dfrac{500 \sin 20°}{250}$

$\angle D = \sin^{-1}\left(\dfrac{500 \sin 20°}{250}\right)$

$\angle D = 43.160...$

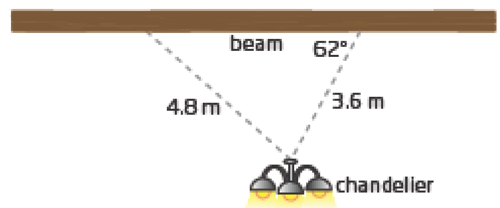
Then, in ΔCDH:

$$\cos 43.160...° = \frac{DH}{250}$$
$$DH = 250\cos 43.160...°$$
$$DH = 182.361...$$

Since CA = CD, ΔACH ≈ ΔDCH and AH + HD.
So AD = 2(182.361…) or 364.722…
The length of shoreline that is illuminated by the spotlight is 364.7 m, to the nearest tenth of a metre.

Section 2.3 Page 109 Question 12

Let A and B be the points where the chains attach to the beam and C the chandelier.

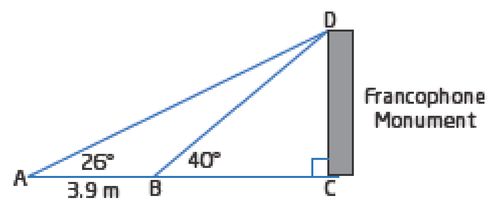


$$\frac{\sin A}{3.6} = \frac{\sin 62}{4.8}$$
$$\sin A = \frac{3.6\sin 62}{4.8}$$
$$\angle A = \sin^{-1}\left(\frac{3.6\sin 62}{4.8}\right)$$
$$\angle A = 41.468...$$

The second chain makes an angle of 41° with the beam, to the nearest degree.

Section 2.3 Page 109 Question 13

In ΔABD, ∠ABD = 180° − 40° = 140°.
Then, ∠BDA = 180° − (140° + 26°) = 14°.



$$\frac{BD}{\sin 26°} = \frac{3.9}{\sin 14°}$$
$$BD = \frac{3.9\sin 26°}{\sin 14°}$$
$$BD = 7.066...$$

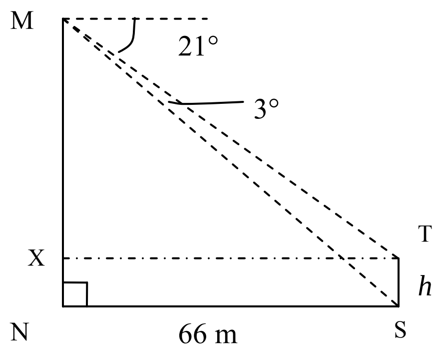
Then, in ΔBCD

$$\sin 40° = \frac{CD}{7.066...}$$
$$CD = (7.066...)\sin 40°$$
$$CD = 4.542...$$

The height of the monument is 4.5 m, to the nearest tenth of a metre.

Section 2.3 Page 110 Question 14

a)



b) Let N be at the base of the hotel, S be the base of the statue, and T the top of the statue.
In ΔMNS, ∠MSN = 21° + 3°, because of equal alternate angles.

$$\tan 24° = \frac{\text{MN}}{66}$$
$$\text{MN} = 66 \tan 24°$$
$$\text{MN} = 29.385...$$

Then, in ΔMTX, ∠MTX = 21°, because of equal alternate angles.

$$\tan 21° = \frac{\text{MX}}{66}$$
$$\text{MX} = 66 \tan 21°$$
$$\text{MX} = 25.335...$$

The height of the statue is MN − MX = 29.385 − 25.335 = 4.05, or 4.1 m to the nearest tenth.

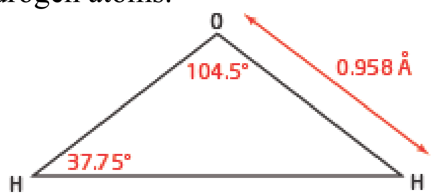
c) In ΔMNS,

$$\cos 24° = \frac{66}{\text{MS}}$$
$$\text{MS} = \frac{66}{\cos 24°}$$
$$\text{MS} = 72.245...$$

The line of sight distance from Max to the foot of the statue is 72.2 m, to the nearest metre.

Section 2.3 Page 110 Question 15

a) Let *d* represent the distance between the two hydrogen atoms.

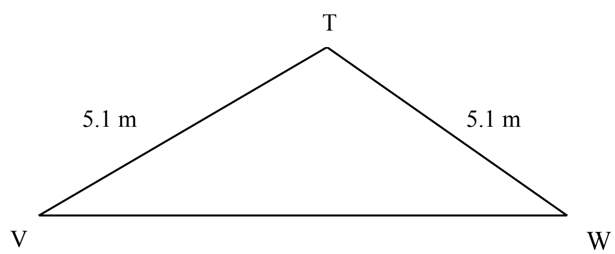


$$\frac{d}{\sin 104.5°} = \frac{0.958}{\sin 37.75°}$$
$$d = \frac{0.958 \sin 104.5°}{\sin 37.75°}$$
$$d = 1.514...$$

The distance between the two hydrogen atoms is 1.51 Å, to the nearest hundredth.

b) Given 1 Å = 0.01 mm, the distance between two hydrogen atoms is 1.51(0.01) or 0.0151 mm.

Section 2.3 Page 110 Question 16



For the greatest wingspan, ∠T = 132°.
Since the triangle is isosceles, ∠V = ∠W = 24°.

$$\frac{\text{VW}}{\sin 132°} = \frac{5.1}{\sin 24°}$$
$$\text{VW} = \frac{5.1 \sin 132°}{\sin 24°}$$
$$\text{VW} = 9.318...$$

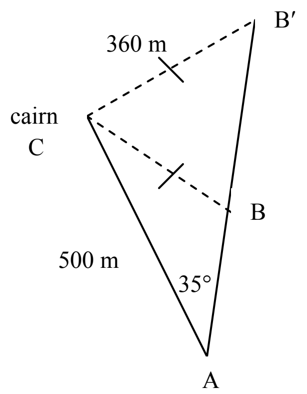
For the least wingspan, ∠T = 127°.
Since the triangle is isosceles, ∠V = ∠W = 26.5°.

$$\frac{\text{VW}}{\sin 127°} = \frac{5.1}{\sin 26.5°}$$
$$\text{VW} = \frac{5.1 \sin 127°}{\sin 26.5°}$$
$$\text{VW} = 9.128...$$

Then, to the nearest tenth of a metre, the greatest wingspan is 9.3 m and the least is 9.1 m.

Section 2.3 Page 110 Question 17

a)



500 sin 35° = 286.788…
Since this is less than 360 m, there are two possible triangles.
Determine possible measures of ∠B:

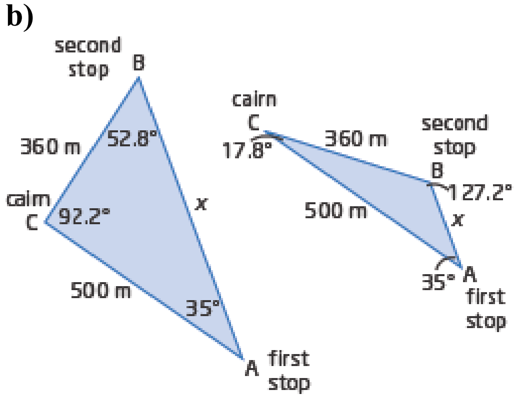
$$\frac{\sin B}{500} = \frac{\sin 35°}{360}$$

$$\angle B = \sin^{-1}\left(\frac{500 \sin 35°}{360}\right)$$

$$\angle B = 52.809...$$

So, ∠B ≈ 52.8° or
∠B ≈ 180° − 52.8° = 127.2°
Then, ∠C ≈ 92.2° or ∠C ≈ 17.8°.
Note: for accuracy in the next step, angles are recorded to the nearest tenth.

b)



c) $\frac{x}{\sin 92.2°} = \frac{360}{\sin 35°}$ or $\frac{x}{\sin 17.8°} = \frac{360}{\sin 35°}$

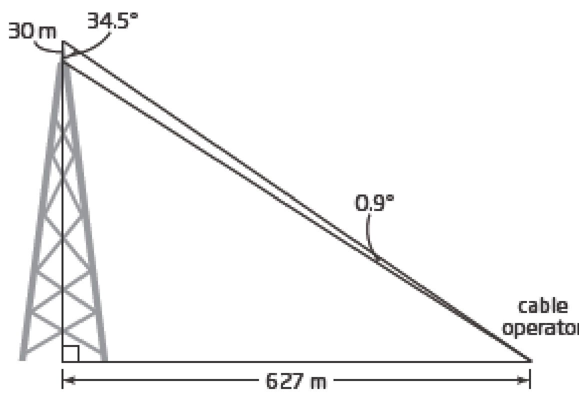
$$x = \frac{360 \sin 92.2°}{\sin 35°} \qquad x = \frac{360 \sin 17.8°}{\sin 35°}$$

$$x = 627.178... \qquad x = 191.866...$$

The possible distances between Armand's first and second stops are 191.9 m or 627.2 m, to the nearest tenth of a metre.

Section 2.3 Page 111 Question 18

First, use the oblique triangle to determine the distance, *d*, from the cable operator to the top of the tower.



$$\frac{d}{\sin 34.5°} = \frac{30}{\sin 0.9°}$$

$$d = \frac{30 \sin 34.5°}{\sin 0.9°}$$

$$d = 1081.800...$$

Then, use the Pythagorean Theorem in the smaller right triangle to determine the height, *h*, of the tower.

$$d^2 = h^2 + 627^2$$

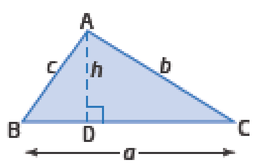
$$h^2 = \left(\frac{30 \sin 34.5°}{\sin 0.9°}\right)^2 - 627^2$$

$$h = \sqrt{\left(\frac{30 \sin 34.5°}{\sin 0.9°}\right)^2 - 627^2}$$

$$h = 881.568....$$

The height of the structure is 881.6 + 30 or 911.6 m, to the nearest tenth of a metre.

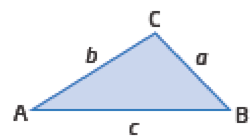
Section 2.3 Page 111 Question 19



Statements	Reasons
$\sin C = \frac{h}{b}$ $\sin B = \frac{h}{c}$	sin C ratio in △ACD sin B ratio in △ABD
$h = b \sin C$ $h = c \sin B$	Solve each ratio for *h*.
$b \sin C = c \sin B$	Equivalence property or substitution
$\frac{\sin C}{c} = \frac{\sin B}{b}$	Divide both sides by *bc*.

Section 2.3 Page 111 Question 20

Given ∠A = ∠B, prove $a = b$.



Using the sine law:

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

If ∠A = ∠B, then sin A = sin B.

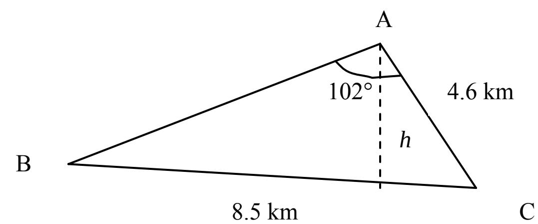
So, $\frac{a}{\sin A} = \frac{b}{\sin A}$

$$a = \frac{b \sin A}{\sin A}$$

$$a = b$$

Section 2.3 Page 111 Question 21

First, determine the height, h, of ΔABC.



$$\frac{\sin B}{4.6} = \frac{\sin 102°}{8.5}$$

$$\angle B = \sin^{-1}\left(\frac{4.6 \sin 102°}{8.5}\right)$$

$$\angle B = 31.961...$$

Then, ∠C ≈ 180° − (102° + 32°) = 46°.

$$\sin 46° = \frac{h}{4.6}$$

$$h = 4.6 \sin 46°$$

$$h = 3.308...$$

Now calculate the area.

$$A = \frac{1}{2}bh$$

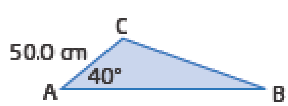
$$A = \frac{1}{2}(8.5)(3.308...)$$

$$A = 14.063...$$

The area of the oil spill was 14.1 km^2, to the nearest tenth.

Section 2.3 Page 111 Question 22

a)

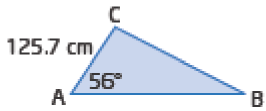


h = 50 sin 40°

h = 32.139…

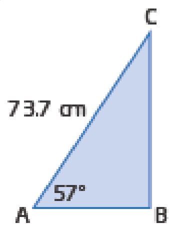
There are two possible solutions for ΔABC if 32.1 cm < a < 50.0 cm.

b)



$h = 125.7 \sin 56°$
$h = 104.210\ldots$
There are no possible solutions for ΔABC if $a < 104.2$ cm.

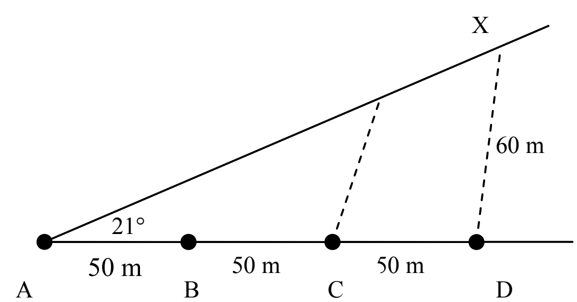
c)



$h = 73.7 \sin 57°$
$h = 61.810\ldots$
There is only one possible solution for ΔABC when $a = 61.8$ cm.

Section 2.3 Page 111 Question 23

First, determine whether the light from D will reach the pathway. Let h represent the shortest distance from D to the pathway.
$h = 150 \tan 21°$
$h = 57.579\ldots$
Since this distance is less than 60 m, the light from D will reach the pathway.



Check whether the light from the next streetlight will reach the pathway.
$h = 200 \tan 21°$
$h = 76.772\ldots$
Since this distance is more than 60 m, the light from the streetlight at 200 m will not reach the pathway.

Focus on ΔADX, where X is the point on the pathway that is 60 m from D.

$$\frac{\sin X}{150} = \frac{\sin 21°}{60}$$

$$\sin X = \frac{150 \sin 21°}{60}$$

$$\angle X = \sin^{-1}\left(\frac{150 \sin 21°}{60}\right)$$

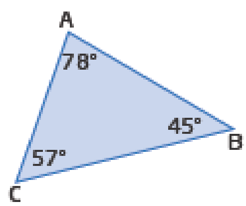
$$\angle X = 63.626...$$

Then, $\angle D = 180° - (21° + 63.6°) = 95.4°$

$$\frac{AX}{\sin 95.4°} = \frac{60}{\sin 21°}$$
$$AX = \frac{60 \sin 95.4°}{\sin 21°}$$
$$AX = 166.682...$$
The farthest point on the pathway that is lit is 166.7 m from A, to the nearest tenth of a metre.

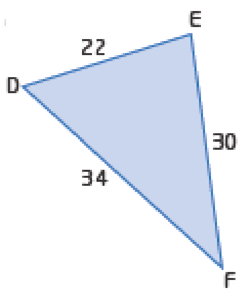
Section 2.3 Page 112 Question 24

a)



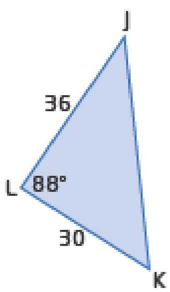
No side length is given, so in the sine law there will always be two unknowns.

b)



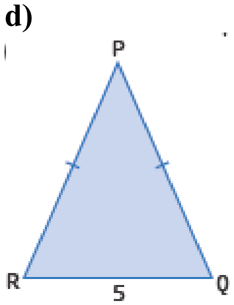
No angle measure is given, so in the sine law there will always be two unknowns.

c)



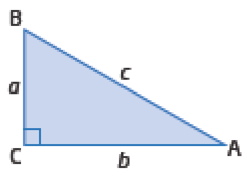
The side opposite the given angle cannot be matched with another side and angle.

d)



No angle measure is given and only one side is given, so in the sine law there will be three unknowns.

Section 2.3 Page 112 Question 25



Use the sine ratio twice:

$$\sin A = \frac{a}{c} \quad \text{and} \quad \sin B = \frac{b}{c}$$

$$c = \frac{a}{\sin A} \qquad c = \frac{b}{\sin B}$$

So, since both equal *c*:

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

Section 2.3 Page 112 Question 26

a) $$\frac{AB}{\sin 72°} = \frac{8}{\sin 36°}$$
$$AB = \frac{8\sin 72°}{\sin 36°}$$
$$AB = 12.944...$$

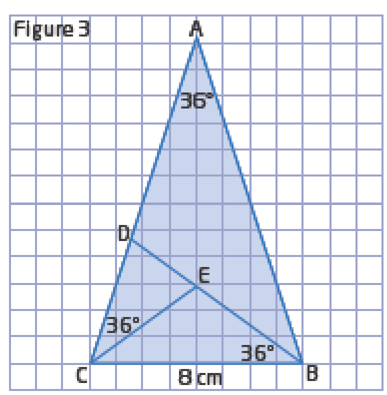
Using the sine law, the length of each equal side is 12.9 cm, to the nearest tenth of a centimetre.

b) $$\frac{AB}{8} = \frac{\frac{\sqrt{5}+1}{2}}{1}$$
$$AB = 8\left(\frac{\sqrt{5}+1}{2}\right)$$
$$AB = 4(\sqrt{5}+1)$$

Using the golden ratio, the exact length of each equal side is $4(\sqrt{5}+1)$ cm.

c) $$\frac{CD}{\sin 36°} = \frac{8}{\sin 72°}$$
$$CD = \frac{8\sin 36°}{\sin 72°}$$
$$CD = 4.944......$$

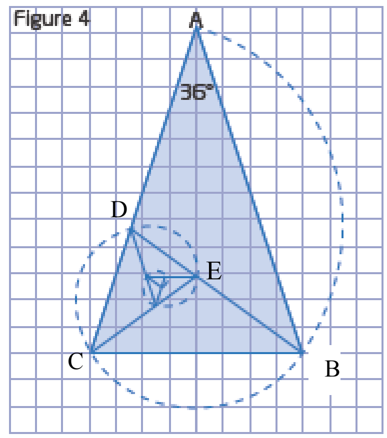
CD = 4.9 cm, to the nearest tenth of a centimetre.



d) $$\frac{DE}{\sin 36°} = \frac{4.944...}{\sin 72°}$$
$$DE = \frac{(4.944...)\sin 36°}{\sin 72°}$$
$$DE = 3.055...$$

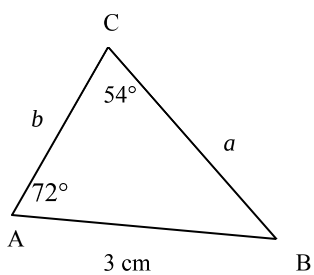
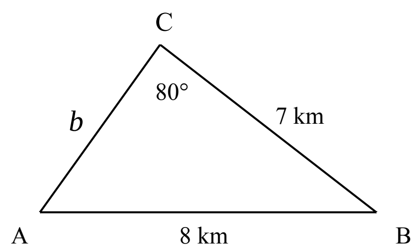
DE = 3.1 cm, to the nearest tenth of a centimetre.

e) The spiral is created by drawing arcs of a circle on one of the equal sides with centre at one of the equal angles in the next step. For example, the first arc is $\overset{\frown}{AB}$ with centre at D. Then, the second arc is $\overset{\frown}{BC}$ with centre at E.



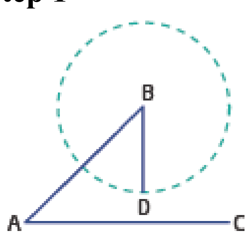
Section 2.3 Page 112 Question 27

Answers may vary. Example:
The sine law can be used to solve an oblique triangle in two cases:

You are given two angles (which means you easily know three angles) and one side.	You are given two sides and one angle that is opposite one of the given sides.
↓↓	↓↓
For example, you can solve the triangle shown. Use angle sum of a triangle to find the measure of ∠B and then use the sine law twice to determine lengths *a* and *b*. 	For example, you can solve the triangle shown. Use the sine law to find the measure of ∠A. Then, use angle sum of a triangle to determine ∠B. Use the sine law again to calculate *b*. 

Section 2.3 Page 112 Question 28

Step 1



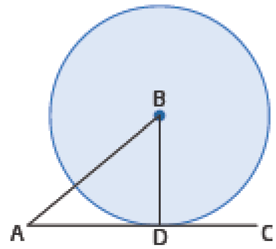
Step 2 a) BD is too short to reach AC, so a triangle cannot be formed.

b) No triangle is formed when BD is less than the perpendicular distance from B to AC.

Step 3

a) Yes, a triangle is formed when the circle just touches the line.

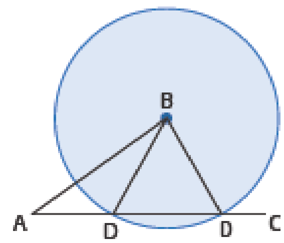
b) Exactly one triangle is formed when BD is the perpendicular distance from B to AC.



Step 4

a) Yes, when the circle cuts the line in two places, two triangles can be formed.

b) Two triangles can be drawn when BD is greater than the perpendicular distance from B to AC but BD must be shorter than BA.



Step 5

a) One triangle can be formed when the string is longer than AB but less than AC.

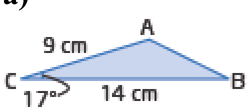
b) When the radius BD is greater than AB but less than AC, one triangle is formed. If the radius is also greater than AC then no triangle is formed.

Step 6 The results will apply so long as ∠A is acute.

Section 2.4 The Cosine Law

Section 2.4 Page 119 Question 1

a)



$c^2 = 14^2 + 9^2 - 2(14)(9) \cos 17°$

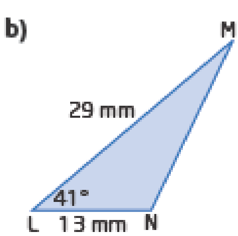
$c^2 = 36.011\ldots$

$c = \sqrt{36.011...}$

$c = 6.000...$

The length of side AB is 6.0 cm to the nearest tenth of a centimetre.

b)



$l^2 = 13^2 + 29^2 - 2(13)(29) \cos 41°$

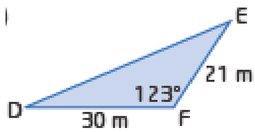
$l^2 = 440.094\ldots$

$l = \sqrt{440.948...}$

$l = 20.998...$

The length of MN is 21.0 mm to the nearest tenth of a millimetre.

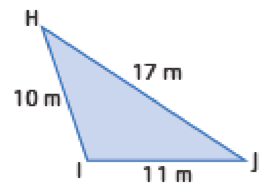
c)



$f^2 = 30^2 + 21^2 - 2(30)(21)\cos 123°$
$f^2 = 36.011\ldots$
$c = \sqrt{2027.245\ldots}$
$c = 45.024\ldots$
The length of DE is 45.0 m to the nearest tenth of a metre.

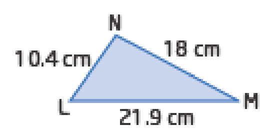
Section 2.4 Page 119 Question 2

a)



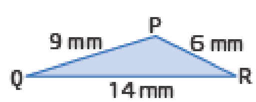
$10^2 = 17^2 + 11^2 - 2(17)(11)\cos J$
$100 = 289 + 121 - 374\cos J$
$374\cos J = 289 + 121 - 100$
$\cos J = \frac{310}{374}$
$\angle J = \cos^{-1}\left(\frac{310}{374}\right)$
$\angle J = 34.016...$
$\angle J = 34°$, to the nearest degree.

b)



$18^2 = 10.4^2 + 21.9^2 - 2(10.4)(21.9)\cos L$
$324 = 108.16 + 479.61 - 455.52\cos L$
$455.52\cos L = 587.77 - 324$
$\cos L = \frac{263.77}{455.52}$
$\angle L = \cos^{-1}\left(\frac{263.77}{455.52}\right)$
$\angle L = 54.616...$
$\angle L = 55°$, to the nearest degree.

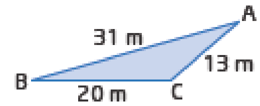
c)



$14^2 = 9^2 + 6^2 - 2(9)(6)\cos P$
$196 = 81 + 36 - 108\cos P$
$108\cos P = 117 - 196$
$\cos P = -\frac{79}{108}$
$\angle P = \cos^{-1}\left(-\frac{79}{108}\right)$
$\angle P = 137.010...$
$\angle P = 137°$, to the nearest degree.

d)
$31^2 = 20^2 + 13^2 - 2(20)(13)\cos C$
$961 = 400 + 169 - 520\cos C$
$520\cos C = 569 - 961$

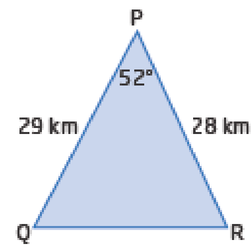
$$\cos C = -\frac{392}{520}$$
$$\angle C = \cos^{-1}\left(-\frac{392}{520}\right)$$
$$\angle C = 138.924...$$
$\angle C = 139°$, to the nearest degree.



Section 2.4 Page 120 Question 3

a) Use the cosine law to solve for *p*.

$p^2 = 29^2 + 28^2 - 2(29)(28) \cos 52°$
$p^2 = 625.165\ldots$
$p = \sqrt{625.165...}$
$p = 25.003...$
$p = 25.0$ km, to the nearest tenth of a kilometre.



Use the sine law to solve for ∠Q.

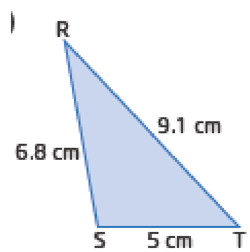
$$\frac{\sin Q}{28} = \frac{\sin 52°}{25.003...}$$
$$\sin Q = \frac{28 \sin 52°}{25.003...}$$
$$\angle Q = \sin^{-1}\left(\frac{28 \sin 52°}{25.003...}\right)$$
$$\angle Q = 61.939...$$
$\angle Q = 62°$, to the nearest degree.

Use the angle sum of a triangle to determine ∠R.

$\angle R = 180° - (52° + 62°)$
$\angle R = 66°$

b) Use the cosine law to solve for ∠S.

$9.1^2 = 6.8^2 + 5^2 - 2(6.8)(5) \cos S$
$82.81 = 46.24 + 25 - 68 \cos S$
$68 \cos S = 71.24 - 82.81$
$$\cos S = -\frac{11.57}{68}$$
$$\angle S = \cos^{-1}\left(-\frac{11.57}{68}\right)$$
$$\angle S = 99.796...$$
$\angle S = 100°$, to the nearest degree.



Use the sine law to solve for ∠T.

$$\frac{\sin T}{6.8} = \frac{\sin 99.796...°}{9.1}$$
$$\sin T = \frac{6.8 \sin 99.796...°}{9.1}$$
$$\angle T = \sin^{-1}\left(\frac{6.8 \sin 99.796...°}{9.1}\right)$$
$$\angle T = 47.421...$$

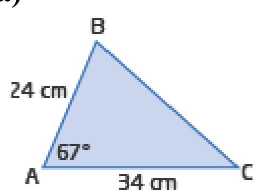
∠T = 47°, to the nearest degree.

Use the angle sum of a triangle to determine ∠R.

$\angle R = 180° - (100° + 47°)$
$\angle R = 33°$

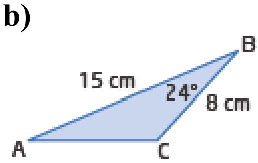
Section 2.4 Page 120 Question 4

a)



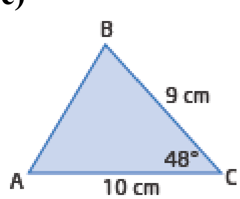
$BC^2 = 24^2 + 34^2 - 2(24)(34) \cos 67°$
$BC^2 = 1094.326…$
$BC^2 = \sqrt{1094.326...}$
$BC = 33.080...$
BC = 33.1 cm, to the nearest tenth of a centimetre.

b)



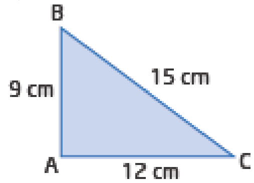
$AC^2 = 15^2 + 8^2 - 2(15)(8) \cos 24°$
$AC^2 = 69.749…$
$AC^2 = \sqrt{69.749...}$
$AC = 8.351...$
AC = 8.4 cm, to the nearest tenth of a centimetre.

c)



$AB^2 = 10^2 + 9^2 - 2(10)(9) \cos 48°$
$AB^2 = 60.556…$
$AB^2 = \sqrt{60.556...}$
$AB = 7.781...$
AB = 7.8 cm, to the nearest tenth of a centimetre.

d)



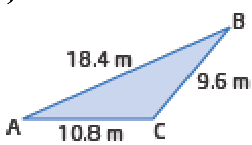
ΔABC is a right triangle, because $15^2 = 9^2 + 12^2$.

Use the sine ratio.

$$\sin B = \frac{12}{15}$$
$$\angle B = \sin^{-1}\left(\frac{12}{15}\right)$$
$$\angle B = 53.130...$$

$\angle$B = 53°, to the nearest degree.

e)

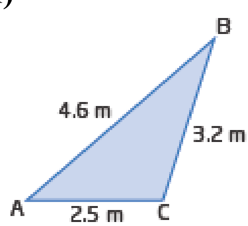


$9.6^2 = 10.8^2 + 18.4^2 - 2(10.8)(18.4) \cos A$
$92.19 = 116.64 + 338.56 - 397.44 \cos A$
$397.44 \cos A = 455.2 - 92.19$

$$\cos A = \frac{363.01}{397.44}$$
$$\angle A = \cos^{-1}\left(\frac{363.01}{397.44}\right)$$
$$\angle A = 24.024...$$

$\angle$A = 24°, to the nearest degree.

f)



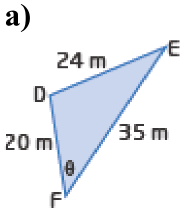
$4.6^2 = 3.2^2 + 2.5^2 - 2(3.2)(2.5) \cos C$
$21.16 = 10.24 + 6.25 - 16 \cos C$
$16 \cos C = 16.49 - 21.16$

$$\cos C = -\frac{4.67}{16}$$
$$\angle C = \cos^{-1}\left(-\frac{4.67}{16}\right)$$
$$\angle C = 106.970...$$

$\angle$C = 107°, to the nearest degree.

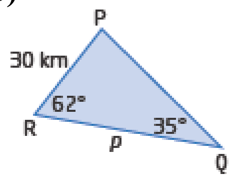
Section 2.4 Page 120 Question 5

a)



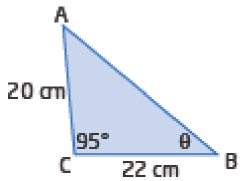
Use the cosine law to determine $\angle$F because three sides are given, but no angle.

b)



Since two angles and one side are given, you can use the sine law to find *p*. First use the angle sum of a triangle to determine ∠P.

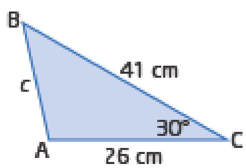
c)



This case will need both, since two sides and the angle contained between them is given. First use the cosine law to determine AB. Then, use the sine law to determine ∠B.

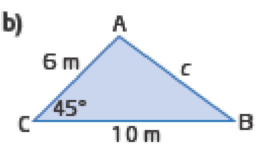
Section 2.4 Page 120 Question 6

a)



$$c^2 = 26^2 + 41^2 - 2(26)(41)\cos 30°$$
$$c^2 = 676 + 1681 - 2(26)(41)\frac{\sqrt{3}}{2}$$
$$c^2 = 2357 - 1066\sqrt{3}$$
$$c = \sqrt{2357 - 1066\sqrt{3}} \text{ cm}$$

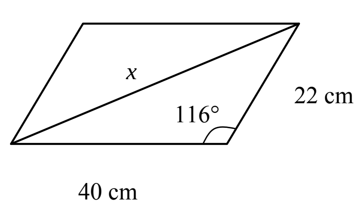
b)



$$c^2 = 6^2 + 10^2 - 2(6)(10)\cos 45°$$
$$c^2 = 36 + 100 - 120\left(\frac{\sqrt{2}}{2}\right)$$
$$c^2 = 136 - 60\sqrt{2}$$
$$c^2 = 4(34 - 15\sqrt{2})$$
$$c = \sqrt{4(34 - 15\sqrt{2})}$$
$$c = 2\sqrt{34 - 15\sqrt{2}} \text{ m}$$

Section 2.4 Page 120 Question 7

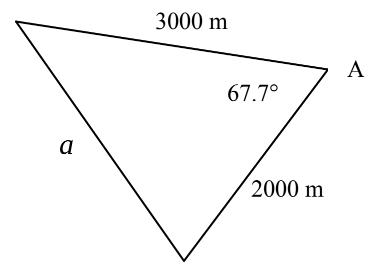
$$x^2 = 40^2 + 22^2 - 2(40)(22)\cos 116°$$
$$x^2 = 2855.553\ldots$$
$$x = \sqrt{2855.553...}$$
$$x = 53.437\ldots$$



The longest diagonal is 53.4 cm long, to the nearest tenth of a centimetre.

Section 2.4 Page 120 Question 8

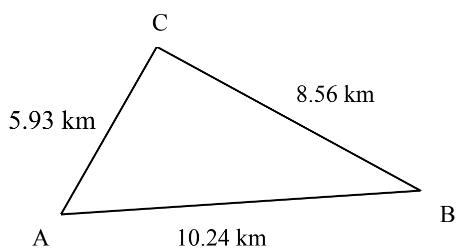
Let a represent the length of the tunnel.



$a^2 = 2000^2 + 3000^2 - 2(2000)(3000) \cos 67.7°$
$a^2 = 8\ 446\ 526.086\ldots$
$a = \sqrt{8\ 446\ 526.086...}$
$a = 2906.290\ldots$

The length of the tunnel would be 2906 m, to the nearest metre.

Section 2.4 Page 120 Question 9



$8.56^2 = 5.93^2 + 10.24^2 - 2(5.93)(10.24) \cos A$
$121.4464 \cos A = 66.7489$
$\cos A = \dfrac{66.7489}{121.4464}$
$\angle A = \cos^{-1}\left(\dfrac{66.7489}{121.4464}\right)$
$\angle A = 56.659...$
$\angle A = 57°$, to the nearest degree.

Next, use the sine law to determine $\angle B$.

$\dfrac{\sin B}{5.93} = \dfrac{\sin 56.659...°}{8.56}$
$\angle B = \sin^{-1}\left(\dfrac{5.93 \sin 56.659...°}{8.56}\right)$
$\angle B = 35.362...$
$\angle B = 35°$, to the nearest degree.

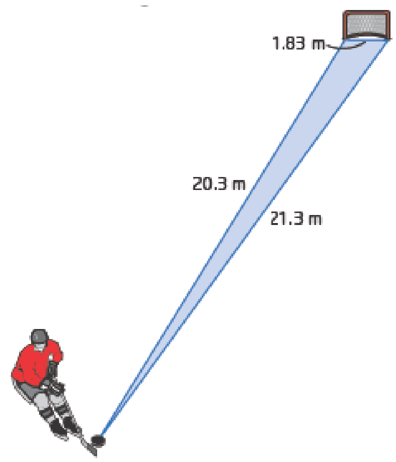
Use the angle sum of a triangle to determine $\angle C$.
$\angle C = 180° - (57° + 35°)$
$\angle C = 88°$

Section 2.4 Page 121 Question 10

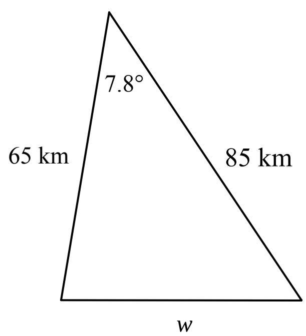
Let $\angle P$ represent the required angle.

$$1.83^2 = 20.3^2 + 21.3^2 - 2(20.3)(21.3) \cos P$$
$$864.78 \cos P = 862.4311$$
$$\cos P = \frac{862.4311}{864.78}$$
$$\angle P = \cos^{-1}\left(\frac{862.4311}{864.78}\right)$$
$$\angle P = 4.223...$$

She must shoot within an angle of 4.2° to hit the net.



Section 2.4 Page 121 Question 11



Let w represent the width at the base of the bay.

$w^2 = 65^2 + 85^2 - 2(65)(85) \cos 7.8°$

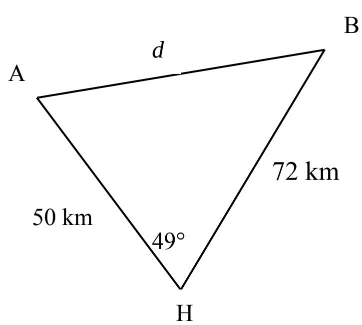
$w^2 = 502.236…$

$w = \sqrt{502.236...}$

$w = 22.410…$

The width of the bay at its base is 22.4 km, to the nearest tenth of a kilometre.

Section 2.4 Page 121 Question 12



Let d represent the distance between the two aircraft.

$d^2 = 50^2 + 72^2 - 2(50)(72) \cos 49°$

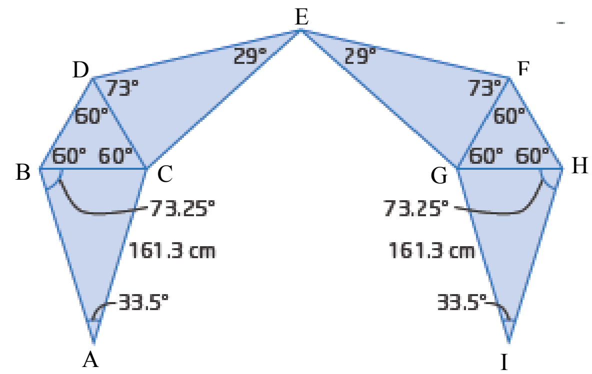
$d^2 = 2960.374…$

$d = \sqrt{2960.374...}$

$d = 54.409…$

The distance between the two aircraft is 54.4 km, to the nearest tenth of a kilometre.

Section 2.4 Page 121 Question 13



The maximum width is BH,
which is BC + CG + GH.
In ΔABC:

$$\frac{BC}{\sin 33.5°} = \frac{161.3}{\sin 73.25°}$$

$$BC = \frac{161.3 \sin 33.5°}{\sin 73.25°}$$

$$BC = 92.972...$$

Since ΔGHI ≅ ΔABC,
GH = 92.972… also.

ΔBCD is equilateral, so CD = BC = 92.972… also. Likewise, FG = 92.972…
In ΔCDE:

$$\frac{CE}{\sin 73°} = \frac{92.972}{\sin 29°}$$

$$CE = \frac{92.972 \sin 73°}{\sin 29°}$$

$$CE = 183.390...$$

Since ΔEFG ≅ ΔCDE, EG = 183.390… also.
$\angle DCE = 180° - (29° + 73°)$
$\angle DCE = 78°$
Then, in ΔCEG:
$\angle ECG = 180° - (60° + 78°)$
$\angle ECG = 42°$
Since the triangle is isosceles, $\angle EGC = 42°$ also.
Then, $\angle CEG = 180° - (42° + 42°)$
$\angle CEG = 96°$

$$\frac{CG}{\sin \angle CEG} = \frac{EC}{\sin \angle EGC}$$

$$\frac{CG}{\sin 96°} = \frac{183.390}{\sin 42°}$$

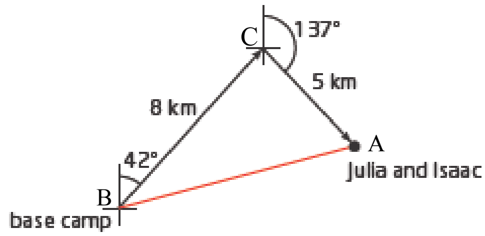
$$CG = \frac{183.390 \sin 96°}{\sin 42°}$$

$$CG = 272.570...$$

Finally, BC + CG + GH = 92.97 + 272.57 + 92.97 = 458.51.
So, the maximum width of Moondog is 458.5 cm.

Section 2.4 Page 121 Question 14

a)



b) $\angle C = 42° + 43°$ (alternate angle + supplement of the exterior angle)

$\angle C = 85°$

$AB^2 = 8^2 + 5^2 - 2(8)(5) \cos 85°$

$AB^2 = 82.027\ldots$

$AB = \sqrt{82.027...}$

$AB = 9.056\ldots$

Julia and Isaac are 9.1 km from the base camp, to the nearest tenth of a kilometre.

c) $\frac{\sin A}{8} = \frac{\sin 85°}{9.056...}$

$\sin A = \frac{8 \sin 85°}{9.056...}$

$\angle A = \sin^{-1}\left(\frac{8 \sin 85°}{9.056...}\right)$

$\angle A = 61.635...$

Then the heading that they must travel, from South, is 137° − 62°, or S75°W. This is a bearing of 180° + 75° or 255°.

Section 2.4 Page 122 Question 15

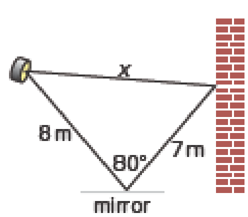
$x^2 = 8^2 + 7^2 - 2(8)(7) \cos 80°$

$x^2 = 93.551\ldots$

$x = \sqrt{93.551...}$

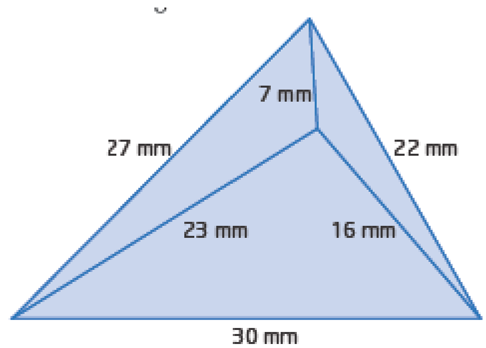
$x = 9.672\ldots$

The distance from the spotlight to the point on the wall where the light is reflected is 9.7 m, to the nearest tenth of a metre.



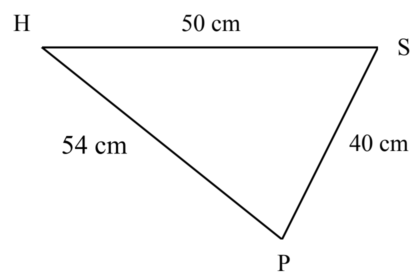
Section 2.4 Page 122 Question 16

Answers may vary. Example:
Use the cosine law in each of the smaller triangles to determine the measure of each of the three angles that meet in the middle of the design. The sum of those three angles should be 360°.



Section 2.4 Page 122 Question 17

The diagram shows the triangular part of the frame. ∠H is the head tube angle, ∠S is the seat angle, and ∠P is the angle at the peddle.



Use the cosine law to determine ∠S.

$$54^2 = 50^2 + 40^2 - 2(50)(40) \cos S$$
$$2916 = 4100 - 4000 \cos S$$
$$4000 \cos S = 4100 - 2916$$
$$\cos S = \frac{1184}{4000}$$
$$\angle S = \cos^{-1}\left(\frac{1184}{4000}\right)$$
$$\angle S = 72.782...$$

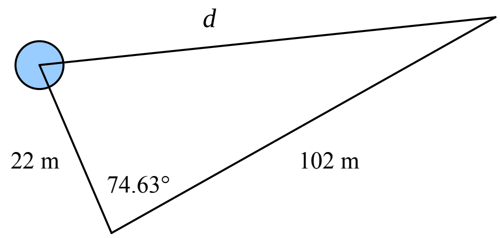
Use the sine law to determine ∠H.

$$\frac{\sin H}{40} = \frac{\sin 72.782...}{54}$$
$$\sin H = \frac{40 \sin 72.782...}{54}$$
$$\angle H = \sin^{-1}\left(\frac{40 \sin 72.782...}{54}\right)$$
$$\angle H = 45.035...$$

Use angle sum of a triangle to determine ∠P.
$\angle P = 180° - (45° + 73°)$
$\angle P = 62°$
The interior angles of the bike frame are 73°, 45°, and 62°.

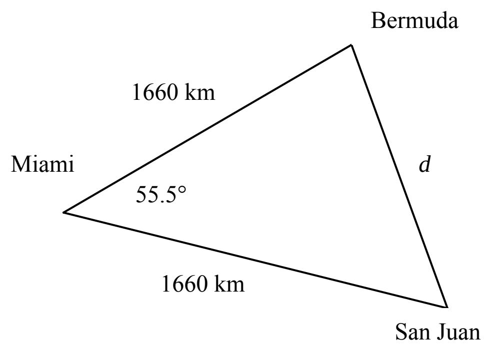
Section 2.4 Page 122 Question 18

$d^2 = 22^2 + 102^2 - 2(22)(102) \cos 74.63°$
$d^2 = 9698.449\ldots$
$d = \sqrt{9698.449...}$
$d = 98.480\ldots$
Barbora Spotakova threw the javelin 98.48 m, to the nearest hundredth of a metre.



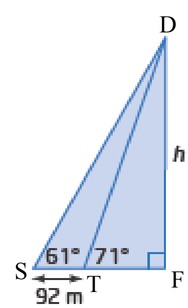
Section 2.4 Page 122 Question 19

$d^2 = 1660^2 + 1660^2 - 2(1660)(1660) \cos 55.5°$
$d^2 = 2\ 389\ 621.947\ldots$
$d = \sqrt{2\ 389\ 621.947...}$
$d = 1545.840\ldots$
The distance from Bermuda to San Juan is 1546 km, to the nearest kilometre.



Section 2.4 Page 123 Question 20

In ΔDTS,
$\angle DTS = 180° - 71°$ (supplementary angles)
$\angle DTS = 109°$
Then, by angle sum of a triangle,
$\angle RDS = 180° - (61° + 109°)$
$\angle RDS = 10°$



Now use the sine law to determine DT.

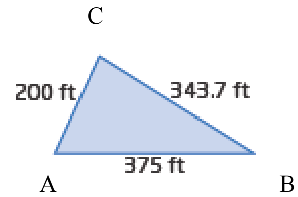
$$\frac{DT}{\sin 61°} = \frac{92}{\sin 10°}$$
$$DT = \frac{92 \sin 61°}{\sin 10°}$$
$$DT = 463.379...$$

Then, in ΔDFT:

$$\sin 71° = \frac{h}{463.379...}$$
$$h = (463.379...) \sin 71°$$
$$h = 438.133...$$

The height of Della Falls is 438.1 m, to the nearest tenth of a metre.

Section 2.4 Page 123 Question 21



First use the cosine law to determine ∠A.

$$343.7^2 = 200^2 + 375^2 - 2(200)(375) \cos A$$
$$150\ 000 \cos A = 62\ 495.31$$
$$\cos A = \frac{62\ 495.31}{150\ 000}$$
$$\angle A = \cos^{-1}\left(\frac{62\ 495.31}{150\ 000}\right)$$
$$\angle A = 65.377...$$

∠A is 65°, to the nearest degree.

Next use the sine law to determine ∠B.

$$\frac{\sin B}{200} = \frac{\sin 65.377...^\circ}{343.7}$$
$$\sin B = \frac{200 \sin 65.377...^\circ}{343.7}$$
$$\angle B = \sin^{-1}\left(\frac{200 \sin 65.377...^\circ}{343.7}\right)$$
$$\angle B = 31.937...$$

∠B is 32°, to the nearest degree.

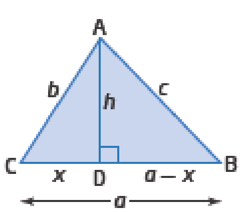
Then use angle sum of a triangle to determine ∠C.

$\angle C = 180^\circ - (65^\circ + 32^\circ)$

$\angle C = 83^\circ$

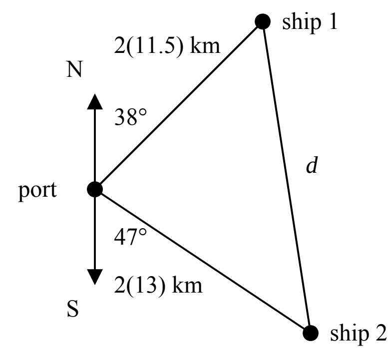
The interior angles of the building are 65°, 32°, and 83°.

Section 2.4 Page 123 Question 22



Statement	Reason
$c^2 = (a - x)^2 + h^2$	Use Pythagorean Theorem in ΔABD.
$c^2 = a^2 - 2ax + x^2 + h^2$	Expand the square of a binomial.
$b^2 = x^2 + h^2$	Use Pythagorean Theorem in ΔACD.
$c^2 = a^2 - 2ax + b^2$	Substitute b^2 for $x^2 + h^2$.
$\cos C = \frac{x}{b}$	Use the cosine ratio in ΔACD.
$x = b \cos C$	Multiply both sides by b.
$c^2 = a^2 - 2ab \cos C + b^2$	Substitute $b \cos C$ for x in step 4.
$c^2 = a^2 + b^2 - 2ab \cos C$	Rearrange the terms.

Section 2.4 Page 123 Question 23

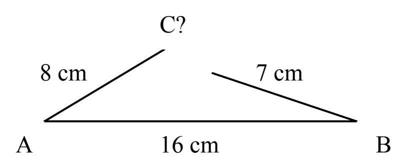


From 4 p.m. until 6 p.m. is 2 h. In this time, the first ships will have travelled 2(11.5) or 23 km. The second ship will have travelled 2(13) or 26 km.

$\angle SPT = 180° - (38° + 47°)$
$\angle SPT = 95°$
$d^2 = 23^2 + 26^2 - 2(23)(26) \cos 95°$
$d^2 = 1309.238\ldots$
$d = \sqrt{1309.238...}$
$d = 36.183\ldots$

At 6 p.m. the two ships are 36.2 km apart, to the nearest tenth of a kilometre.

Section 2.4 Page 123 Question 24



It is not possible to draw a triangle with side lengths 7 cm, 8 cm, and 16 cm because the sum of the two shorter sides is less than the longest side.

Try substituting into the cosine law:

$$16^2 = 7^2 + 8^2 - 2(7)(8) \cos C$$

$112 \cos C = -143$

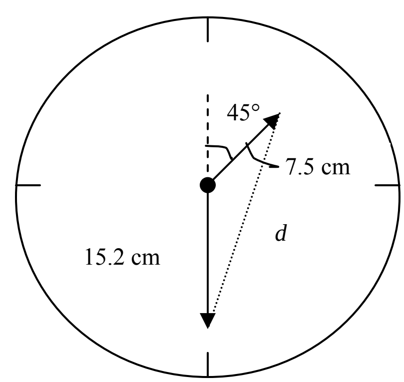
$$\cos C = -\frac{143}{112}$$

$$\angle C = \cos^{-1}\left(-\frac{143}{112}\right)$$

$$\angle C = \text{error}$$

An error message is returned, because cosine cannot be less than −1.

Section 2.4 Page 123 Question 25



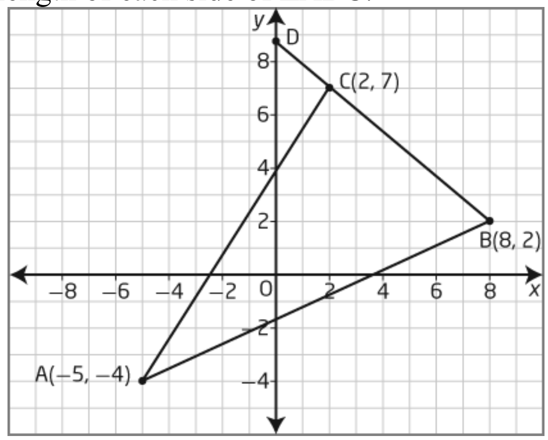
At 1:30 the hour hand will be half way between 12:00 and 3:00, so the angle from 12:00 is 45°. The angle between the two hands will be 135°.

$d^2 = 7.5^2 + 15.2^2 - 2(7.5)(15.2) \cos 135°$
$d^2 = 448.510$
$d = \sqrt{448.510...}$
$d = 21.178\ldots$

At 1:30 p.m. the distance between the tip of the minute and hour hands is 21.2 cm, to the nearest tenth of a centimetre.

Section 2.4 Page 123 Question 26

Use the Pythagorean Theorem to determine the length of each side of ΔABC.



$AB^2 = (2 - (-4))^2 + (8 - (-5))^2$
$AB^2 = 36 + 169$
$AB = \sqrt{205}$

$BC^2 = (7 - 2)^2 + (2 - 8)^2$
$BC^2 = 25 + 36$
$BC = \sqrt{61}$

$CA^2 = (7 - (-4))^2 + (2 - (-5))^2$
$CA^2 = 121 + 49$
$CA = \sqrt{170}$

Use the cosine law to determine $\angle$ABC.

$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$170 = 61 + 205 - 2(\sqrt{61})(\sqrt{205}) \cos B$$
$$2(\sqrt{61})(\sqrt{205}) \cos B = 266 - 170$$
$$\cos B = \frac{96}{2\sqrt{61}\sqrt{205}}$$
$$\angle B = \cos^{-1}\left(\frac{96}{2\sqrt{61}\sqrt{205}}\right)$$
$$\angle B = 64.580...$$

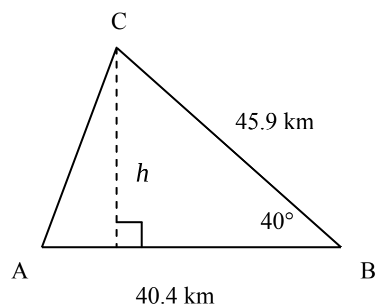
The measure of interior angle $\angle$ABC is 65°, to the nearest degree.

Now use the sine law to determine $\angle$BCA.

$$\frac{\sin C}{\sqrt{205}} = \frac{\sin 64.580...°}{\sqrt{170}}$$
$$\sin C = \frac{\sqrt{205}\sin 64.580...°}{\sqrt{170}}$$
$$\angle C = \sin^{-1}\left(\frac{\sqrt{205}\sin 64.580...°}{\sqrt{170}}\right)$$
$$\angle C = 82.665...$$

Then, $\angle ACD = 180° - 83°$
$\angle ACD = 97°$, to the nearest degree.

Section 2.4 Page 124 Question 27



First determine the height.

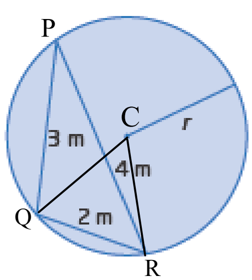
$$\sin 40° = \frac{h}{45.9}$$
$$h = 45.9 \sin 40° h.$$
$$h = 29.503...$$

Now, use the formula for area of a triangle.

$$A = \frac{bh}{2}$$
$$A = \frac{(40.4)(29.503...)}{2}$$
$$A = 595.979...$$

The area of the region is 596 km^2, to the nearest square kilometre.

Section 2.4 Page 124 Question 28



Join the centre, C, to two vertices of the triangle, Q and R. CR and CQ are radii. Use the cosine law to determine ∠QPR.

$$2^2 = 3^2 + 4^2 - 2(3)(4) \cos P$$
$$24 \cos P = 21$$
$$\cos P = \frac{21}{24}$$
$$\angle P = \cos^{-1}(0.875)$$
$$\angle P = 28.955...$$

The angle at the centre is twice the angle subtended from the same arc.

So, $\angle QCR = 2(\angle QPR)$
$$\angle QCR = 2(29°)$$
$$\angle QCR = 58°$$

ΔCRQ is isosceles, so $\angle CQR = \angle CRQ = \frac{1}{2}(180° - 58°)$ or 61°.

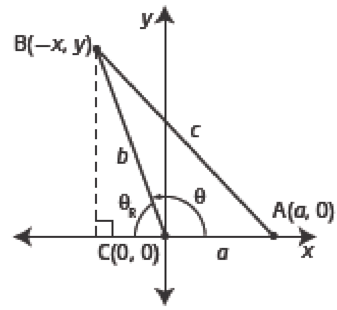
Use the sine law in ΔCQR to determine *r*.

$$\frac{r}{\sin 61°} = \frac{2}{\sin 58°}$$
$$r = \frac{2\sin 61°}{\sin 58°}$$
$$r = 2.062...$$

The radius of the circle is 2.1 m, to the nearest tenth of a metre.

Section 2.4 Page 124 Question 29

$\cos\ \theta_R = -\cos\theta = -\dfrac{x}{\sqrt{x^2+y^2}}$



$b = \sqrt{x^2+y^2}$

$c = \sqrt{(a+x)^2+y^2}$

Prove $c^2 = a^2 + b^2 - 2ab\cos C$:

$$\begin{aligned}\text{Left Side} &= \left(\sqrt{(a+x)^2+y^2}\right)^2 \\ &= (a+x)^2+y^2 \\ &= a^2+2ax+x^2+y^2\end{aligned}$$

$$\begin{aligned}\text{Right Side} &= a^2+\left(\sqrt{x^2+y^2}\right)^2 - 2a\left(\sqrt{x^2+y^2}\right)\left(-\frac{x}{\sqrt{x^2+y^2}}\right) \\ &= a^2+x^2+y^2+2ax \\ &= a^2+2ax+x^2+y^2\end{aligned}$$

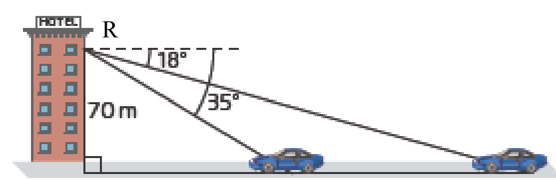
Left Side = Right Side

Therefore, the cosine law is true.

Section 2.4 Page 124 Question 30

In right triangle ABR:

$\angle RBA = 35°$ (alternate angles)



$$\sin 35° = \frac{70}{RB}$$

$$RB = \frac{70}{\sin 35°}$$

$$RB = 122.041...$$

In ΔRBC:

$\angle CRB = 35° - 18° = 17°$

$\angle RCB = 18°$ (alternate angles)

Use the sine law to determine BC.

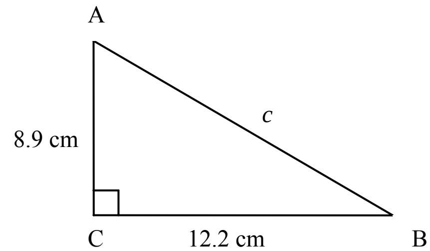
$$\frac{BC}{\sin 17°} = \frac{122.041...}{\sin 18°}$$

$$BC = \frac{122.041...\sin 17°}{\sin 18°}$$

$$BC = 115.467...$$

The car has travelled 115.5 m, to the nearest tenth of a metre.

Section 2.4 Page 124 Question 31



a) $c^2 = 12.2^2 + 8.9^2 - 2(12.2)(8.9)\cos 90°$
$c^2 = 228.05$

b) $c^2 = 12.2^2 + 8.9^2$
$c^2 = 228.05$

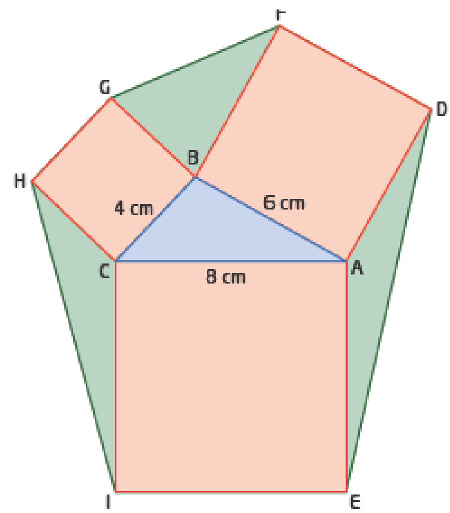
c) The cosine law is similar to the Pythagorean Theorem, but it has the extra term $-2ab\cos C$. The equations are the same when $\angle C = 90°$.

d) The two formulas are the same in a right triangle because $\cos 90° = 0$, so the extra term has value 0.

Section 2.4 Page 124 Question 32

Concept Summary for Solving a Triangle	
Given	**Begin by Using the Method of**
Right triangle	A
Two angles and any side	B
Three sides	C
Three angles	D
Two sides and the included angle	C
Two sides and the angle opposite one of them	B

Section 2.4 Page 125 Question 33



Step 1
a)-c) Construct the figure, actual size, on a full sheet of paper.

Step 2
a) Use the cosine law to determine ∠A.
$4^2 = 6^2 + 8^2 - 2(6)(8) \cos A$
$96 \cos A = 84$
$$\cos A = \frac{84}{96}$$
$$\angle A = \cos^{-1}(0.875)$$
$$\angle A = 28.955...$$

Use the sine law to determine ∠C.
$$\frac{\sin C}{6} = \frac{\sin 28.955...}{4}$$
$$\sin C = \frac{6 \sin 28.955...}{4}$$
$$\angle C = \sin^{-1}(1.5 \sin 28.955...)$$
$$\angle C = 46.567...$$
Use the angle sum of a triangle to determine ∠B.
$\angle B = 180° - (29° + 47°)$
$\angle B = 104°$

b) There are four angles meeting at each vertex of ΔABC. Two of the angles are corners of squares, so they are each 90°. At any vertex, A, B, or C, the other two angles must have a sum of 360° − 2(90°) or 180°.

$\angle GBF = 180° - \angle ABC$	$\angle HCI = 180° - \angle BCA$	$\angle DAE = 180° - \angle BAC$
$\angle GBF = 180° - 104°$	$\angle HCI = 180° - 47°$	$\angle DAE = 180° - 29°$
$\angle GBF = 76°$	$\angle HCI = 133°$	$\angle DAE = 151°$

c) In ΔBGF, BG = BC = 4 cm, BF = BA = 6 cm.
Use the cosine law to determine GF.
$GF^2 = 4^2 + 6^2 - 2(4)(6)\cos 76°$
$GF^2 = 40.387\ldots$
$GF = \sqrt{40.387...}$
$GF = 6.355\ldots$
The length of GF is 6.4 cm, to the nearest tenth of a centimetre.

In ΔADE, AD = AB = 6 cm, AE = AC = 8 cm.
Use the cosine law to determine GF.
$DE^2 = 6^2 + 8^2 - 2(6)(8)\cos 151°$

$DE^2 = 183.963\ldots$

$DE = \sqrt{183.963...}$

$DE = 13.563\ldots$

The length of DE is 13.6 cm, to the nearest tenth of a centimetre.

In ΔCHI, CH = CB = 4 cm, CI = AC = 8 cm.

Use the cosine law to determine HI.

$HI^2 = 4^2 + 8^2 - 2(4)(8)\cos 133°$

$HI^2 = 123.647\ldots$

$HI = \sqrt{123.647...}$

$HI = 11.119\ldots$

The length of HI is 11.1 cm, to the nearest tenth of a centimetre.

Step 3

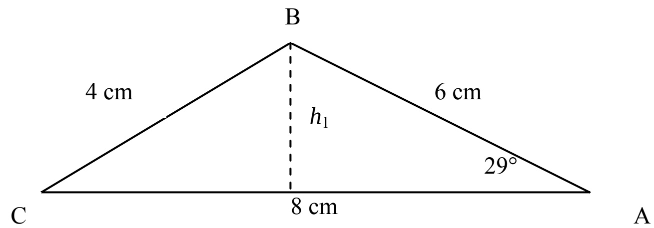
a) and b)

In ΔABC, draw an altitude from B to AC. Let its height be h_1.

$$\sin A = \frac{h_1}{AB}$$

$$h_1 = 6\sin 29°$$

$$h_1 = 2.908...$$



Now, use the formula for area of a triangle.

$$A = \frac{bh}{2}$$

$$A = \frac{(8)(2.908...)}{2}$$

$$A = 11.635...$$

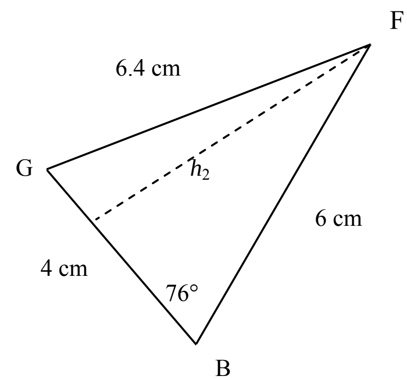
The altitude of ΔABC from B to AC is 2.9 cm, to the nearest tenth of a centimetre and the area of ΔABC is 11.6 cm^2, to the nearest tenth of a square centimetre.

In ΔBFG, draw an altitude from F to BG. Let its height be h_2.

$$\sin B = \frac{h_2}{BF}$$

$$h_2 = 6\sin 76°$$

$$h_2 = 5.821...$$



Now, use the formula for area of a triangle.

$$A = \frac{bh}{2}$$

$$A = \frac{(4)(5.821...)}{2}$$

$$A = 11.643...$$

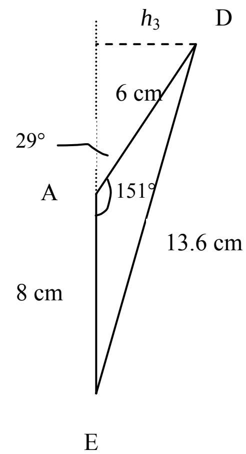
An altitude of ΔBFG is 5.8 cm, to the nearest tenth of a centimetre and the area of ΔBFG is 11.6 cm^2, to the nearest tenth of a square centimetre.

In ΔADE, draw an altitude from D to EA extended. Let its height be h_3.

$$\sin A = \frac{h_3}{DA}$$
$$h_3 = 6 \sin 29°$$
$$h_3 = 2.908...$$

Now, use the formula for area of a triangle.

$$A = \frac{bh}{2}$$
$$A = \frac{(8)(2.908...)}{2}$$
$$A = 11.635...$$



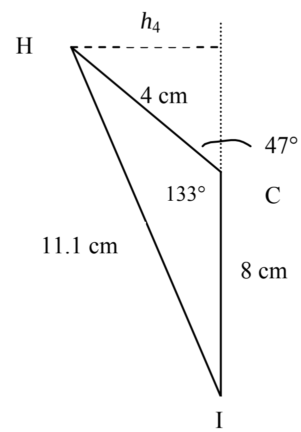
An altitude of ΔDAE is 2.9 cm, to the nearest tenth of a centimetre and the area of ΔADE is 11.6 cm^2, to the nearest tenth of a square centimetre.

In ΔCHI, draw an altitude from H to IC extended. Let its height be h_4.

$$\sin C = \frac{h_4}{CH}$$
$$h_4 = 4 \sin 47°$$
$$h_4 = 2.925...$$

Now, use the formula for area of a triangle.

$$A = \frac{bh}{2}$$
$$A = \frac{(8)(2.925...)}{2}$$
$$A = 11.701...$$



An altitude of ΔCHI is 2.9 cm, to the nearest tenth of a centimetre and the area of ΔBFG is 11.7 cm^2, to the nearest tenth of a square centimetre.

Step 4

Within rounding errors, the areas of the triangles are the same. This is because the area of ΔABC can be found by using an altitude from any vertex to its opposite side. For example, using the altitude from vertex B, the height of ΔABC could be either 6 sin 29° or 4 sin 47° with base 8. Notice each of these possible combinations was used to calculate the areas of ΔADE and ΔCHI, respectively. If an altitude from vertex A in ΔABC is used, the height would be 6 sin 76° with base 4, which is the same as the area calculation for ΔBFG. This would hold true for any ΔABC.

Chapter 2 Review

Chapter 2 Review Page 126 Question 1

a) E An angle in standard position is an angle whose vertex is at the origin and whose arms are the *x*-axis and the terminal arm.

b) D The reference angle is the acute angle formed by the terminal arm and the *x*-axis.

c) B An exact value is is not an approximation and may involve a radical.

d) A The sine law is a formula that relates the lengths of the side of a triangle to the sine values of its angles.

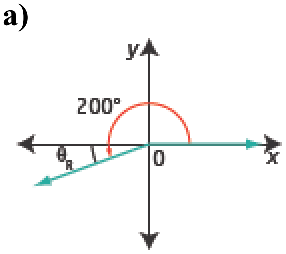
e) F The cosine law is a formula that relates the lengths of the side of a triangle to the cosine value of one of its angles.

f) C A terminal arm is the final position of the rotating arm of an angle is standard position.

g) G An ambiguous case is a situation that is open to two or more interpretations.

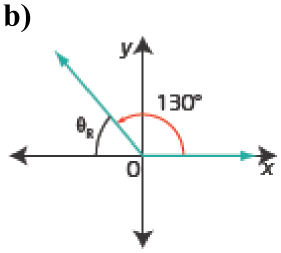
Chapter 2 Review Page 126 Question 2

a)



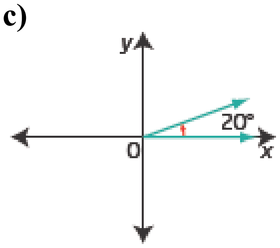
quadrant III,
reference angle is 200° − 180° = 20°

b)



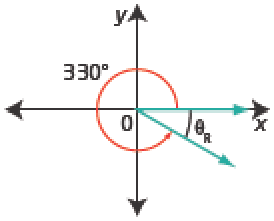
quadrant II,
reference angle is 180° − 130° = 50°

c)



quadrant I,
reference angle is 20°

d)



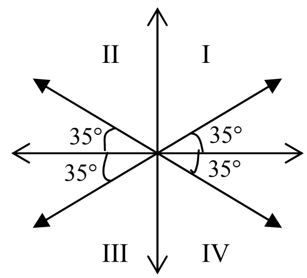
quadrant IV,
reference angle is 360° − 330° = 30°

Chapter 2 Review Page 126 Question 3

The 30° angle is not a reference angle because it is measured from the vertical. In this situation, the reference angle would be 60°.

Chapter 2 Review Page 126 Question 4

Consider a reference angle of 35° in each quadrant as shown.



In quadrant II, the angle in standard position is 180° − 35° = 145°.

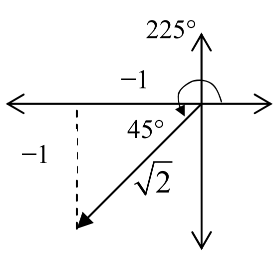
In quadrant III, the angle in standard position is 180° + 35° = 215°

In quadrant IV, the the angle in standard position is 360° − 35° = 325º

The angles, in standard position, $0° \le \theta < 360°$, that have 35° for their reference angle are 35°, 145°, 215° and 325°.

Chapter 2 Review Page 126 Question 5

a)

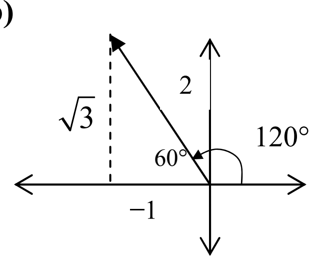


$$\sin 225° = \frac{y}{r} = \frac{-1}{\sqrt{2}} \text{ or } -\frac{\sqrt{2}}{2}$$

$$\cos 225° = \frac{x}{r} = \frac{-1}{\sqrt{2}} \text{ or } -\frac{\sqrt{2}}{2}$$

$$\tan 225° = \frac{y}{x} = \frac{-1}{-1} = 1$$

b)

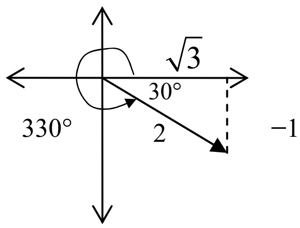


$$\sin 120° = \frac{y}{r} = \frac{\sqrt{3}}{2}$$

$$\cos 120° = \frac{x}{r} = \frac{-1}{2}$$

$$\tan 120° = \frac{y}{x} = \frac{\sqrt{3}}{-1} = -\sqrt{3}$$

c)

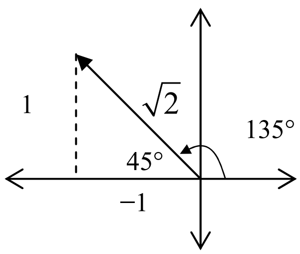


$$\sin 330° = \frac{y}{r} = \frac{-1}{2}$$

$$\cos 120° = \frac{x}{r} = \frac{\sqrt{3}}{2}$$

$$\tan 120° = \frac{y}{x} = \frac{-1}{\sqrt{3}} \text{ or } -\frac{\sqrt{3}}{3}$$

d)



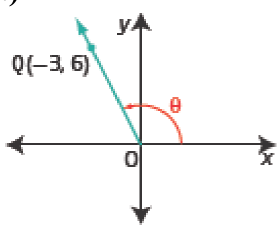
$$\sin 135° = \frac{y}{r} = \frac{1}{\sqrt{2}} \text{ or } \frac{\sqrt{2}}{2}$$

$$\cos 135° = \frac{x}{r} = \frac{-1}{\sqrt{2}} \text{ or } -\frac{\sqrt{2}}{2}$$

$$\tan 135° = \frac{y}{x} = \frac{1}{-1} = -1$$

Chapter 2 Review Page 126 Question 6

a)



b)
$$r^2 = x^2 + y^2$$
$$r^2 = (-3)^2 + 6^2$$
$$r^2 = 45$$
$$r = \sqrt{45}$$
$$r = 3\sqrt{5}$$

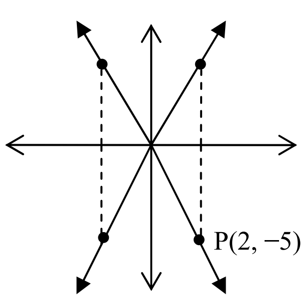
c)
$$\sin \theta = \frac{y}{r} = \frac{6}{3\sqrt{5}} = \frac{2}{\sqrt{5}} \text{ or } \frac{2\sqrt{5}}{5}$$

$$\cos \theta = \frac{x}{r} = \frac{-3}{3\sqrt{5}} = \frac{-1}{\sqrt{5}} \text{ or } -\frac{\sqrt{5}}{5}$$

$$\tan \theta = \frac{y}{x} = \frac{6}{-3} = -2$$

d)
$$\theta = \cos^{-1}\left(\frac{-1}{\sqrt{5}}\right)$$
$$\theta = 116.565\ldots$$
$\theta = 117°$, to the nearest degree.

Chapter 2 Review Page 126 Question 7



P(2, −5) is in quadrant IV.
Points with the same reference angle, in the other quadrants, are (2, 5), (−2, 5), and (−2, −5).

Chapter 2 Review Page 127 Question 8

a) $\sin \theta = \frac{y}{r}$ $\cos \theta = \frac{x}{r}$ $\tan \theta = \frac{y}{x}$

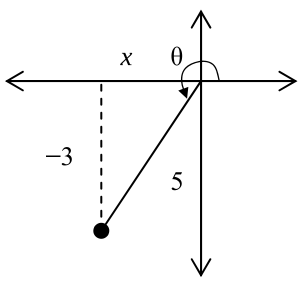
$\sin 90° = \frac{1}{1} = 1$ $\cos 90° = \frac{0}{1}$ $\tan 90° = \frac{1}{0}$, which is undefined

b) $\sin \theta = \frac{y}{r}$ $\cos \theta = \frac{x}{r}$ $\tan \theta = \frac{y}{x}$

$\sin 180° = \frac{0}{3} = 0$ $\cos 180° = \frac{-3}{3} = -1$ $\tan 180° = \frac{0}{-3} = 0$

Chapter 2 Review Page 127 Question 9

a) Since both sine and cosine are negative, ∠θ is in quadrant III.

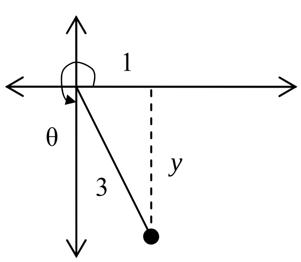


$$\begin{aligned} x^2 + y^2 &= r^2 \\ x^2 + (-3)^2 &= 5^2 \\ x^2 &= 25 - 9 \\ x^2 &= 16 \\ x &= -4 \end{aligned}$$

Then, $\cos \theta = \frac{x}{r} = \frac{-4}{5}$

$\tan \theta = \frac{y}{x} = \frac{-3}{-4} = \frac{3}{4}$

b) Since cosine is positive and tangent negative, ∠θ is in quadrant IV.

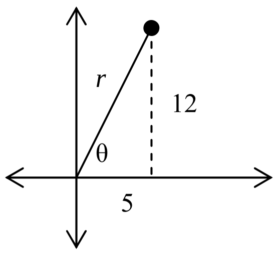


$$\begin{aligned} x^2 + y^2 &= r^2 \\ 1^2 + y^2 &= 3^2 \\ y^2 &= 9 - 1 \\ y^2 &= 8 \\ y &= -\sqrt{8} \text{ or } -2\sqrt{2} \end{aligned}$$

Then, $\sin \theta = \frac{y}{r} = \frac{-2\sqrt{2}}{3}$

$\tan \theta = \frac{y}{x} = \frac{-2\sqrt{2}}{1} = -2\sqrt{2}$

c) Since both tangent and sine are positive, $\angle\theta$ is in quadrant I.



$$x^2 + y^2 = r^2$$
$$5^2 + 12^2 = r^2$$
$$25 + 144 = r^2$$
$$r^2 = 169$$
$$r = \sqrt{169}$$
$$r = 13$$

Then, $\sin\theta = \frac{y}{r} = \frac{12}{13}$

$\cos\theta = \frac{x}{r} = \frac{5}{13}$

Chapter 2 Review Page 127 Question 10

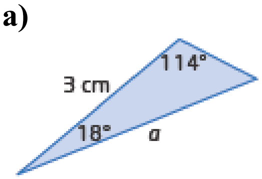
a) Given $\tan\theta = -1.1918$.
Since $\tan\theta$ is negative, $\angle\theta$ is in quadrant II or IV.
$\tan^{-1}(1.1918) = 50.001\ldots$
So, the reference angle is 50°, to the nearest degree.
In quadrant II, $\angle\theta = 180° - 50° = 130°$.
In quadrant IV, $\angle\theta = 360° - 50° = 310°$.

b) Given $\sin\theta = -0.3420$.
Since $\sin\theta$ is negative, $\angle\theta$ is in quadrant III or IV.
$\sin^{-1}(0.3420) = 19.998\ldots$
So, the reference angle is 20°, to the nearest degree.
In quadrant III, $\angle\theta = 180° + 20° = 200°$.
In quadrant IV, $\angle\theta = 360° - 20° = 340°$.

c) Given $\cos\theta = 0.3420$.
Since $\cos\theta$ is positive, $\angle\theta$ is in quadrant I or IV.
$\cos^{-1}(0.3420) = 70.001\ldots$
So, the reference angle is 70°, to the nearest degree.
In quadrant I, $\angle\theta = 70°$.
In quadrant IV, $\angle\theta = 360° - 70° = 290°$.

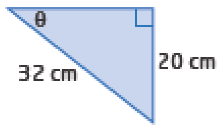
Chapter 2 Review Page 127 Question 11

a)



Yes. One side and its corresponding angle are given, and you know the measures of all the angles, so you can determine the other two sides using the sine law.

b)



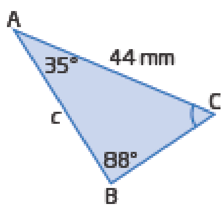
You could use the sine law to determine ∠θ, by using $\frac{\sin\theta}{20}=\frac{\sin 90°}{32}$. However, since this is a right triangle it is more efficient to use the sine ratio to determine ∠θ and the Pythagorean Theorem to determine the unknown side length.

c)

No. You are not given the right pairs of sides and angles to use the sine law.

Chapter 2 Review Page 127 Question 12

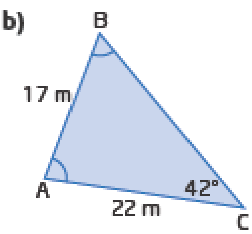
a)



$\angle C = 180° - (35° + 88°)$
$\angle C = 57°$
Then, using the sine law:

$$\frac{c}{\sin 57°}=\frac{44}{\sin 88°}$$
$$c=\frac{44\sin 57°}{\sin 88°}$$
$$c = 36.923...$$

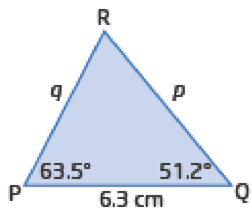
c = 36.9 mm, to the nearest tenth of a millimetre.



$$\frac{\sin B}{22}=\frac{\sin 42°}{17}$$
$$\sin B=\frac{22\sin 42°}{17}$$
$$\angle B=\sin^{-1}\left(\frac{22\sin 42°}{17}\right)$$
$$\angle B = 59.989...$$

∠B = 60°, to the nearest degree.
Then, $\angle A = 180° - (42° + 60°)$
$\angle A = 78°$

Chapter 2 Review Page 127 Question 13



$\angle R = 180° - (63.5° + 51.2°)$
$\angle R = 65.3°$

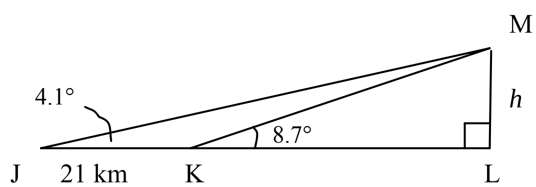
Then, using the sine law:

$$\frac{p}{\sin 63.5°} = \frac{6.3}{\sin 65.3°}$$
$$p = \frac{6.3 \sin 63.5°}{\sin 65.3°}$$
$$p = 6.205...$$

$$\frac{q}{\sin 51.2°} = \frac{6.3}{\sin 65.3°}$$
$$q = \frac{6.3 \sin 51.2°}{\sin 65.3°}$$
$$q = 5.404...$$

The unknown sides are RQ = 6.2 cm and PR = 5.4 cm, to the nearest tenth of a centimetre.

Chapter 2 Review Page 127 Question 14



In ΔJKM,
$\angle JMK = 8.7° - 4.1°$ (exterior angle of a triangle)
$\angle JMK = 4.6°$
Then, using the sine law:

$$\frac{MK}{\sin 4.1°} = \frac{21}{\sin 4.6°}$$
$$MK = \frac{21 \sin 4.1°}{\sin 4.6°}$$
$$MK = 18.721...$$

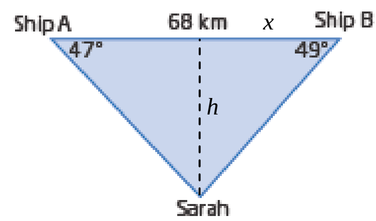
In ΔKLM, *h* represents the height of the mountain.

$$\sin 8.7° = \frac{h}{18.721...}$$
$$h = 18.721... \sin 8.7°$$
$$h = 2.831...$$

The height of the mountain is approximately 2.8 km, to the nearest tenth of a kilometre. This is 2800 m.

Chapter 2 Review Page 127 Question 15

a) Ship B is closer to Sarah.



$$\frac{\text{BS}}{\sin 47°} = \frac{68}{\sin 84°} \qquad \text{Check AS: } \frac{\text{AS}}{\sin 49°} = \frac{68}{\sin 84°}$$

$$\text{BS} = \frac{68 \sin 47°}{\sin 84°} \qquad \text{AS} = \frac{68 \sin 49°}{\sin 84°}$$

$$\text{BS} = 50.005... \qquad \text{AS} = 51.602...$$

Sarah is 50.0 km from ship B.

b) Let h represent the perpendicular distance from Sarah to the line from ship A to ship B. Let x represent the distance from the foot of the perpendicular to B.
Use the tangent ratio in each right triangle:

$$\tan 49° = \frac{h}{x} \qquad \tan 47° = \frac{h}{68 - x}$$

$$h = x \tan 49° \qquad h = \tan 47°(68 - x)$$

So, $x \tan 49° = \tan 47°(68 - x)$
$x(\tan 49° + \tan 47°) = 68 \tan 47°$

$$x = \frac{68 \tan 47°}{\tan 49° + \tan 47°}$$

$$x = 32.806...$$

Then, $\cos 49° = \frac{32.806...}{BS}$ and $\cos 47° = \frac{68 - 32.806...}{\text{AS}}$

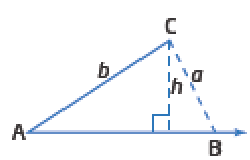
$$\text{BS} = \frac{32.806...}{\cos 49°} \qquad \text{AS} = \frac{68 - 32.806...}{\cos 47°}$$

$$\text{BS} = 50.005... \qquad \text{AS} = 51.602...$$

This verifies the answer in part a), that ship B is closer to Sarah.

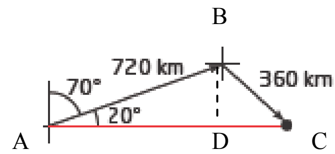
Chapter 2 Review Page 127 Question 16

There is one solution if $a = b \sin A$, because this creates a right triangle. If $a \geq b$ then only one triangle can be drawn. If $b \sin A < a < b$, then two possible triangles can be drawn, one with an acute angle at B, and one with an obtuse angle at B. If $a < b \sin A$ then no triangle exists.



Chapter 2 Review Page 128 Question 17

a) On the first leg, the plane goes for 1 h at 720 km/h, so AB = 720 km.
On the second part, the plane travels for 30 min before reaching the original eastward path. So, BC = 360 km.



b) Let D be at the foot of the perpendicular from B to the East-West line AC.
Then, in ΔABD:

$$\sin 20° = \frac{BD}{720}$$

$$BD = 720(\sin 20°)$$

Now in ΔBCD:

$$\cos \angle CBD = \frac{720(\sin 20°)}{360}$$

$$\angle CBD = \cos^{-1}\left(\frac{720(\sin 20°)}{360}\right)$$

$$\angle CBD = 46.839\ldots$$

The heading of the second part is S47°E, to the nearest degree.

c) In ΔBCD, $\angle C = 180° - (46.839\ldots° + 90°) = 43.161\ldots°$
In ΔABC, $\angle B = 180° - (20° + 43.161\ldots°)$

$$\angle B = 116.839\ldots°$$

Use the sine law to determine AC.

$$\frac{AC}{\sin 116.839...°} = \frac{360}{\sin 20°}$$

$$AC = \frac{360 \sin 116.839...°}{\sin 20°}$$

$$AC = 939.185...$$

The jet resumes its original path 939.2 km east of the point where it changed course.

Chapter 2 Review Page 128 Question 18

a) Given $a = 7$, $b = 2$ and $c = 4$, the triangle cannot exist because the two shorter sides are less than the longest side i.e., the three sides will not meet to make a triangle.

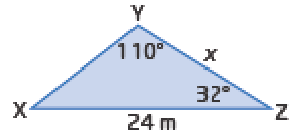
b) Given ∠A = 85°, $b = 10$, ∠C = 98°, the triangle cannot exist because the two given angles sum to more than 180°.

c) Given $a = 12$, $b = 20$ and $c = 8$, the triangle cannot exist because the two shorter sides equal the longest side i.e., the three sides will not meet to make a triangle.

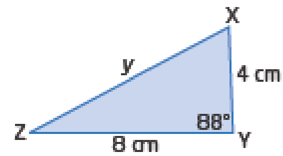
d) ∠A = 65°, ∠B = 52°, ∠C = 35°, the triangle cannot exist because the sum of the three angles is less than 180°.

Chapter 2 Review Page 128 Question 19

a) You can use the sine law. First you need to find the measure of ∠X using angle sum of a triangle. Then you can solve $\frac{x}{\sin 38°} = \frac{24}{\sin \text{Y}}$.

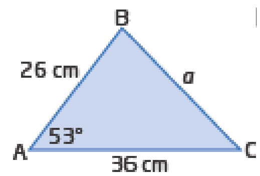


b) You can use the cosine law directly:
$y^2 = 4^2 + 8^2 - 2(4)(8) \cos 88°$

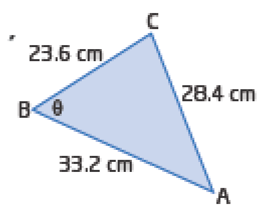


Chapter 2 Review Page 128 Question 20

a) $a^2 = 26^2 + 36^2 - 2(26)(36) \cos 53°$
$a^2 = 845.402…$
$a = \sqrt{845.402...}$
$a = 29.075…$
a = 29.1 cm, to the nearest tenth of a centimetre.

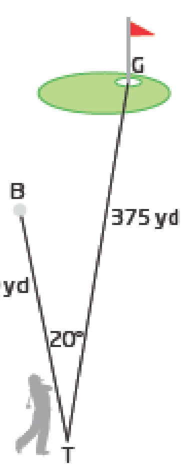


b) $28.4^2 = 23.6^2 + 33.2^2 - 2(23.6)(33.2) \cos \theta$
$1567.04 \cos \theta = 852.64$
$\cos \theta = \frac{852.64}{1567.04}$
$\theta = \cos^{-1}\left(\frac{852.64}{1567.04}\right)$
$\theta = 57.036…$
∠B = 57°, to the nearest degree.



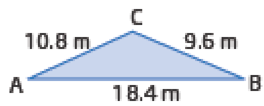
Chapter 2 Review Page 128 Question 21

$\text{BG}^2 = 240^2 + 375^2 - 2(240)(375) \cos 20°$
$\text{BG}^2 = 29\ 080.328…$
$\text{BG} = \sqrt{29\ 080.328...}$
$\text{BG} = 170.529…$
The distance of the ball from the centre of the hole is 170.5 yd, to the nearest tenth of a yard.



Chapter 2 Review Page 128 Question 22

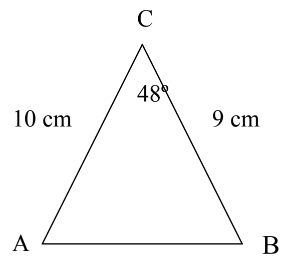
a)



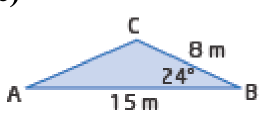
$$9.6^2 = 10.8^2 + 18.4^2 - 2(10.8)(18.4)\cos A$$
$$397.44\cos A = 363.04$$
$$\cos A = \frac{363.04}{397.44}$$
$$\angle A = \cos^{-1}\left(\frac{363.04}{397.44}\right)$$
$$\angle A = 24°$$

b)

$AB^2 = 10^2 + 9^2 - 2(10)(9)\cos 48°$
$AB^2 = 60.556\ldots$
$AB = \sqrt{60.556...}$
$AB = 7.781\ldots$
The length of AB is 7.8 cm, to the nearest tenth of a centimetre.



c)



Use the cosine law to determine *b*.
$b^2 = 8^2 + 15^2 - 2(8)(15)\cos 24°$
$b^2 = 69.749\ldots$
$b = \sqrt{69.749...}$
$b = 8.351\ldots$

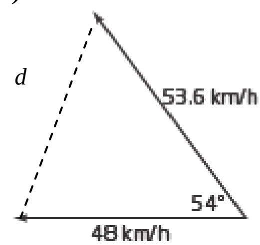
Next, use the sine law to determine ∠A.

$$\frac{\sin A}{8} = \frac{\sin 24°}{8.351...}$$
$$\sin A = \frac{8\sin 24°}{8.351...}$$
$$\angle A = \sin^{-1}\left(\frac{8\sin 24°}{8.351...}\right)$$
$$\angle A = 22.930...$$

Now, use angle sum of a triangle to determine ∠C.
$\angle C = 180° - (24° + 23°)$
$\angle C = 133°$
In ΔABC, AC = 8.4 m, to the nearest tenth of a metre, ∠A = 23° and ∠C = 133°, both to the nearest degree.

Chapter 2 Review Page 128 Question 23

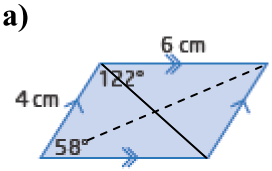
a)



b) Determine the distance travelled by each boat in 4 h.
4(48) = 192
4(53.6) = 214.4
Use the cosine law to find their distance apart, *d*.
$d^2 = 192^2 + 214.4^2 - 2(192)(214.4) \cos 54°$
$d^2 = 34\ 439.235\ldots$
$d = \sqrt{34\ 439.235...}$
$d = 185.578\ldots$
After 4 h, the boats are 185.6 km apart, to the nearest tenth of a kilometre.

Chapter 2 Review Page 128 Question 24

a)



b) Let s represent the length of the shorter diagonal.
$s^2 = 4^2 + 6^2 - 2(4)(6) \cos 58°$
$s^2 = 26.563\ldots$
$s = \sqrt{26.563...}$
$s = 5.154\ldots$
Let *d* represent the length of the longer diagonal.
$d^2 = 4^2 + 6^2 - 2(4)(6)\cos 122°$
$d^2 = 77.436\ldots$
$d = \sqrt{77.436...}$
$d = 8.799\ldots$
The lengths of the two diagonals are 5.2 cm and 8.8 cm, to the nearest tenth of a centimetre.

Chapter 2 Practice Test

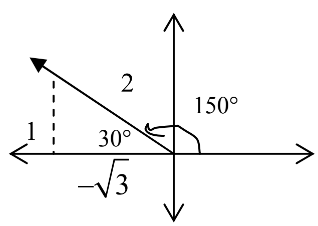
Chapter 2 Practice Test Page 129 Question 1

Angle Reference Angle
A 125° $180° - 125° = 55°$
B 155° $180° - 155° = 25°$
C 205° $205° - 180° = 25°$
D 335° $360° - 335° = 25°$
Angle **A** has a different reference angle than the others.

Chapter 2 Practice Test Page 129 Question 2

Angle Reference Angle
A 35° 35°
B 125° $180° - 125° = 55°$
C 235° $235° - 180° = 55°$
D 305° $360° - 305° = 55°$
Angle **A** does not have a reference angle of 55°.

Chapter 2 Practice Test Page 129 Question 3



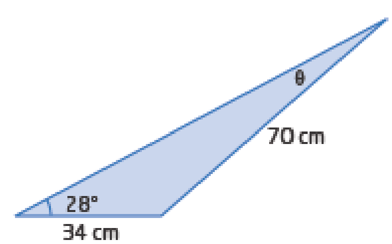
$\cos\theta = \frac{x}{r}$

$\cos 150° = \frac{-\sqrt{3}}{2}$

C is the exact value of cos 150°.

Chapter 2 Practice Test Page 129 Question 4

From the diagram,
$\frac{\sin\theta}{34} = \frac{\sin 28°}{70}$
The equation **B** can be used to determine the measure of angle θ.



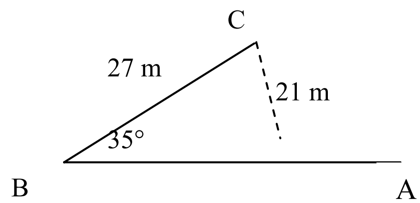
Chapter 2 Practice Test Page 129 Question 5

A Since three sides are given, only one triangle can be drawn.

B Since ∠D is given as an obtuse angle only one such triangle can be drawn.

C Check $h = 27 \sin 35° = 15.486…$. Since $h <$ 21, two possible triangles can be drawn.

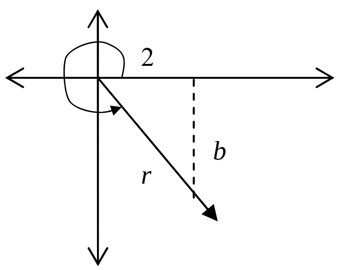
For the triangle in **C**, you must consider the ambiguous case.



D Here the three angles and one side are known, so only one such triangle can be drawn.

Chapter 2 Practice Test Page 129 Question 6

Since cos θ is positive and tan θ is negative, the angle is in quadrant IV.



$$\cos\theta = \frac{x}{r}$$

$$\frac{1}{\sqrt{10}} = \frac{2}{r}$$

$$r = 2\sqrt{10}$$

Then, using the Pythagorean Theorem:

$$r^2 = x^2 + b^2$$

$$\left(2\sqrt{10}\right)^2 = 2^2 + b^2$$

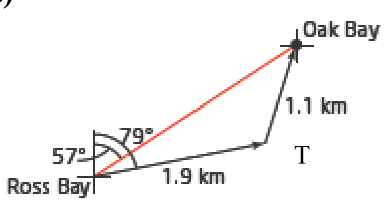
$$40 - 4 = b^2$$

$$b^2 = 36$$

$$b = -6$$

Chapter 2 Practice Test Page 129 Question 7

a)

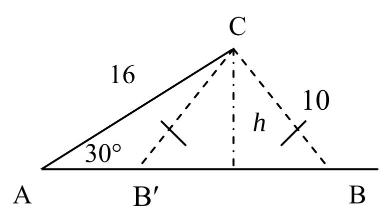


b) Consider the vertices of the triangle to be O, R, and T.
$\angle R = 79° - 57° = 22°$
Use the sine law to determine a second angle inside ΔORT.

$$\frac{\sin O}{1.9} = \frac{\sin 22°}{1.1}$$
$$\sin O = \frac{1.9 \sin 22°}{1.1}$$
$$\angle O = \sin^{-1}\left(\frac{1.9 \sin 22°}{1.1}\right)$$
$$\angle O = 40.319...$$
$\angle O = 40°$, to the nearest degree.
Then, $\angle T = 180° - (22° + 40°)$
$\angle T = 118°$
Now, use the cosine law to determine the length RO.
$t^2 = 1.1^2 + 1.9^2 - 2(1.1)(1.9) \cos 118°$
$t^2 = 6.782\ldots$
$t = 2.604\ldots$
The distance between Ross Bay and Oak Bay is 2.6 km, to the nearest tenth of a kilometre.

Chapter 2 Practice Test Page 129 Question 8

a) $h = 16 \sin 30° = 8$
Since $8 < 10 < 16$, this is the ambiguous case. Two distinct triangles can be drawn.



b) $$\frac{\sin B}{16} = \frac{\sin 30°}{10}$$
$$\sin B = \frac{16 \sin 30°}{10}$$
$$\angle B = \sin^{-1}\left(\frac{16 \sin 30°}{10}\right)$$
$$\angle B = 53.130...$$
So, $\angle B = 53°$ or $\angle B = 180° - 53° = 127°$.

Case 1: $\angle B = 53°$.
$\angle C = 180° - (30° + 53°)$
$\angle C = 97°$
Use the sine law to determine c.

Case 2: $\angle B = 127°$
$\angle C = 180° - (30° + 127°)$
$\angle C = 23°$

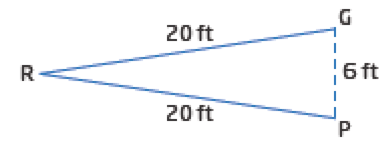
$$\frac{c}{\sin 97°} = \frac{16}{\sin 53°}$$
$$c = \frac{16 \sin 97°}{\sin 53°}$$
$$c = 19.884...$$

$$\frac{c}{\sin 23°} = \frac{16}{\sin 53°}$$
$$c = \frac{16 \sin 23°}{\sin 53°}$$
$$c = 7.827...$$

The unknown measures in ΔABC are ∠B = 53°, ∠C = 97°, and AB = 19.9 or ∠B = 127°, ∠C = 23°, and AB = 7.8, where angles are given to the nearest degree and lengths to the nearest tenth.

Chapter 2 Practice Test Page 129 Question 9

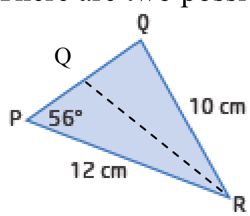
$$6^2 = 20^2 + 20^2 - 2(20)(20) \cos R$$
$$800 \cos R = 764$$
$$\cos R = \frac{764}{800}$$
$$\angle R = \cos^{-1}\left(\frac{764}{800}\right)$$
$$\angle R = 17.253...$$



Rudy must fire the puck within an angle of 17°, to the nearest degree.

Chapter 2 Practice Test Page 129 Question 10

a) h = 12 sin 56° = 9.948…, and 9.948…< 10 and <12. So this is an ambiguous case. There are two possible solutions.



b) Use the sine law to determine ∠Q.

$$\frac{\sin Q}{12} = \frac{\sin 56°}{10}$$
$$\sin Q = \frac{12 \sin 56°}{10}$$
$$\angle Q = \sin^{-1}\left(\frac{12 \sin 56°}{10}\right)$$
$$\angle Q = 84.179....$$

Or ∠Q = 180° − 84.179…° = 95.820…°

Then, use angle sum of a triangle to determine ∠R.

∠R = 180° − (56° + 84°) or ∠R = 180° − (56° + 96°)

∠R = 40° ∠R = 28°

Use the sine law again to determine side *r*.

Case I: $\frac{r}{\sin 40°} = \frac{10}{\sin 56°}$ or Case II: $\frac{r}{\sin 28°} = \frac{10}{\sin 56°}$

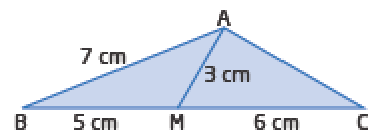
$$r = \frac{10\sin 40°}{\sin 56°} \qquad r = \frac{10\sin 28°}{\sin 56°}$$

$$r = 7.753... \qquad r = 5.662...$$

So, case I: the unknown side length is *r* = 7.8 cm, to the nearest tenth of a centimetre, and the unknown angles are ∠Q = 84° and ∠R = 40°, to the nearest degree.
case II: the unknown side length is *r* = 5.7 cm, to the nearest tenth of a centimetre, and the unknown angles are ∠Q = 96° and ∠R = 28°, to the nearest degree.

Chapter 2 Practice Test Page 130 Question 11

Use the cosine law in ΔABM to determine ∠ABM.



$$3^2 = 7^2 + 5^2 - 2(7)(5)\cos B$$

$$70\cos B = 65$$

$$\cos B = \frac{65}{70}$$

$$\angle B = \cos^{-1}\left(\frac{65}{70}\right)$$

$$\angle B = 21.786...$$

Use the cosine law in ΔABC to determine AC.

$$AC^2 = 7^2 + 11^2 - 2(7)(11)\cos 21.786\ldots$$

$$AC^2 = 27$$

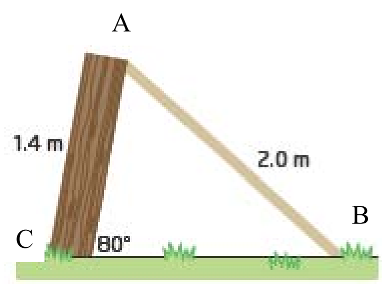
$$AC = \sqrt{27}$$

$$AC = 5.196\ldots$$

The length of AC is 5.2 cm, to the nearest tenth of a centimetre.

Chapter 2 Practice Test Page 130 Question 12

a) Use the sine law to determine ∠B.



$$\frac{\sin B}{1.4} = \frac{\sin 80°}{2.0}$$

$$\sin B = \frac{1.4\sin 80°}{2.0}$$

$$\angle B = \sin^{-1}\left(\frac{1.4\sin 80°}{2.0}\right)$$

$$\angle B = 43.579...$$

The board makes an angle of 44° with the ground.

b) Use the angle sum of a triangle to determine ∠A.

$\angle A = 180° - (80° + 44°)$
$\angle A = 56°$
The board makes an angle of 56° with the top of the fence.

c) Use the sine law to determine BC.

$$\frac{BC}{\sin 56°} = \frac{2.0}{\sin 80°}$$
$$BC = \frac{2.0 \sin 56°}{\sin 80°}$$
$$BC = 1.683...$$

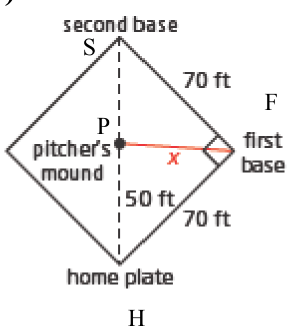
The bottom of the board is 1.7 m from the base of the fence.

Chapter 2 Practice Test Page 130 Question 13

If $0° \le \theta_R \le 90°$, then the angle is in the first quadrant and $\theta = \theta_R$.
If $90° < \theta_R \le 180°$, then the angle is in the second quadrant and $\theta = 180° - \theta_R$.
If $180° < \theta_R \le 270°$, then the angle is in the third quadrant and $\theta = 180° + \theta_R$.
If $270° < \theta_R < 360°$, then the angle is in the fourth quadrant and $\theta = 360° - \theta_R$.

Chapter 2 Practice Test Page 130 Question 14

a)



b) In ΔPHF, $\angle PHF = 45°$.
$x^2 = 50^2 + 70^2 - 2(50)(70) \cos 45°$
$x^2 = 2450.252\ldots$
$x = \sqrt{2450.252...}$
$x = 49.500\ldots$
But using the Pythagorean Theorem in ΔHFS:
$HS^2 = 70^2 + 70^2$
$HS^2 = 9800$
$HS = 98.994\ldots$
Then, PS = 99 − 50 or 49 ft.
The distance from the pitcher's mound to first base is 49.5 ft, which is 0.5 ft more than the 49 ft distance from the pitcher's mound to second base.

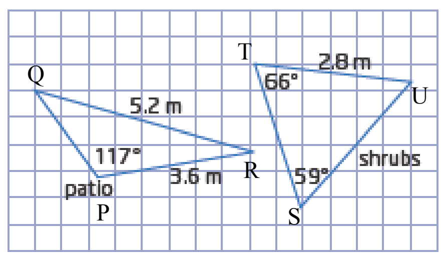
Chapter 2 Practice Test Page 130 Question 15

You use the sine law or cosine law to solve oblique triangles.
Use the cosine law first if
- you are given two sides and their contained angle, then you can find the third side using the cosine law
- you are given three sides but no angle, then you can use the cosine law to determine one angle.

Use the sine law if
- you are given two angles and one side
- you are given two sides and one angle that is not contained by the two given sides

Chapter 2 Practice Test Page 130 Question 16

Label the vertices to form ΔPQR and ΔSTU.



In ΔPQR:

$$\frac{\sin Q}{3.6} = \frac{\sin 117°}{5.2}$$

$$\sin Q = \frac{3.6 \sin 117°}{5.2}$$

$$\angle Q = \sin^{-1}\left(\frac{3.6 \sin 117°}{5.2}\right)$$

$$\angle Q = 38.086...$$

Then, $\angle R = 180° - (117° + 38°)$

$$\angle R = 25°$$

Use the sine law again, to determine side PQ.

$$\frac{PQ}{\sin 25°} = \frac{5.2}{\sin 117°}$$

$$PQ = \frac{5.2 \sin 25°}{\sin 117°}$$

$$PQ = 2.466...$$

In ΔPQR, the patio triangle, the unknown side is 2.5 m, to the nearest tenth of a metre.
The unknown angles are 38° and 25°, to the nearest degree.

In ΔSTU:

$\angle U = 180° - (66° + 59°)$

$\angle U = 55°$

Use the sine law to determine the two unknown sides.

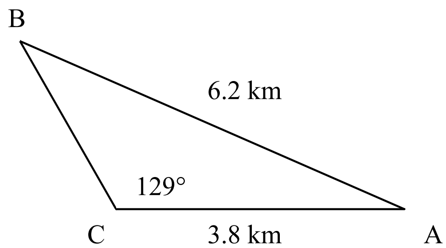
$$\frac{SU}{\sin 66°} = \frac{2.8}{\sin 59°} \quad \text{and} \quad \frac{ST}{\sin 55°} = \frac{2.8}{\sin 59°}$$

$$SU = \frac{2.8 \sin 66°}{\sin 59°} \qquad ST = \frac{2.8 \sin 55°}{\sin 59°}$$

$$SU = 2.984... \qquad ST = 2.675...$$

In ΔSTU, the shrubs triangle, the unknown sides are 3.0 m and 2.7 m, to the nearest tenth of a metre. The unknown angle is 55°, to the nearest degree.

Chapter 2 Practice Test Page 130 Question 17



First use the sine law to determine ∠B. then, use it again to determine the distance CB.

$$\frac{\sin B}{3.8} = \frac{\sin 129°}{6.2}$$
$$\sin B = \frac{3.8 \sin 129°}{6.2}$$
$$\angle B = \sin^{-1}\left(\frac{3.8 \sin 129°}{6.2}\right)$$
$$\angle B = 28.445...$$

Then, $\angle A = 180° - (129° + 28°)$

$$\angle A = 23°$$
$$\frac{BC}{\sin 23°} = \frac{6.2}{\sin 129°}$$
$$BC = \frac{6.2 \sin 23°}{\sin 129°}$$
$$BC = 3.117...$$

Before losing contact with the Alpha group, the Beta group can walk 3.1 km, to the nearest tenth of a kilometre.

Cumulative Review, Chapters 1−2

Cumulative Review Page 133 Question 1

a) A 3, 7, 11, 15, 19, …is an arithmetic sequence with $t_1 = 3$ and $d = 4$.

b) D 1, 3, 9, 27, 81, … is a geometric sequence with $t_1 = 1$ and $r = 3$.

c) E is an arithmetic series with $t_1 = 2$ and $d = 3$.

d) C is a geometric series with $t_1 = 1$ and $r = 2$.

e) B is a convergent series with $t_1 = 5$ and $r = \frac{1}{5}$.

Cumulative Review Page 133 Question 2

a) 27, 18, 12, 8, … is a geometric sequence because successive terms have a common ratio. The common ratio is $\frac{18}{27}$ or $\frac{2}{3}$. The next three terms are $\frac{16}{3}$, $\frac{32}{9}$, $\frac{64}{27}$.

b) 17, 14, 11, 8,… is an arithmetic sequence because successive terms have a common difference of −3. The next three terms are 5, 2, −1.

c) −21, −16, −11, −6, … is an arithmetic sequence because successive terms have a common difference of 5. The next three terms are −1, 4, 9.

d) 3, −6, 12, −24, … is a geometric sequence because successive terms have a common ratio. The common ratio is −2. The next three terms are 48, −96, 192.

Cumulative Review Page 133 Question 3

a) Substitute $t_1 = 18$ and $d = -3$ into $t_n = t_1 + (n - 1)d$.
$t_n = 18 + (n - 1)(-3)$
$t_n = 21 - 3n$

b) Substitute $t_1 = 1$ and $d = \frac{3}{2}$ into $t_n = t_1 + (n - 1)d$.

$t_n = 1 + (n - 1)\frac{3}{2}$

$t_n = \frac{3}{2}n - \frac{1}{2}$

Cumulative Review Page 133 Question 4

Substitute $t_1 = 2$, $r = -2$, and $n = 20$ into $t_n = t_1r^{n-1}$.
$t_{20} = 2(-2)^{19}$
$t_{20} = -1\ 048\ 576$

Cumulative Review Page 133 Question 5

a) First find t_1. Substitute $d = 3$, $n = 12$, and $t_{12} = 31$ into $t_n = t_1 + (n - 1)d$.
$31 = t_1 + (12 - 1)3$
$31 - 33 = t_1$
$t_1 = -2$

Now substitute into $S_n = \frac{n}{2}(t_1 + t_n)$.

$S_{12} = \frac{12}{2}(-2 + 31)$

$S_{12} = 174$

b) First find r. Substitute $t_{10} = 78\ 732$ and $t_1 = 4$ into $t_n = t_1r^{n-1}$.

$78\ 732 = 4(r^9)$

$r^9 = 19\ 683$

$r = \sqrt[9]{19\ 683}$

$r = 3$

Now substitute into $S_n = \frac{t_1(r^n - 1)}{r - 1}$.

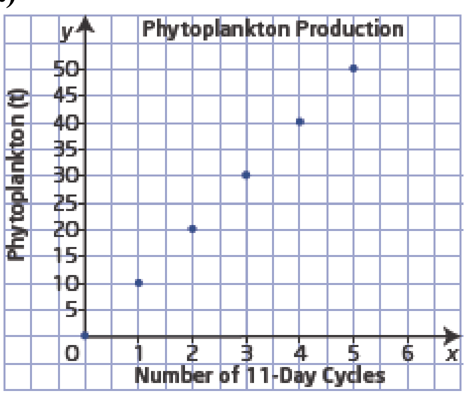
$S_5 = \frac{4(3^5 - 1)}{3 - 1}$

$S_5 = 2(242)$

$S_5 = 484$

Cumulative Review Page 133 Question 6

a)



b) $t_n = 10n$, where n is the number of 11-day cycles.

c) The coefficient 10 is the slope of the linear function.

Cumulative Review Page 133 Question 7

$h = 5.8 + 61(3.2)$

$h = 201$

The building is 201 m tall.

Cumulative Review Page 134 Question 8

a) $t_1 = \frac{9}{10}$ and $r = \frac{1}{10}$

$S_\infty = \frac{t_1}{1-r}$

$S_\infty = \frac{\frac{9}{10}}{1-\frac{1}{10}}$

$S_\infty = \frac{\frac{9}{10}}{\frac{9}{10}}$

$S_\infty = 1$

b) Answers may vary. In a way they are both correct.

Cumulative Review Page 134 Question 9

Substitute $x = -2$ and $y = 4$ into $r^2 = x^2 + y^2$.

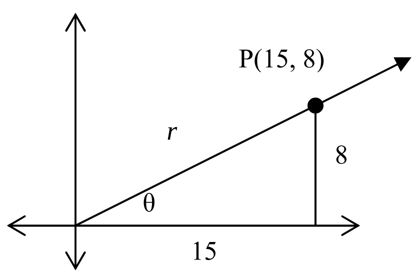
$r^2 = (-2)^2 + (4)^2$

$r^2 = 4 + 16$

$r = \sqrt{20}$ or $2\sqrt{5}$

The exact distance from the origin to the point P(−2, 4) is $2\sqrt{5}$.

Cumulative Review Page 134 Question 10



First determine r. Substitute $x = 15$ and $y = 8$ into $r^2 = x^2 + y^2$.

$r^2 = (15)^2 + (8)^2$

$r^2 = 225 + 64$

$r = \sqrt{289}$

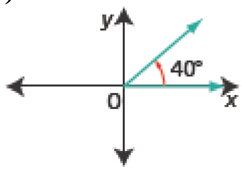
$r = 17$

$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$

$\sin\theta = \frac{8}{17}$ $\cos\theta = \frac{15}{17}$ $\tan\theta = \frac{8}{15}$

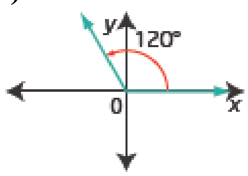
Cumulative Review Page 134 Question 11

a)



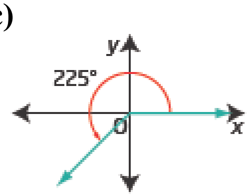
Since 40° is in quadrant I, the reference angle is 40°.

b)



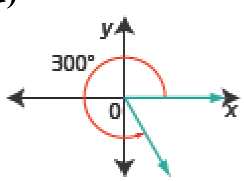
120° is in quadrant II.
Its reference angle is 180° − 120° = 60°.

c)



225° is in quadrant III.
Its reference angle is 225° − 180° = 45°.

d)

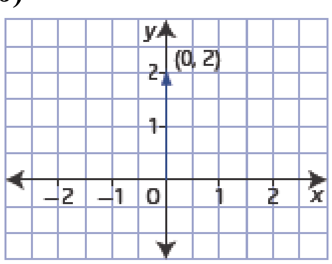


300° is in quadrant IV.
Its reference angle is 360° − 300° = 60°.

Cumulative Review Page 134 Question 12

a) At 3 o'clock the hands form an angle of 90°.

b)

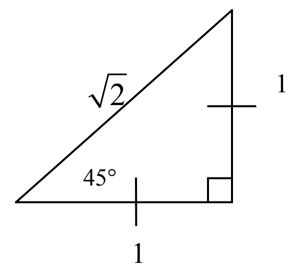


c) $\sin\theta = \dfrac{y}{r}$ $\cos\theta = \dfrac{x}{r}$ $\tan\theta = \dfrac{y}{x}$

$\sin\theta = \dfrac{2}{2} = 1$ $\cos\theta = \dfrac{0}{2} = 0$ $\tan\theta = \dfrac{2}{0}$, which is undefined

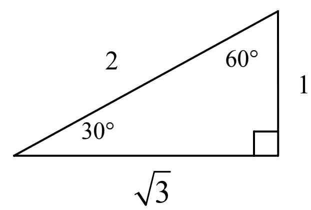
Cumulative Review Page 135 Question 13

a) sin 405° = sin 45°, since 405° is coterminal with 45°. Use the special triangle for 45°.



$\sin 45° = \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$

b) The reference angle for 330° is 30° and the angle is in quadrant IV. Use the special triangle for 30° and 60°.



$$\cos\theta = \frac{x}{r}$$

$$\cos 330° = \frac{\sqrt{3}}{2}$$

c) The reference angle for 225° is 45° and the angle is in quadrant III. Use the special triangle shown is part a).

$$\tan\theta = \frac{y}{x}$$

$$\tan 225° = \frac{-1}{-1} = 1$$

d) The point (−1, 0) is on the terminal arm of 180°.

$$\cos\theta = \frac{x}{r}$$

$$\cos 180° = \frac{-1}{1} = -1$$

e) The reference angle for 150° is 30° and the angle is in quadrant II. Use the special triangle for 30° and 60° as shown in part b).

$$\tan\theta = \frac{y}{x}$$

$$\tan 150° = \frac{1}{-\sqrt{3}} \text{ or } -\frac{\sqrt{3}}{3}$$

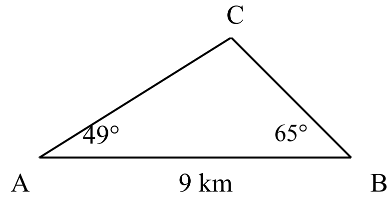
f) The point (0, −1) is on the terminal arm of 270°.

$$\sin\theta = \frac{y}{r}$$

$$\sin 270° = \frac{-1}{1} = -1$$

Cumulative Review Page 135 Question 14

First use angle sum of a triangle to determine ∠C.
$\angle C = 180° - (49° + 65°)$
$\angle C = 66°$
Use the sine law to determine each distance, AC and BC.



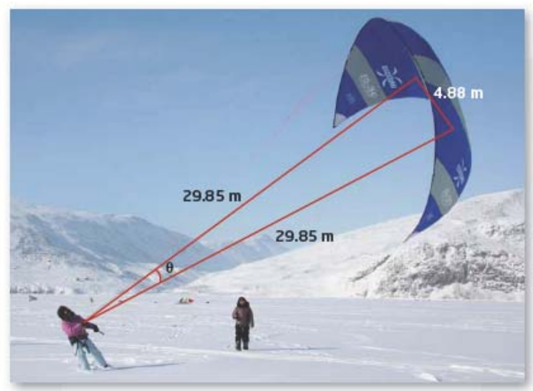
$$\frac{BC}{\sin 49°} = \frac{9}{\sin 66°}$$
$$BC = \frac{9 \sin 49°}{\sin 66°}$$
$$BC = 7.435...$$

$$\frac{AC}{\sin 65°} = \frac{9}{\sin 66°}$$
$$AC = \frac{9 \sin 65°}{\sin 66°}$$
$$AC = 8.928...$$

The polar bear is 8.9 km from station A and 7.4 km from station B, both to the nearenst tenth of a kilometre.

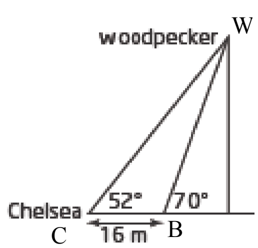
Cumulative Review Page 135 Question 15

Use the cosine law.
$4.88^2 = 29.85^2 + 29.85^2 - 2(29.85)(29.85) \cos \theta$
$1782.045 \cos \theta = 1758.2306$
$$\cos\theta = \frac{1758.2306}{1782.045}$$
$$\angle\theta = \cos^{-1}\left(\frac{1758.2306}{1782.045}\right)$$
$$\angle\theta = 9.377...$$
The measure of angle θ is 9.4°, to the nearest tenth of a degree.



Cumulative Review Page 135 Question 16

a)



b) In ΔBCW:
$\angle WBC = 180° - 70° = 110°$
$\angle BWC = 70° - 52° = 18°$
Chelsea is closest to the bird when she is at B.

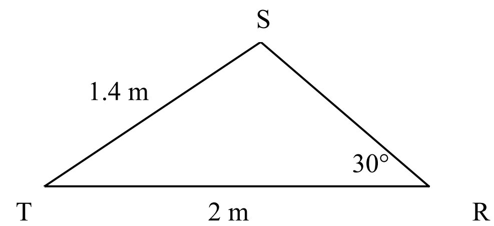
$$\frac{\text{BW}}{\sin 52°} = \frac{16}{\sin 18°}$$

$$\text{CW} = \frac{16 \sin 52°}{\sin 18°}$$

$$\text{CW} = 40.800...$$

Chelsea is 40.8 m from the bird, to the nearest tenth of a metre.

Cumulative Review Page 135 Question 17



Use the sine law to determine the acute measure of ∠S.

$$\frac{\sin \text{S}}{2} = \frac{\sin 30°}{1.4}$$

$$\sin \text{S} = \frac{2(0.5)}{1.4}$$

$$\angle\text{S} = \sin^{-1}\left(\frac{1}{1.4}\right)$$

$$\angle\text{S} = 45.584...$$

So, obtuse ∠S = 180° − 45.6° = 134.4°, to the nearest tenth of a degree.

Unit 1 Test

Unit 1 Test Page 136 Question 1

The sequence 8, 4, 0…. is arithmetic with $t_1 = 8$ and $d = -4$.
Substitute in $t_n = t_1 + (n - 1)d$

$t_n = 8 + (n - 1)(-4)$

$t_n = 8 - 4(n - 1)$

Therefore, option **B** is best.

Unit 1 Test Page 136 Question 2

Observe the pattern in the terms: $t_1 = x$, $t_2 = x^3$, $t_3 = x^5$, …Then, $t_n = x^{2n-1}$.
So $t^{14} = x^{27}$.
Option **C** is best.

Unit 1 Test Page 136 Question 3

The series 6 + 18 + 54 + … is geometric, with $t_1 = 6$ and $r = 3$. Substitute $S_n = 2184$ into

$$S_n = \frac{t_1(r^n - 1)}{r - 1}.$$

$$2184 = \frac{6(3^n - 1)}{3 - 1}$$
$$2184 = 3(3^n - 1)$$
$$728 = 3^n - 1$$
$$729 = 3^n$$
$$3^6 = 3^n$$
$$n = 6$$
There are 6 terms in the series. Option **D** is the best answer.

Unit 1 Test Page 136 Question 4

235° has a reference angle of 55°, because 235° − 180° = 55°.
Option **C** is the best answer.

Unit 1 Test Page 136 Question 5

$$\sin\theta = \frac{y}{r}$$
$$\frac{\sqrt{5}}{5} = \frac{y}{r}$$
Then, substitute $y = \sqrt{5}$ and $r = 5$ in $x^2 + y^2 = r^2$.
$$x^2 + 5 = 25$$
$$x^2 = 20$$
$$x = \pm\sqrt{20} \text{ or } \pm 2\sqrt{5}$$
Then, for $90° \le \theta \le 180°$,
$$\cos\theta = \frac{x}{r}$$
$$\cos\theta = \frac{-2\sqrt{5}}{5}$$
Option **D** is the best answer.

Unit 1 Test Page 136 Question 6

Let x represent the amount, in dollars, collected per cup of coffee.
$$350 = 20 + 2200x$$
$$330 = 2200x$$
$$x = \frac{330}{2200}$$
$$x = 0.15$$
To raise $350, they must collect **$0.15** per cup of coffee.

Unit 1 Test Page 136 Question 7

An angle of 315° drawn in standard position has a reference angle of 360° − 315°, or **45°**.

Unit 1 Test Page 136 Question 8

Given that $\sin\theta = -\frac{\sqrt{3}}{2}$, the reference angle for θ is 60°. Since θ is in quadrant IV, its measure is 360° − 60° or **300°**.

Unit 1 Test Page 136 Question 9

This is an arithmetic series with $t_1 = 150$, $d = 5$, and $n = 15$. Substitute into

$S_n = \frac{n}{2}[2t_1 + (n-1)d]$

$S_n = \frac{15}{2}[2(150) + (15-1)5]$

$S_n = 2775$

The total number of people that attended the first 15 concerts was 2775.

Unit 1 Test Page 136 Question 10

a) Substitute for $t_3 = 4$ and $t_7 = 24$ into $t_n = t_1 + (n - 1)d$.

$4 = t_1 + (3 - 1)d$ and $24 = t_1 + (7 - 1)d$

$4 = t_1 + 2d$ and $24 = t_1 + 6d$

Subtract the two equations:

$20 = 4d$

$d = 5$

The common difference is 5.

b) Substitute $d = 5$ to find t_1.

$4 = t_1 + 2(5)$

$t_1 = -6$

c) Substitute $d = 5$ and $t_1 = -6$ into $t_n = t_1 + (n - 1)d$.

$t_n = -6 + (n - 1)5$

$t_n = 5n - 11$

d) Substitute $n = 10$, $d = 5$, and $t_1 = -6$ into $S_n = \frac{n}{2}[2t_1 + (n-1)d]$.

$S_{10} = \frac{10}{2}[2(-6) + (10-1)5]$

$S_{10} = 165$

The sum of the first 10 terms is 165.

Unit 1 Test Page 137 Question 11

Value at end of first year = 35 000(0.8) = 28 000
Each successive year the car depreciates 10%, so 6 years later
Value = $28\ 000(0.9)^6$
= 14 880.35
After 7 years, the car will be worth $14 880.35.

Unit 1 Test Page 137 Question 12

The additional amounts above the $100 are an arithmetic series with $t_1 = 5$ and $d = 5$.

Substitute into $S_n = \frac{n}{2}[2t_1 + (n-1)d]$.

$$50 = \frac{n}{2}[2(5) + (n-1)5]$$
$$50 = \frac{n}{2}[10 + 5n - 5]$$
$$100 = 5n + 5n^2$$
$$n^2 + n - 20 = 0$$
$$(n+5)(n-4) = 0$$
$$n = 4$$
This walker would need to walk 4 km to earn $150.

Unit 1 Test Page 137 Question 13

a) The first four terms of the geometric sequence are: 64, 32, 16, 8.

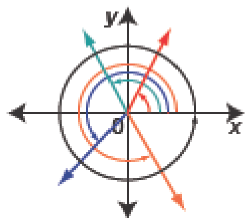
b) $t_n = t_1(r^{n-1})$

$$t_n = 64\left(\frac{1}{2}\right)^{n-1}$$

c) The number of games needed to determine the winner is 32 + 16 + 8 + 4 + 2 + 1 or 63 games.

Unit 1 Test Page 137 Question 14

a)



b) 60, 120, 180, 240, 300, 360

c) Substitute $t_1 = 60$ and $d = 60$ into
$t_n = t_1 + (n - 1)d$.
$t_n = 60 + (n - 1)60$
$t_n = 60n$

Unit 1 Test Page 137 Question 15

a) CD = CB = 5 (equal radii)

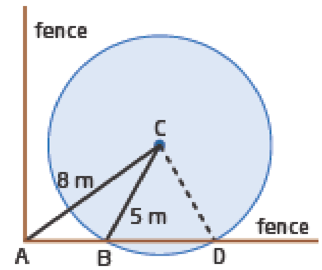
In ΔACD,

$$\frac{\sin D}{8} = \frac{\sin 32°}{5}$$

$$\sin D = \frac{8\sin 32°}{5}$$

$$\angle D = \sin^{-1}\left(\frac{8\sin 32°}{5}\right)$$

$$\angle D = 57.980...$$



The measure of ∠CDA is 58°, to the nearest degree.

b) In ΔBCD, ∠CBD = ∠ CDB = 58°. (isosceles triangle)

Then, ∠BCD = 180° − (58° + 58°) = 64°.

$$\frac{BD}{\sin 64°} = \frac{5}{\sin 58°}$$

$$BD = \frac{5\sin 64°}{\sin 58°}$$

$$BD = 5.299...$$

The length of fence that would get wet is 5.3 m, to the nearest tenth of a metre.

Unit 1 Test Page 137 Question 16

The smallest angle will be opposite the shortest side. Use the cosine law.

$$38^2 = 61^2 + 43^2 - 2(61)(43)\cos\theta$$

$$5246\cos\theta = 4126$$

$$\cos\theta = \frac{4126}{5246}$$

$$\angle\theta = \cos^{-1}\left(\frac{4126}{5246}\right)$$

$$\angle\theta = 38.140...$$

The smallest angle in the triangle measures 38°, to the nearest degree.